Critical Soft Matter

Soft Matter In and Out of Equilibrium

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$: NSF, Simons Foundation

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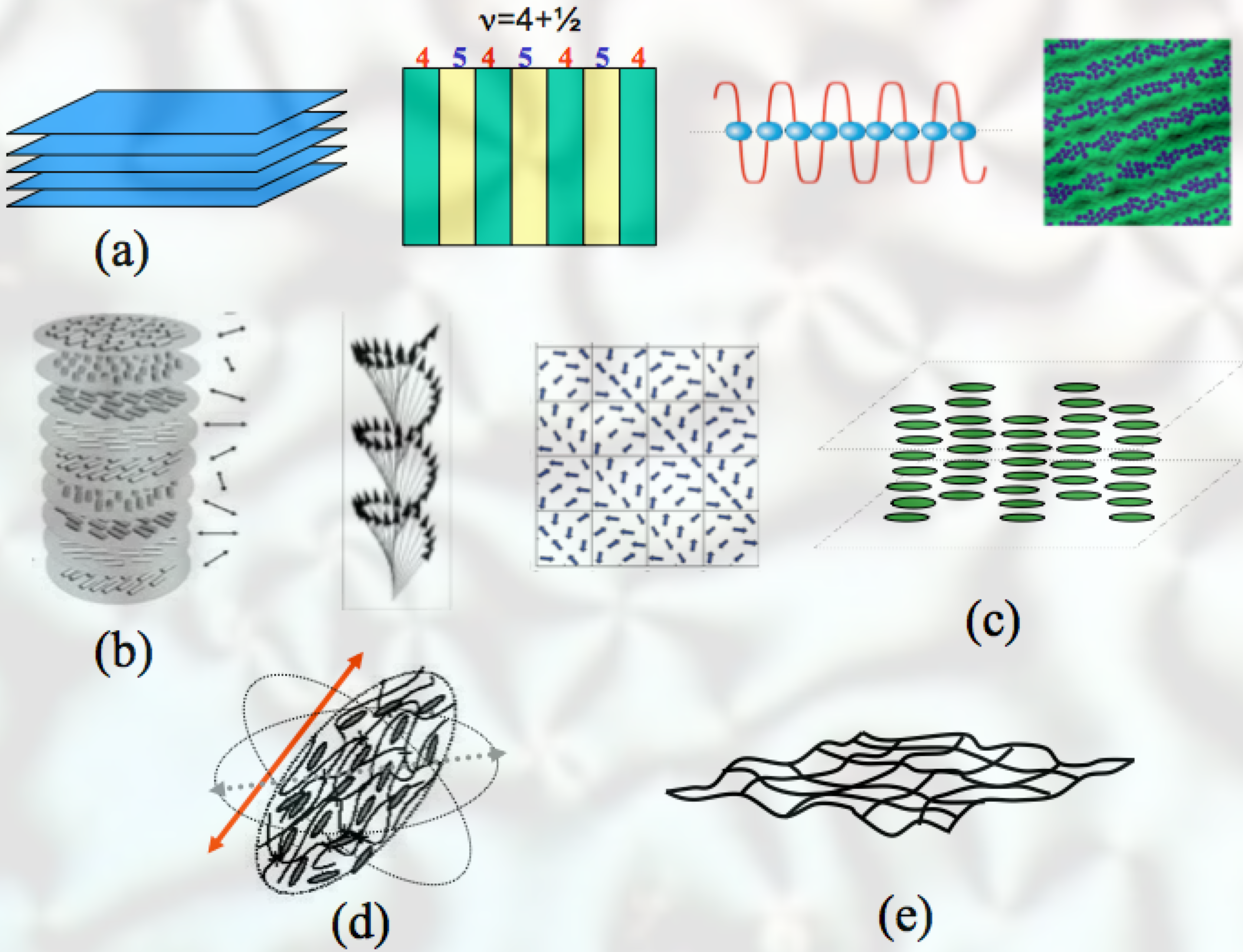
Outline

- Introduction and motivation
- Critical states of matter
- Smectics
- Cholesterics
- Columnar phase
- Polymerized membranes
- Elastomers

Critical soft matter

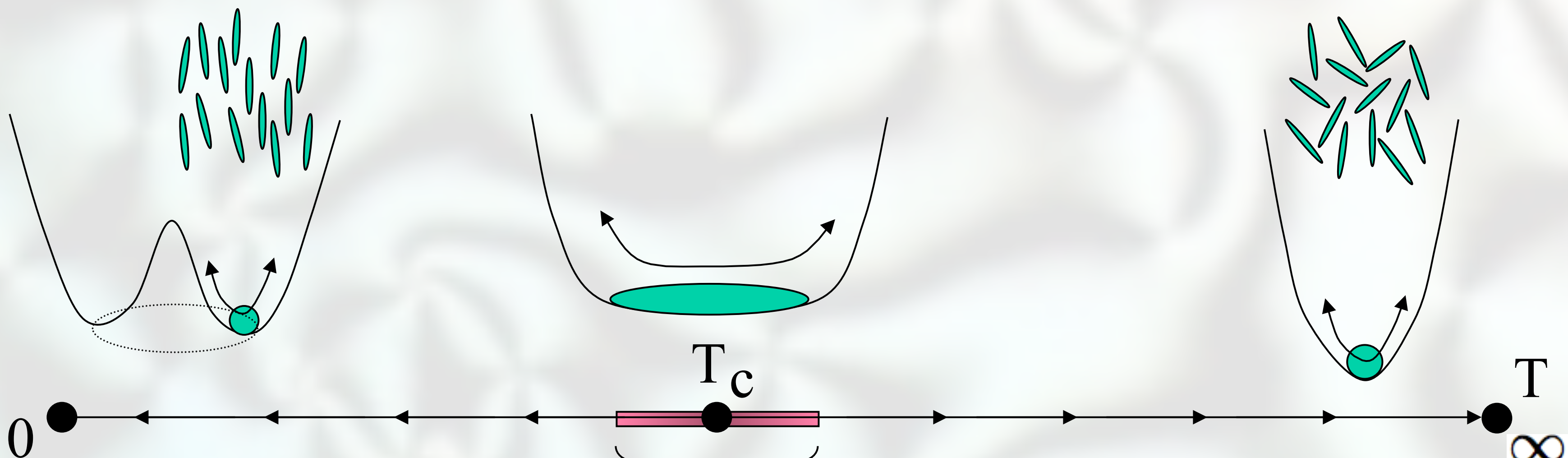


Fluctuations, nonlinearities and phase transitions

Upshot of 50 years of research on fluctuations and critical phenomena:

usually

Fluctuations <u>and</u> nonlinearities are only important near isolated critical points (continuous phase transition)



ordered

(e.g. ferromagnet)

