University of Colorado
Department of Mathematics

2008/2009 Semester 2 | Math 6360 Complex Variables 2 | Assignment 3

Due Monday March 9, 2009

1. Read Ahlfors Chapter 5, Section 2.2, 2.3, 2.4, 2.5. 3.1, 3.2, 4.1, 4.2.
2. (a) Let $0 < |a| < 1$ and $|z| \leq r < 1$; show that
$$|\frac{a+|a|z}{(1-\overline{a}z)a}| \leq \frac{1+r}{1-r}.$$
   (b) Let $\{a_n\}$ be a sequence of complex numbers with $0 < |a_n| < 1$ and $\sum(1-|a_n|) < \infty$. Show that the infinite product
$$B(z) = \prod_{n=1}^{\infty} \frac{|a_n|}{a_n}(\frac{a_n - z}{1-\overline{a}_n z})$$
   coverges uniformly on compact subsets of $U = \{z \in \mathbb{C} : |z| < 1\}$ and that $|B(z)| \leq 1$. What are the zeroes of B? ($B(z)$ is called a *Blaschke Product.*)
3. Show that
$$\int_0^\infty \sin t^2 dt = \frac{1}{2}\sqrt{\frac{\pi}{2}}.$$
4. Do problems 1, 2 and 3 of Section 2.2 in Chapter 5 of Ahlfors.
5. If you want extra credit, try to state and prove the correct version of problem 2 of Section 2.3 in Chapter 5 of Ahlfors.
6. Do problems 1 and 3 of Section 2.4 in Chapter 5 of Ahlfors.
7. Let $\xi(z) = z(z-1)\pi^{-\frac{1}{2}\cdot z}\zeta(z)\Gamma(\frac{z}{2})$. Show that ξ is an enitire function satisfying the functional equtional $\xi(z) = \xi(1-z)$.
(OK, I've found that this is actually a Corollary on p. 217 of the Ahlfors book – however, now your job is to fill in all the gaps that you don't think Ahlfors explains in a crystal clear fashion.)