1. (12) For each of the following series, determine whether it is absolutely convergent, conditionally convergent, or divergent. Be sure to explain your reasoning.

(i)

$$\sum_{n=1}^{\infty} (-1)^n \frac{2^n + 1}{3^n - \cos n};$$

(ii)

$$\sum_{n=2}^{\infty} (-1)^n \frac{1}{(n\log(n))^a}, \ a > 1;$$

(iii)

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^n}{(n+1)^n}.$$

2. (13)

(i) Suppose that $\sum_{n=1}^{\infty} a_n$ is convergent series of real numbers. Define $b_n = a_{2n-1}$ and $c_n = a_{2n}$. Is it true that both $\sum_{n=1}^{\infty} b_n$ and $\sum_{n=1}^{\infty} c_n$ are convergent? If so, prove this is the case. If not, provide a counterexample.

(ii) Fix $b > 1$. Prove that the series

$$\sum_{n=1}^{\infty} \frac{1}{1+x^n}$$

is uniformly convergent on $[b, \infty)$.

3. (12) Consider the function $f : [0,1) \to \mathbb{R}$ defined by $f(x) = \frac{1}{1-x}$. Let $f_n : [0,1) \to \mathbb{R}$ be given for $n \geq 0$ by $f_n(x) = \sum_{k=0}^{n} x^k$. Prove that $\{f_n\}$ converges to f pointwise on $[0,1)$. Is this convergence uniform on $[0,1)$? Be sure to justify your answer.

4. (13) Suppose that $\{f_n : S \to \mathbb{R}\}_{n=1}^{\infty}$ is a sequence of functions defined on the interval $S \subset \mathbb{R}$, with each f_n bounded on S by $M_n > 0$. Suppose also that the sequence $\{f_n\}_{n=1}^{\infty}$ converges uniformly on S to a limit function $f : S \to \mathbb{R}$. Prove that f is bounded on S.

Name: ______________________________

University of Colorado

Mathematics 4001: First Midterm Exam

February 19, 2010

Problem	Points	Score
1	12	
2	13	
3	12	
4	13	
Total	50	