$H = (\nabla S)^2$

qualitative effects of fluctuations and nonlinearities

$H = (\nabla S)^2 + \cancel{r} S^2 + S^4$

0

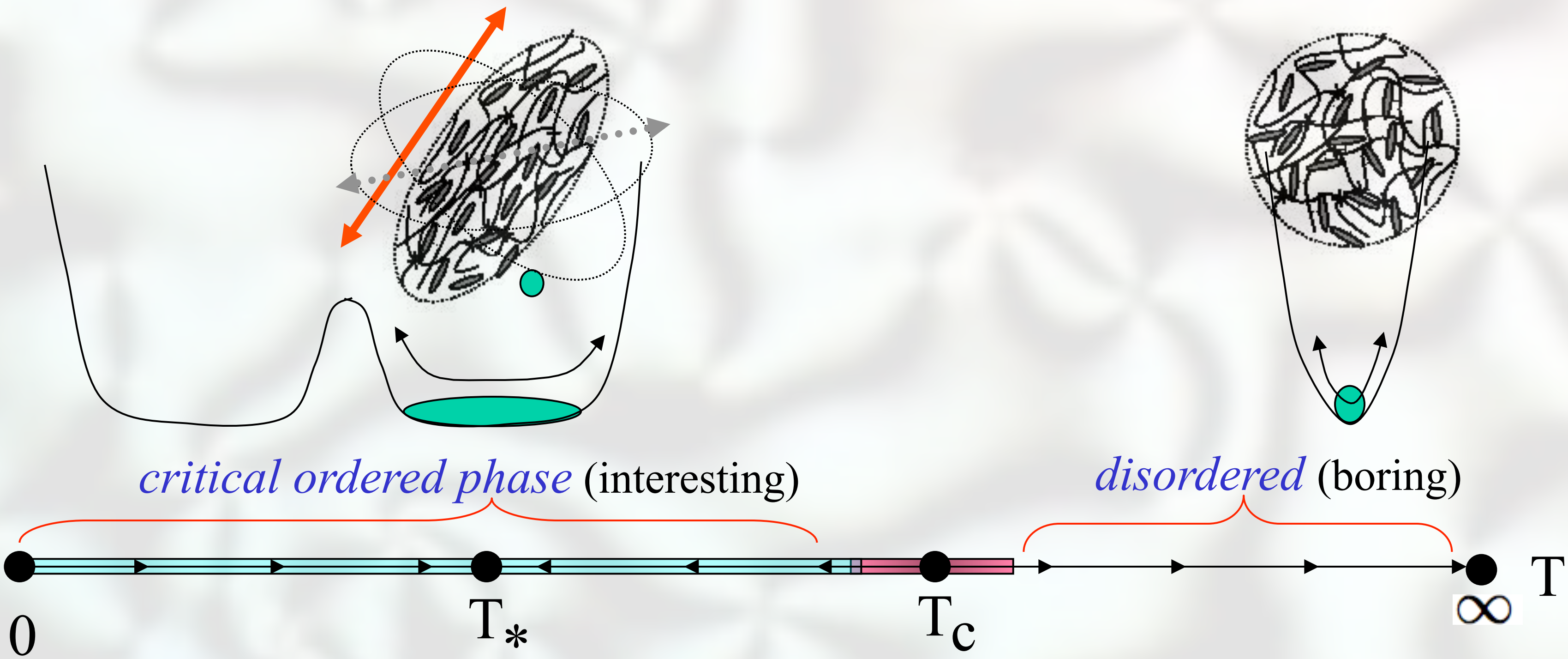
disordered

(e.g. paramagnet)

$H = S^2$

well-known exception: disordered systems, **FQHE,…**

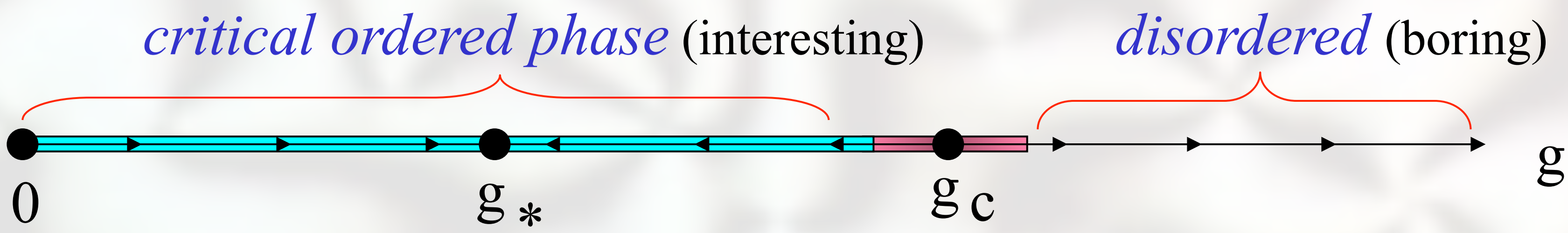
Critical phase



$$H = (\nabla\phi)^2 + \phi^4$$

...ever growing class of systems

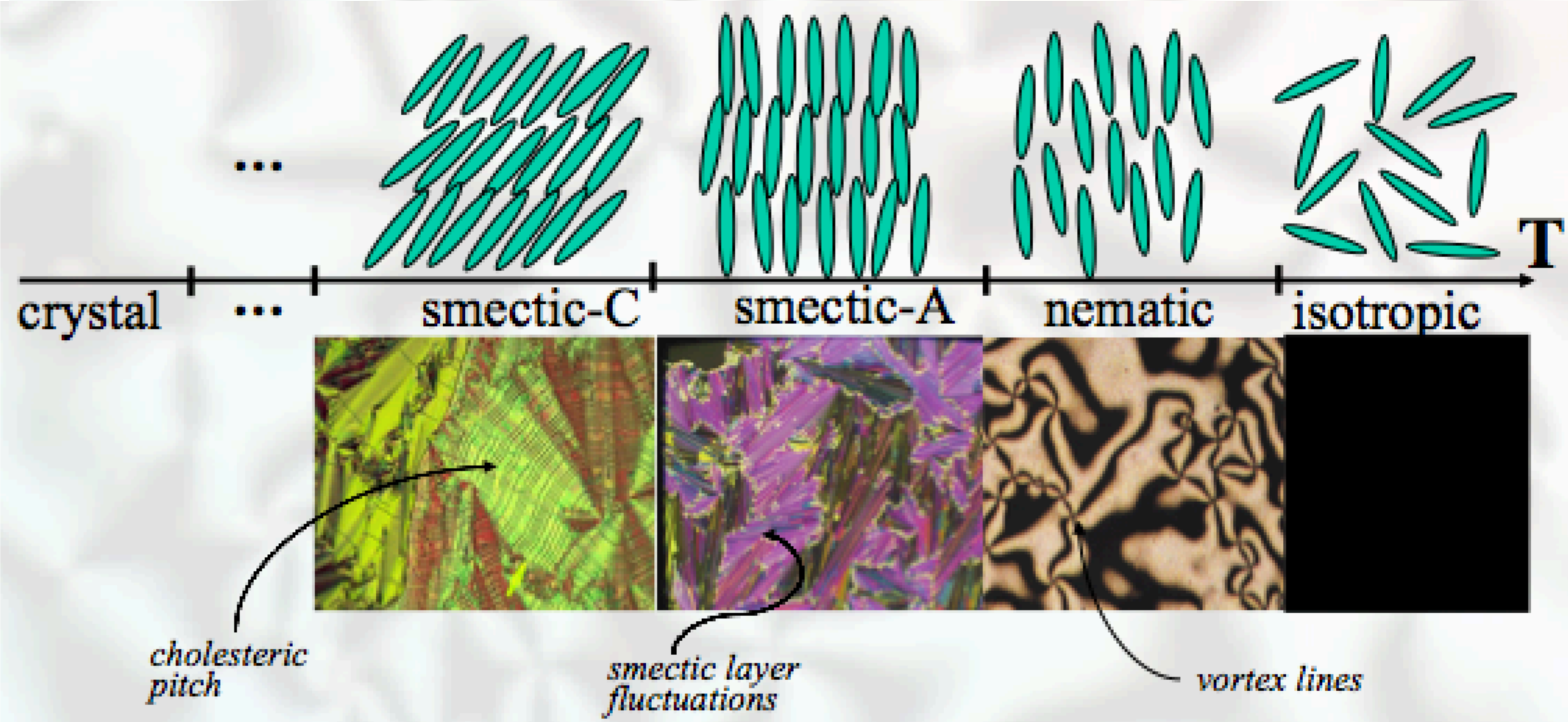
Properties of critical phases

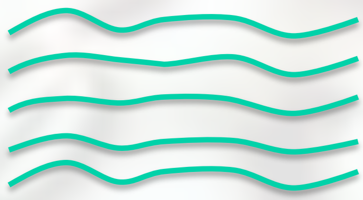


$$H = (\nabla\phi)^2 + \phi^4$$

- *spontaneously broken continuous symmetry*
- *nontrivial fixed point of strongly interacting Goldstone modes (c.f. nonlinear O(N) sigma-model)*
- *universal power-law correlation functions and amplitude ratios (throughout the phase)*
- *no fine-tuning to a critical point required*
- *quantum analogs? road to 3d "Luttinger liquids"?*

