First draft due Friday, April 24, 2015, feedback from peer assessor due Monday April 27, final draft due in class Wednesday April 29, 2015

## DEAL, OR NO DEAL?

The following assignment will be graded with an emphasis on clarity of exposition. You should write in complete sentences, be as precise as possible, and be mathematically correct (both in content and format). Be sure to include a title, a short introduction to the topic, and to define all the relevant mathematical terms.

Have any of you watched the TV game show "Deal or No Deal" with host Howie Mandel? Never mind if you haven't, we'll explain the situation now, and then you will have to make some extremely difficult decisions. You will also need to explain why you made these decisions to family members and/or other people with an extremely rudimentary knowledge of high-school mathematics. You will also be required to explain things in a way that makes sense to your fellow "Discrete Math" students and teachers. It will help you to do some extra reading from the book; in particular, you might find Section 34 (Expectation) of the textbook to be helpful.

Howie Mandel is the host of the show, which begins when 20 beautifully dressed models step out from behind a curtain, each holding a numbered, closed suitcase, given a number from 1 through 20 . Each of these 20 suitcases holds inside it a tag with a different dollar amount representing the value of the suitcase, ranging from $\$ 5$ to $\$ 1,000,000$. The Cukey family (Mom, Dad, and teenage twins Sandy and Kim) are the contestant family.

Dad Cukey chooses Suitcase \#13 for luck and the family holds on to it. The value of this suitcase will not be revealed until the end of the game. For each of the first 16 rounds of the game, the family chooses one-by-one 16 suitcases (other than $\# 13$ ) to be removed from play, and in each round the suitcase they choose to be removed from play is opened and its value is revealed. The Cukeys refuse every single bid that is made by the "Banker" to stop the game and give up their suitcase. The Banker, as ever, is hidden in the shadows, and communicates his bids only to Howie. The fact that they refuse the Banker's early (first 16) offers is not that surprising, since all die-hard fans of the show know that in the early rounds, the Banker's offers are considerably less than the expected value of the suitcases remaining in play.

Now, after 16 rounds, with 16 suitcases removed from play, we are left with three models in designer gowns holding the suitcases labeled $\# 3, \# 8$, and $\# 15$, and the Cukey family, holding tight to $\# 13$. The big flashing board show that the only values left for these four remaining suitcases to have are: $\$ 5, \$ 2000, \$ 50,000$ and $\$ 100,000$. Each of these values is in precisely one of the remaining suitcases, and is equally likely to be in any of the suitcases. The Banker phones up Howie and gives his bid: He will give the Cukey family $\$ 39,000$ if they stop playing the game now, give back Suitcase \#13 to Howie, and go home.
(a) We all know that the Cukeys were chosen to appear on the show because they are zany and completely illogical. But what is the logical decision for the Cukeys, assuming that they want to maximize their winnings? Should they stop playing and go home with $\$ 39,000$, or continue, and chose yet another suitcase to be opened up by one of the models? Explain your reasoning.
(b) The Cukeys go with their gut instinct, keep playing, and choose Suitcase \#8 to be opened up. "I don't trust even numbers," says Mom Tukey. A model opens up Suitcase \#8, which turns out to contain a label stating it held the $\$ 100,000$ prize. "Never mind!" says Howie. "The $\$ 50,000$ suitcase is still in play! Oh and by the way, the Banker now says he will now pay you $\$ 16,500$ to give back Suitcase \#13 and go home." What is the logical decision for the Cukeys to make, if they want to maximize their winnings? Should they keep playing the game, or take the $\$ 16,500$ and go home?
(c) Of course, the Cukeys decide to keep playing. Howie considers whether to open one of the remaining suitcases randomly (not knowing where any of the prizes are), when suddenly the Banker springs forth out of the shadows. It's Monty Hall, who retired from his game show in the 1980's and lost his savings in the Great Recession of 2008-2009, and he wants revenge! "What a twist!" the models exclaim in unison.
"This is my schtick, Mandel!" he cries. "I know where all the prizes are, and I'm going to randomly open one of the losing suitcases that the Cukeys haven't picked! Ha ha, that'll show you! Open \#15! Behold, $\$ 2000$ !" A model opens suitcase \#15, and everyone sees Monty Hall is right. Chaos ensues; the models choose sides, and the hosts battle it out.
In the confusion, the Cukeys decide to grab one of the two remaining suitcases (\#13 or \#3) and run for it. Which one should they take? Remember, they only have enough time as it takes to read a two-page essay.

Be sure to give precise arguments for all your assertions, and to give a title to your essay. On Friday April 24 you will hand in your essay to your peer assessor for advice and feedback, but after that your final work should be your own. Your essay should be at most two pages, written or typed.

