University of Colorado
Department of Mathematics

2008/2009 Semester 2 | Math 6360 Complex Variables 2 | Assignment 1

Due Monday January 26, 2009

1. Read Ahlfors Chapter 4, Section 6.1, 6.2, 6.3, 6.4,
2. Show that if u is harmonic, then so are $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$. (Be sure to explain why these functions are twice differentiable).
3. Prove that for $r \neq 0$, Laplace's equation in polar coordinates takes the form

$$r\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) + \frac{\partial^2 u}{\partial \theta^2} \ = \ 0.$$

4. Prove that for $0 < r_1 < r_2 < \infty$ and $\Omega \ = \ \{x + iy : (r_1)^2 < x^2 + y^2 < (r_2)^2\}$, $u(x, y) = \log(x^2+y^2)^{\frac{1}{2}}$ is harmonic in Ω. Does u have a harmonic conjugate function defined on Ω? If so, write down a conjugate function for v. If not, explain why not.
5. Let $u(x, y) = e^{-x}(x \sin y - y \cos y)$. Prove that u is harmonic in $\mathbb{C}$ and find a harmonic conjugate function for u.
6. Let $p(x, y) = \sum_{k,l=0}^{n} a_{k,l}x^k y^l$ for all $(x, y) \in \mathbb{R}^2$. Show that p is harmonic if and only if
   (a) $k(k-1)a_{k,l-2} + l(l-1)a_{k-2,l} = 0$ for $2 \leq k, l \leq n$;
   (b) $a_{n-1,l} = a_{n,l} = 0$ for $2 \leq l \leq n$;
   (c) $a_{k,n-1} = a_{k,n} = 0$ for $2 \leq k \leq n$.
7. Do problem 2 of Section 6.2 in Chapter 4 of Ahlfors.
8. Do problems 1 and 2 of Section 6.4 of Ahlfors.