Liquid crystal phases





$u(x)$

$\psi \sim e^{-i\,q\,u}$

Smectics

$$\rho(\mathbf{x}) = \rho_0 + \mathrm{Re}[e^{i\mathbf{q_0}\cdot\mathbf{x}}\psi(\mathbf{x})] = \rho_0 + |\psi|\cos\left[q_0(z - u(\mathbf{x})\right]$$

- elasticity by symmetry or via de Gennes model or :

$$\mathcal{H}_{sm} = \frac{1}{2}J\left[(\nabla^2\rho)^2 - 2q_0^2(\nabla\rho)^2\right] + \frac{1}{2}t\rho^2 - w\rho^3 + v\rho^4 + \ldots,$$

- express in terms of u(**x**):

$$\mathcal{H}_{sm} = J\rho_0^2\left[\tfrac{1}{4}q^2(\nabla^2 u)^2 + \left(q\mathbf{q}\cdot\nabla u - \tfrac{1}{2}q^2(\nabla u)^2\right)^2 + 4(q^2 - q_0^2)\underbrace{\left(q\mathbf{q}\cdot\nabla u - \tfrac{1}{2}q^2(\nabla u)^2\right)}_{\textit{nonlinear strain } u_{qq}}\right]$$

- choose the **q** = q_0 along z:

$$\mathcal{H}_{sm} = \tfrac{1}{2}K(\nabla^2 u)^2 + \tfrac{1}{2}B\left(\partial_z u - \tfrac{1}{2}(\nabla u)^2\right)^2$$

Undulation instability on dilation

Strain-induced instability of monodomain smectic A and cholesteric liquid crystals

Noel A. Clark and Robert B. Meyer*
Gordon McKay Laboratory, Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 9 February 1973)

A mechanism is proposed for the observed mechanical instability of monodomain smectic A and cholesteric liquid crystals subjected to uniaxial dilative stress. The threshold conditions for the instability are derived, and the possible roles of dislocations in controlling the instability and in producing large plastic distortions are discussed.

$$\mathcal{H}_{sm} = \frac{1}{2}K(\nabla^2 u)^2 + \frac{1}{2}B\left(\partial_z u - \frac{1}{2}(\nabla u)^2\right)^2$$

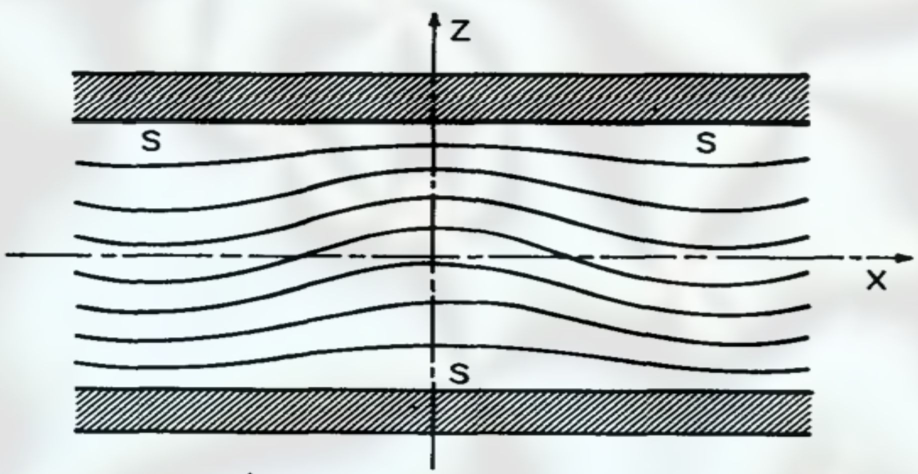
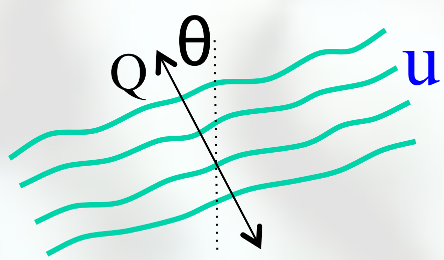
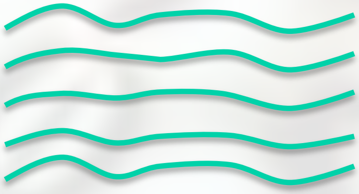


FIG. 1. Periodic undulation of the layers of a dilated smectic A liquid crystal. Regions of maximum dilation are marked S.



$$E[u_0(\mathbf{r})] = 0,$$

for $u_0(\mathbf{r}) = z(\cos\theta - 1) + x\sin\theta$



Sm harmonic properties

- harmonic approximation: $H_{sm} = \frac{1}{2}\int \frac{d^d k}{(2\pi)^d}\left(K k_\perp^4 + B k_z^2\right)|u_{\mathbf{k}}|^2,$

$$\langle u^2 \rangle_0^T = \int_{L_\perp^{-1}}^{\Lambda_\perp} \frac{d^d k}{(2\pi)^d} \frac{T}{B k_z^2 + K k_\perp^4},$$

$$\approx \begin{cases} \frac{T}{2\sqrt{BK}} C_{d-1} L_\perp^{3-d}, & d < 3, \\ \frac{T}{4\pi\sqrt{BK}} \ln q_0 L_\perp, & d = 3, \end{cases}$$

power-law disordered smectic for $x > \xi$, with $u_{rms} \sim a$

$$\xi_\perp \approx \begin{cases} \frac{a^2\sqrt{BK}}{T} \sim \frac{K}{T q_0}, & d = 2, \\ a e^{4\pi a^2 \sqrt{BK}/T} \sim a e^{\frac{cK}{T q_0}}, & d = 3, \end{cases}$$

with $\eta = \frac{q_0^2 T}{8\pi\sqrt{BK}}.$

- density is uniform:

$$\langle \rho_q(\mathbf{x}) \rangle_0 = 2\rho_0 \langle \cos\left[\mathbf{q}_0 \cdot \mathbf{x} - q u(\mathbf{x})\right] \rangle_0,$$

