University of Colorado
Department of Mathematics

2017/18 Semester 2 | Math 8370 Harmonic Analysis | Assignment 3

Due Wednesday March 14, 2018

1. Do problems 3.2, 3.3, 3.5 [hint: use induction], 3.6, 3.7, 3.8, 3.9, 3.11, pp. 56–57 in the Deitmar textbook.
2. Let $F(x) = \frac{1}{2\pi}[\frac{\sin x/2}{x/2}]^2$.
   (a) Using the methods of complex analysis, prove that $\int_{\mathbb{R}} F(x)dx \ = \ 1$.
   (b) For $\lambda \in (0, \infty)$, let $F_\lambda(x) \ = \ \lambda F(\lambda x)$. Prove that $\{F_\lambda\}$ is an approximate identity on $\mathbb{R}$, where here the limit is taken as $\lambda \to \infty$.
   (Recall this means that $\int_{\mathbb{R}} F_\lambda(x)dx \ = \ 1, \ \forall\lambda$; that there exists $M > 0$ with $\int_{\mathbb{R}} |F_\lambda(x)|dx \leq M$, and that $\forall\delta > 0$,
   $$\lim_{\lambda\to\ +\infty} \int_{(-\infty,-\delta)\cup(\delta,\infty)} |F_\lambda(x)|dx \ = \ 0.$$
   (c) Prove that
   $$F_\lambda(x) \ = \ \lambda \int_{-1/2\pi}^{1/2\pi} (1 - 2\pi|y|)e^{2\pi i\lambda yx}dy.$$
   (d) Deduce that if $f \in L^1(\mathbb{R})$, then
   $$\lim_{\lambda\to\infty} \int_{-\lambda/2\pi}^{\lambda/2\pi} (1 - \frac{2\pi|y|}{\lambda})\hat{f}(y)e^{2\pi iyx}dy \ = \ f(x)$$
   in the L^1 norm.
   (e) Use the above to give another proof that if $f, \ \hat{f} \in \ L^1(\mathbb{R})$, then
   $$f(x) \ = \ \int_{\mathbb{R}} \hat{f}(y)e^{2\pi iyx}dy, \ a.e.$$