University of Colorado
Department of Mathematics

2008/2009 Semester 2 Math 4330 Second Midterm Exam, Takehome

Due Wednesday April 8, 2009, 5 p.m. in Math 227.

No late papers will be accepted. You will not receive extra credit for doing the 5330 take-home exam.

INSTRUCTIONS: You are to work by yourself. You are allowed to use the textbook, class notes, previous homework assignments, the previous exam, and any other book you find helpful. If you need any clarification about a problem, you should consult me, and not other students.

1. Give complete solutions to problems 2.7.2 (pp. 120–121), 2.9.3 all parts (p. 131), 3.2.9 (p. 175), 3.7.2 (p. 201), 3.7.6 (a), (b), (c) (p. 202), 5.1.4 (p. 264), 5.1.5 (p. 264), 5.7.4 (p. 295) and 6.2.2 (p. 311) of the Stade textbook. Recall that Exercise 2.9.3 (a) was assigned as part of HW 6.
2. Using the discussion in Proposition 3.7.1 p. 195 as a model, prove that if the real-valued piecewise continuous functions $f,\ g \in L^2[-\pi, \pi]$ have Fourier coefficients $\{a_n\}_{n\geq 0} \cup \{b_n\}_{n\geq 1}$ and $\{\alpha_n\}_{n\geq 0} \cup \{\beta_n\}_{n\geq 1}$, respectively, then

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)g(x)dx \ = \ \frac{a_0\alpha_0}{2} + \sum_{n=1}^{\infty}[a_n\alpha_n + b_n\beta_n].$$

   Be sure to justify your steps.
3. Find the values of the real constants $A_0,\ A_1,\ A_2$ such that the function

$$p(x) \ = \ A_0 + A_1\cos\left(\frac{\pi x}{2}\right) + A_2\cos\left(\pi x\right)$$

   gives the best mean square approximation to $f(x) = x$ over the interval $[0, 2]$.