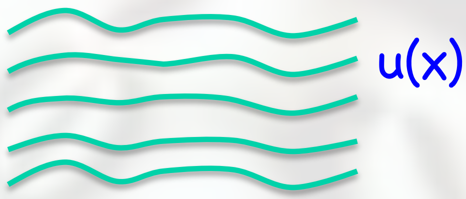
$$= 2\rho_0 e^{-\frac{1}{2} q_0^2 \langle u^2 \rangle_0} \cos\left(\mathbf{q}_0 \cdot \mathbf{x}\right),$$

$$= 2\tilde{\rho}_0(L_\perp) \cos\left(\mathbf{q}_0 \cdot \mathbf{x}\right),$$

with

$$\tilde{\rho}_0(L_\perp) = \rho_0 \begin{cases} e^{-L_\perp/\xi_\perp}, & d = 2, \\ \left(\frac{a}{L_\perp}\right)^{\eta/2}, & d = 3, \end{cases}$$

$$\to 0, \quad \text{for } L_\perp \to \infty,$$



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power-law disordered smectic for x > ξ, with $u_{rms} \sim a$

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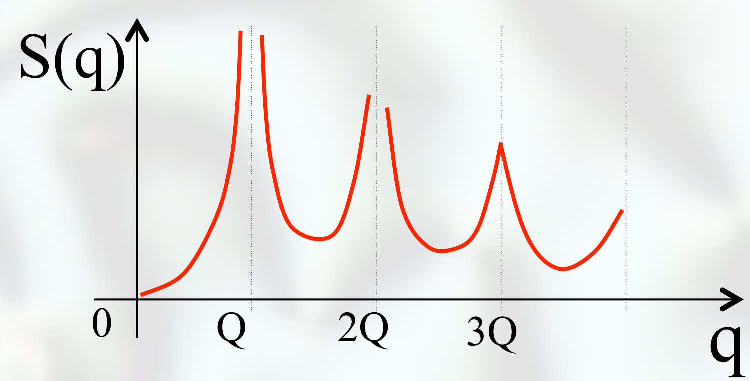
with $\eta = \frac{q_0^2 T}{8\pi\sqrt{BK}}.$

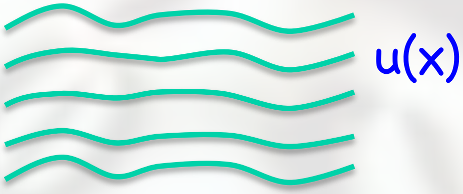
- quasi-Bragg peaks (3d), Lorentzian (2d):

$$S(\mathbf{q}) = \int d^3x \langle \delta\rho(\mathbf{x})\delta\rho(0)\rangle e^{-i\mathbf{q}\cdot\mathbf{x}},$$

$$\approx \frac{1}{2}\sum_{q_n} |\rho_{q_n}|^2 \int_{\mathbf{x}} \langle e^{-iq_n(u(\mathbf{x})-u(0))}\rangle_0 e^{-i(\mathbf{q}-q_n\hat{z})\cdot\mathbf{x}},$$

$$\approx \frac{1}{2}\sum_n \frac{|\rho_{q_n}|^2}{|q_z - nq_0|^{2-n^2\eta}}, \quad \text{for } d = 3,$$





Smectic nonlinearities

Grinstein-Pelcovits '81, L.R. '13

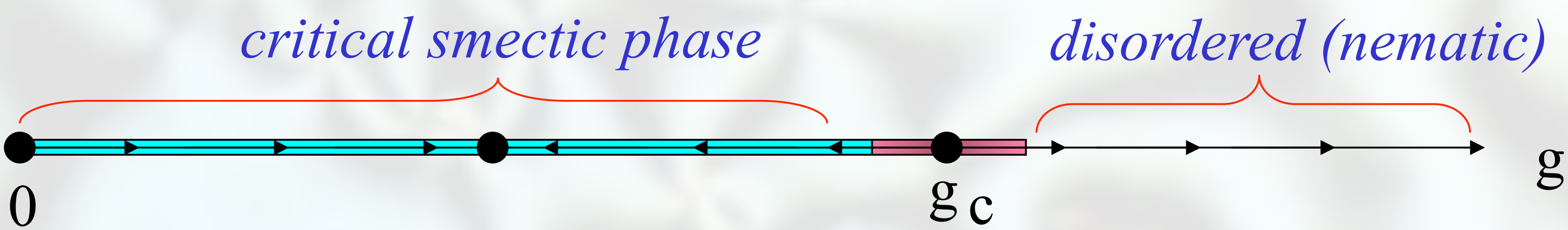
- nonlinearities : $H_{\text{nonlinear}} = -\frac{1}{2}B(\partial_z u)(\nabla u)^2 + \frac{1}{8}B(\nabla u)^4$

- renormalize elastic moduli in length-dependent ways:

$$\delta B \approx -\left[T\left(\frac{B}{K^3}\right)^{1/2} L_\perp^{3-d}\right] B \rightarrow \delta B \approx -\tfrac{1}{8} gB\delta\ell$$

$$\delta K \approx \tfrac{1}{16} gK\delta\ell$$

$$\frac{dg(\ell)}{d\ell} = (3-d)g - \tfrac{5}{32}g^2$$

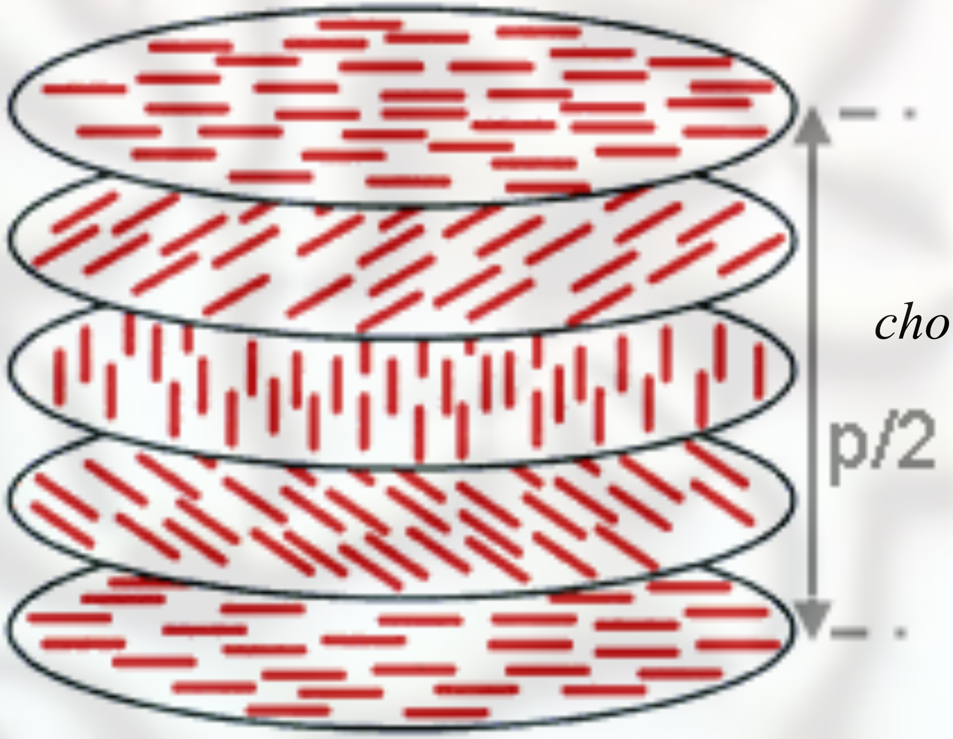


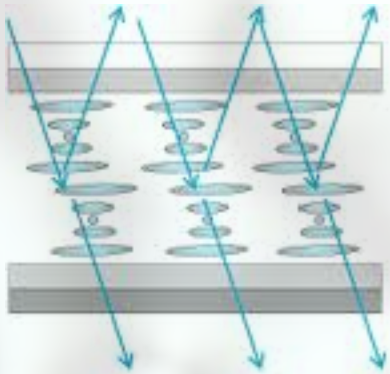
$$\mathcal{E}_k = \left[B(\mathbf{k})k_z^2 + K(\mathbf{k})k_\perp^4\right]|u_\mathbf{k}|^2$$ *with*

