

University of Colorado  
Department of Mathematics

2009/10 Semester 1

Math 8330 Functional Analysis 1

Assignment 5

**Due Monday, November 30, 2009**

1. Recall Problem 4 (b) from Assignment 4. Is  $\mathbf{c}_0$  reflexive? Prove that  $\mathbf{c}_0$  is not isomorphic to the dual of any Banach space.
2. The following was discussed in lectures, but not proved explicitly. Let  $h$  be a tempered function. Prove that  $h$  is integrable over any finite interval. Prove that the function  $k$ , defined by  $k(x) = \int_0^x h(t)dt$  if  $x \geq 0$ , and by  $k(x) = -\int_x^0 h(t)dt$  if  $x \leq 0$ , is a tempered function. Deduce that if  $h$  is a tempered function, there exists a tempered function  $k$  such that  $(u_k)' = u_h$ .
3. Let  $X$  be a locally convex topological vector space over  $\mathbb{F}$ . We say that  $\{x_n\} \subseteq X$  is a **weak Cauchy sequence** if for every  $f \in X^*$ , the sequence  $\{f(x_n)\}$  converges in  $\mathbb{F}$ . We say that  $X$  is **weakly sequentially complete** if whenever  $\{x_n\}$  is a weak Cauchy sequence in  $X$ , there exists  $x_0$  such that  $\lim x_n = x_0$  in the weak topology. Prove that if  $X$  is a reflexive Banach space, then  $X$  is weakly sequentially complete.
4. Let  $X$  be a Banach space over  $\mathbb{F}$ . Let  $S_1 = \{x \in X : \|x\| = 1\}$ . Prove that the closure of  $S_1$  in the weak topology is  $\overline{B}_1 = \{x \in X : \|x\| \leq 1\}$ .
5. Do Exercise 8.2, (a), (b), and (c), on p. 143 of the Baggett textbook.