$$K(\mathbf{k}) \sim k_\perp^{-\eta_K}$$

$$B(\mathbf{k}) \sim k_\perp^{\eta_B}$$

Chiral liquid crystals: cholesterics

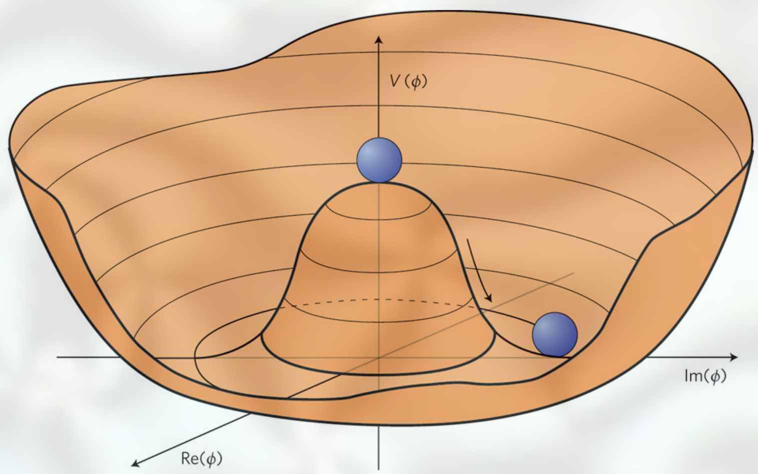




- color selective Bragg reflection from cholesteric planes

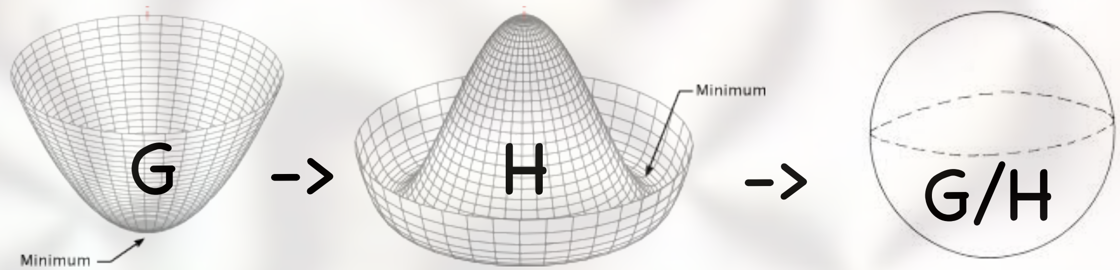
- temperature tunable pitch -> wavelength

Spontaneous symmetry breaking



Counting Goldstone modes

- **Symmetry breaking:**
  (conventional dogma)



number of Goldstone modes = dim[G/H]

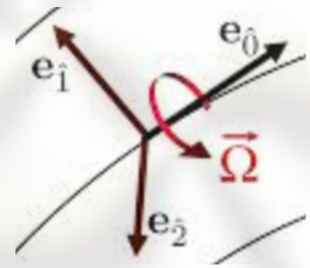
e.g., classical ferromagnet: SO(3) -> SO(2) -> G/H = S_2

- **Exceptions:**
  - *quantum ferromagnet:* *noncommuting symmetry generators* $[S_x, S_y] = i\, S_z$
  - *Higgs mechanism:* *some of the GMs are "eaten" by gauge fields, e.g., superconductor, Standard model, crystals*

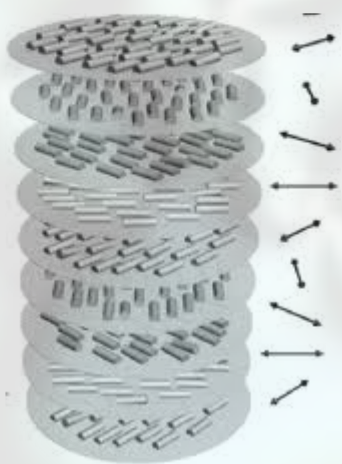
Goldstone modes in a helical state

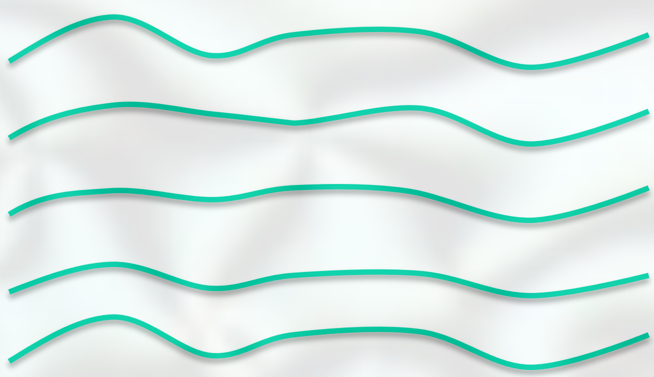
- **Isotropic state**: G = **T** x SO(3)
- **Helical state:** H = $T_x \times T_y \times U(1)$ = Diagonal$[T_z, O_z(2)]$

--> *naively* 3 = dim[G/H = SO(3)] Goldstone modes



--> *actually* 1 phase (ϕ) = phonon (u) mode, defining "layers"



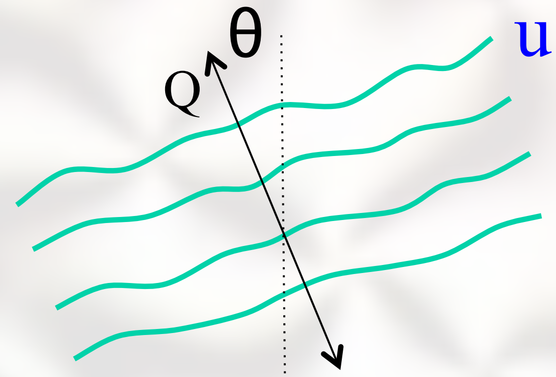


$u(r) = -\phi(r)/q_0$

--> smectic elasticity (proposed by deGennes '72) ?

Goldstone modes via symmetry

- small angle θ rotational invariance

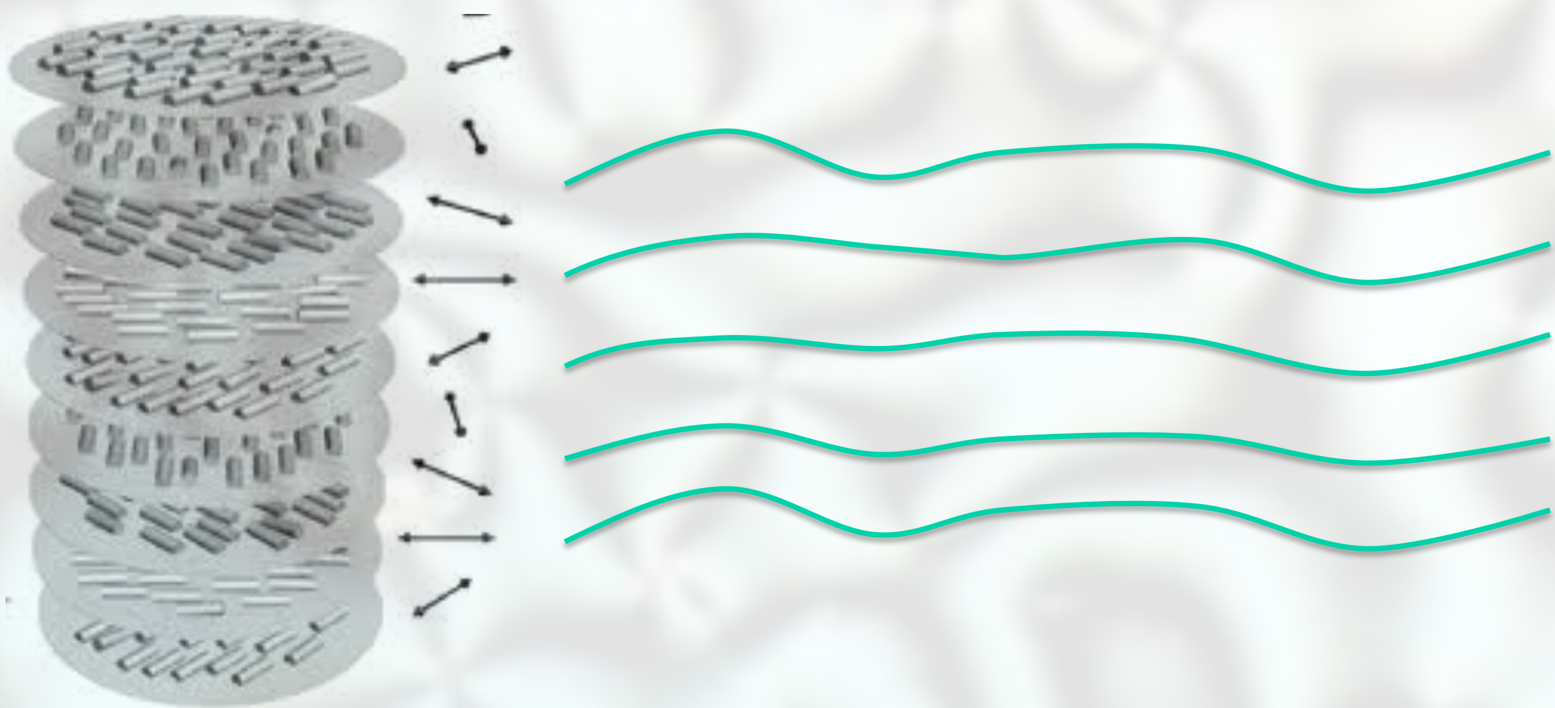


--> cholesteric <--> smectic elasticity (deGennes '72)

--> $$\mathcal{H} = \frac{B}{2}(\partial_z u)^2 + \frac{K}{2}(\partial_\perp^2 u)^2$$

- derivation?
- consequences?
- differences?

Derivation

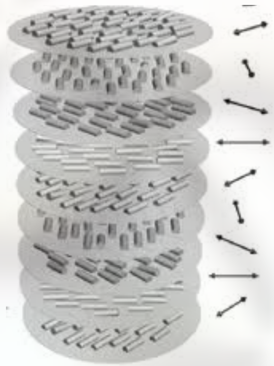


Derivation of Goldstone modes theory

- Frank chiral elasticity or Dzyaloshinkii-Morya interaction

$$\mathcal{H} = K_s(\nabla \cdot \hat{n})^2 + K_b(\hat{n} \times \nabla \times \hat{n})^2 + K_t(\hat{n} \cdot \nabla \times \hat{n} + q_0)^2$$

$$= K \left[(\nabla \hat{n})^2 + 2q_0 \hat{n} \cdot \nabla \times \hat{n} \right] + \ldots$$



- Helical background: $\hat{e}_1(\mathbf{r}) \times \hat{e}_2(\mathbf{r}) = \hat{e}_3(\mathbf{r}) \in S_2$

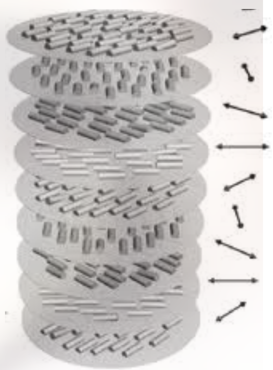
$$\hat{n}(\mathbf{r}) = \hat{e}_1(\mathbf{r}) \cos(\mathbf{q} \cdot \mathbf{r} + \phi(\mathbf{r})) + \hat{e}_2(\mathbf{r}) \sin(\mathbf{q} \cdot \mathbf{r} + \phi(\mathbf{r}))$$

- U(1) gauge theory:

$$\mathcal{H} = \frac{K}{2}(\nabla\phi + \mathbf{a} + \mathbf{q} - q_0 \hat{\mathbf{e}}_3)^2 + \frac{K}{4}\left[(\nabla \hat{e}_3)^2 + 2q_0 \hat{e}_3 \cdot \nabla \times \hat{e}_3 \right]$$

--> spin connection: $\mathbf{a} = \hat{e}_2 \cdot \nabla \hat{e}_1$

Derivation of Goldstone modes theory



- U(1) gauge theory of helical state:

$$\mathcal{H} = \frac{K}{2}(\nabla\phi + \mathbf{a} - q_0\delta\hat{\mathbf{e}}_3)^2 + \frac{K}{4}\left[(\nabla\hat{e}_3)^2 + 2q_0\hat{e}_3\cdot\nabla\times\hat{e}_3\right]$$

- integrate out $\hat{e}_3 = \hat{e}_{3\perp} + \hat{z}\sqrt{1-\hat{e}_{3\perp}^2} \approx \hat{e}_{3\perp} + \hat{z}(1 - \frac{1}{2}\hat{e}_{3\perp}^2)$:

--> Locking helical frame to pitch axis ("Higgs mechanism")

$$\hat{e}_{3\perp} \approx (\nabla_\perp\phi + \mathbf{a}_\perp)/q_0$$

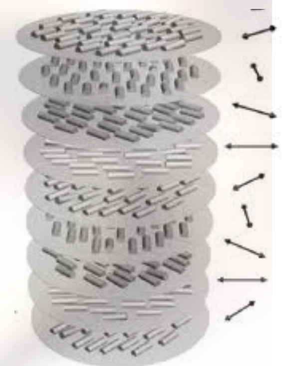
$$\mathcal{H} \approx \frac{K}{2}\left[\partial_z\phi + a_z + \frac{1}{2}(\nabla\phi + \mathbf{a})^2\right]^2 + \frac{K}{4}\left[(\nabla(\nabla\phi + \mathbf{a}))^2 + 2q_0\hat{z}\cdot\nabla\times(\nabla\phi + \mathbf{a})\right]$$

- helical state no defects --> single-valued u and curl-free a

$$\mathcal{H} \approx \frac{B}{2}\left[\partial_z u + \frac{1}{2}(\nabla u)^2\right]^2 + \frac{\overline{K}}{2}(\nabla^2 u)^2$$

- phonon $u(r) = -\phi(r)/q_0$, $B = K q_0^2$, $\overline{K} = K/2$

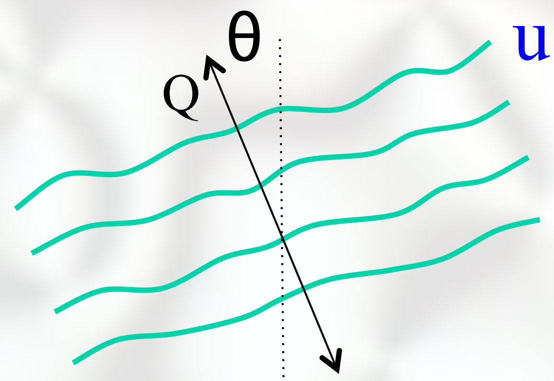
Rotational invariance of helical state



- nonlinear Goldstone modes

$$\mathcal{H} \approx \frac{B}{2}\left[\partial_z u + \frac{1}{2}(\nabla u)^2\right]^2 + \frac{\overline{K}}{2}(\nabla^2 u)^2$$

  - encodes <u>large</u> rotational invariance



$$E[u_0(\mathbf{r})] = 0,$$

$$\text{for } u_0(\mathbf{r}) = z(\cos\theta - 1) + x\sin\theta$$

$$\mathcal{H}_{lin} \approx \frac{B}{2}(\partial_z u)^2 + \frac{\overline{K}}{2}(\nabla^2 u)^2$$

  - important at long length scales --> *<u>anomalous elasticity</u>*

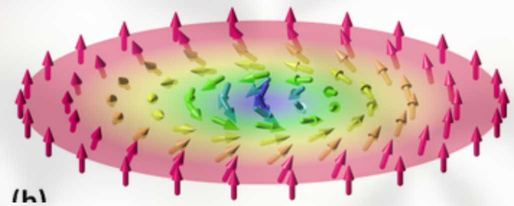
Adjacent phases

$$\mathbf{a} = \hat{e}_2 \cdot \nabla \hat{e}_1$$

- Residual frustration

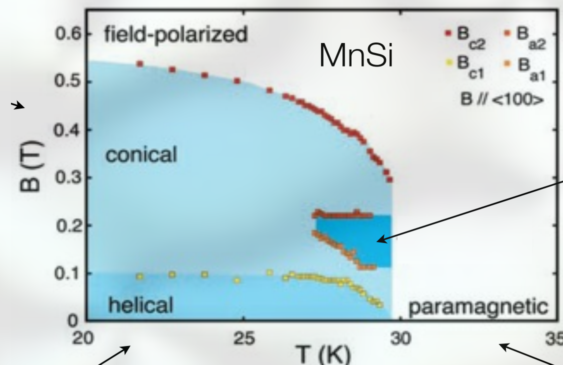
$$\mathcal{H} \approx \frac{K}{2}\left[\partial_z \phi + a_z + \frac{1}{2}(\nabla\phi + \mathbf{a})^2\right]^2 + \frac{K}{4}\left[(\nabla(\nabla\phi + \mathbf{a}))^2 \boxed{+ 2q_0 \hat{z}\cdot\nabla\times\mathbf{a}}\right]$$

- Flux density = Pontryagin index density



$$\nabla \times \mathbf{a} = \epsilon_{ij}\hat{e}_3 \cdot \partial_i \hat{e}_3 \times \partial_j \hat{e}_3$$

- Skyrmion crystal



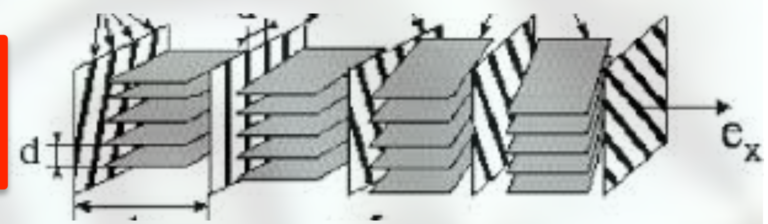
Skyrmion crystal

S. Mühlbauer *et al.* Science (2009)

C. Pfleiderer, et al.

- Twist-Grain Boundary

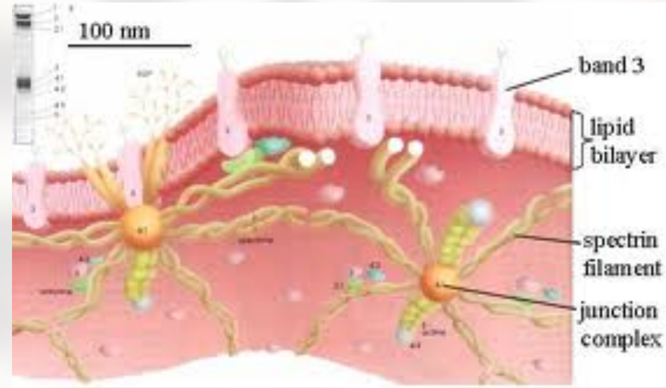
$$\mathcal{H}_{lin} \approx \frac{B}{2}(\partial_z u)^2 + \frac{\overline{K}}{2}(\nabla^2 u)^2 \boxed{- \overline{K} q_0 \hat{z}\cdot\nabla\times\nabla u}$$



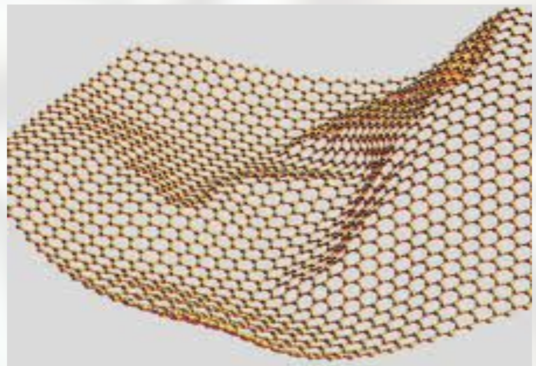
- Melting by dislocation unbinding into Isotropic

Membranes everywhere

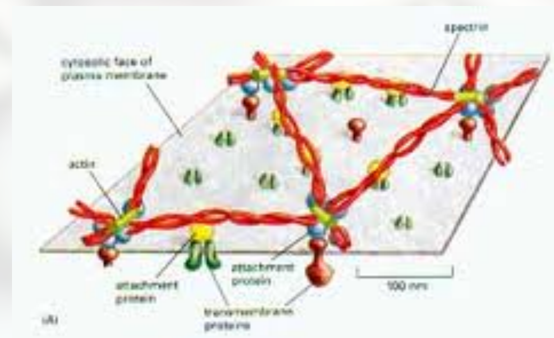
- physical realizations:



biological membranes



graphene



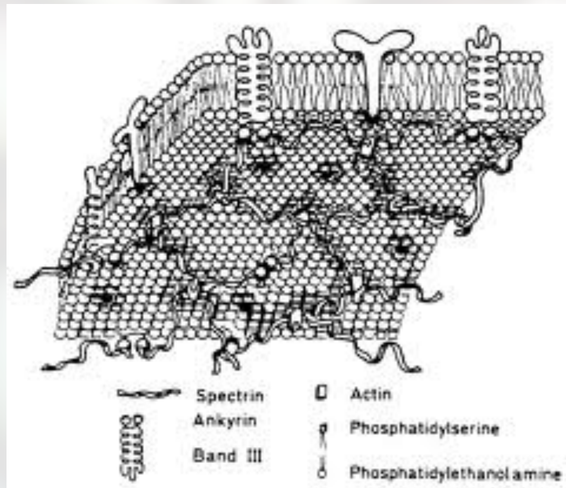
2d polymers and gels

MoS_2, ZrP sheets

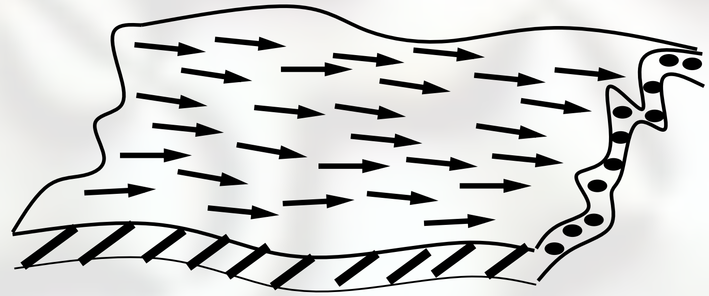
- fascinating interplay of statistical mechanics, field theory and geometry

Ingredients

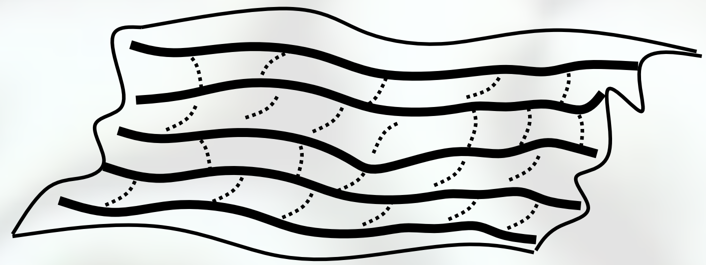
- vanishing surface tension
- bending rigidity



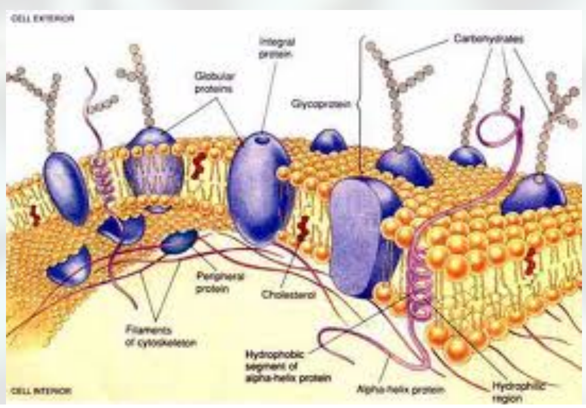
- in-plane elasticity



- in-plane order: anisotropy, hexatic,…

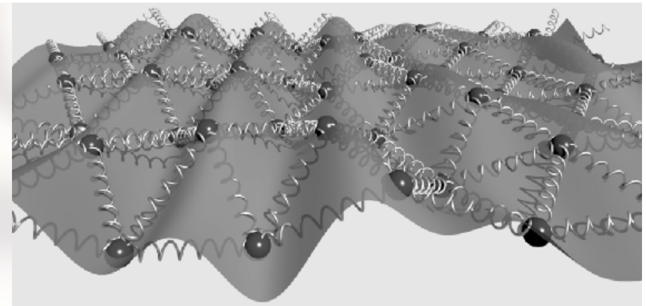


- heterogeneity



Kantor, Kardar, Nelson '86

Model

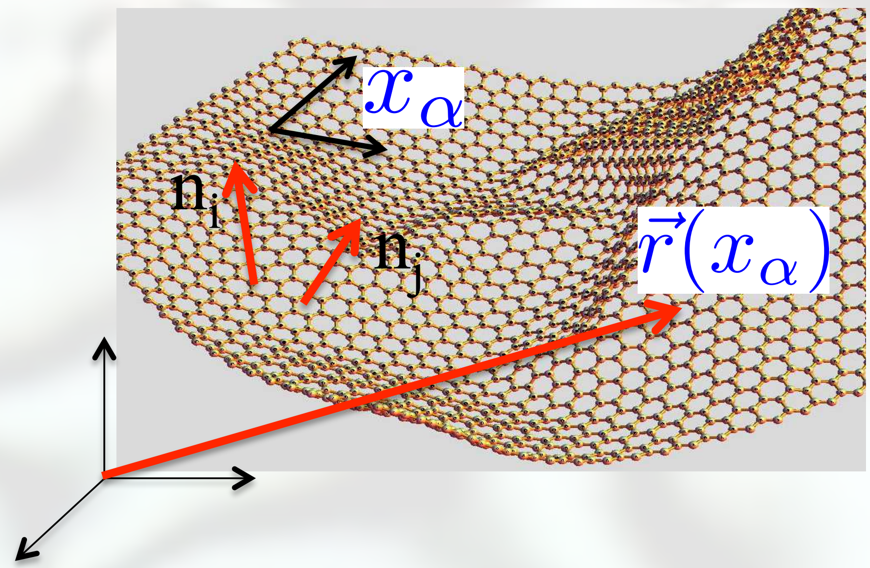


$$H = -\kappa \sum_{<ij>} \hat{n}_i \cdot \hat{n}_j + \sum_{ij} V(|\mathbf{r}_i - \mathbf{r}_j|)$$

- want Landau description

- O(D) x O(d) order parameter:

$$\vec{t}_\alpha = \partial_\alpha \vec{r}$$



$$F[\vec{r}] = \int d^D x \left[\kappa(\partial^2 \vec{r})^2 + \tau_\alpha(\partial_\alpha \vec{r})^2 + g(\partial_\alpha \vec{r})^4\right] + v \int d^D x d^D x' \delta^{(N)}(\vec{r}(\mathbf{x}) - \vec{r}(\mathbf{x}'))$$

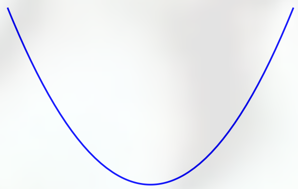
bending rigidity

self-avoidance

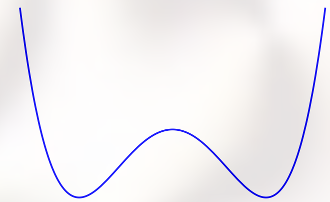
$\vec{t}_\alpha = \partial_\alpha \vec{r}$

Mean-field theory

Kantor, Kardar, Nelson '86
L.R., Toner '97, '99

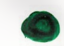


$$f \sim \tau_\alpha (\vec{t}_\alpha)^2 + g(\vec{t}_\alpha)^4$$



ignore $k_B T$, minimize:

- Crumpled phase ($\tau_x > 0$, $\tau_y > 0$): $\langle \vec{t}_x \rangle = \langle \vec{t}_y \rangle = 0$

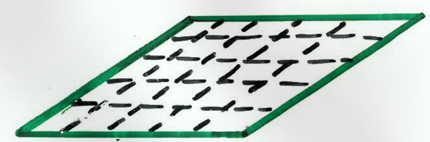


$\vec{r}_c = 0$

- Tubule phase ($\tau_x > 0$, $\tau_y < 0$): $\langle \vec{t}_x \rangle = 0, \langle \vec{t}_y \rangle > 0$

$\vec{r}_t = (0, t_y y, 0)$

- Flat phase ($\tau_x < 0$, $\tau_y < 0$): $\langle \vec{t}_x \rangle > 0, \langle \vec{t}_y \rangle > 0$

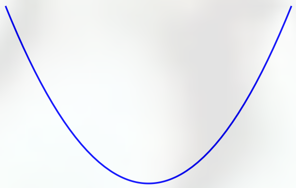


$\vec{r}_t = (t_x x, t_y y, 0)$

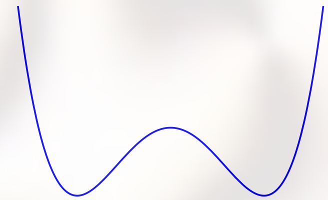
$k_B T$, self-avoidance, heterogeneity, nonlinearities: ???

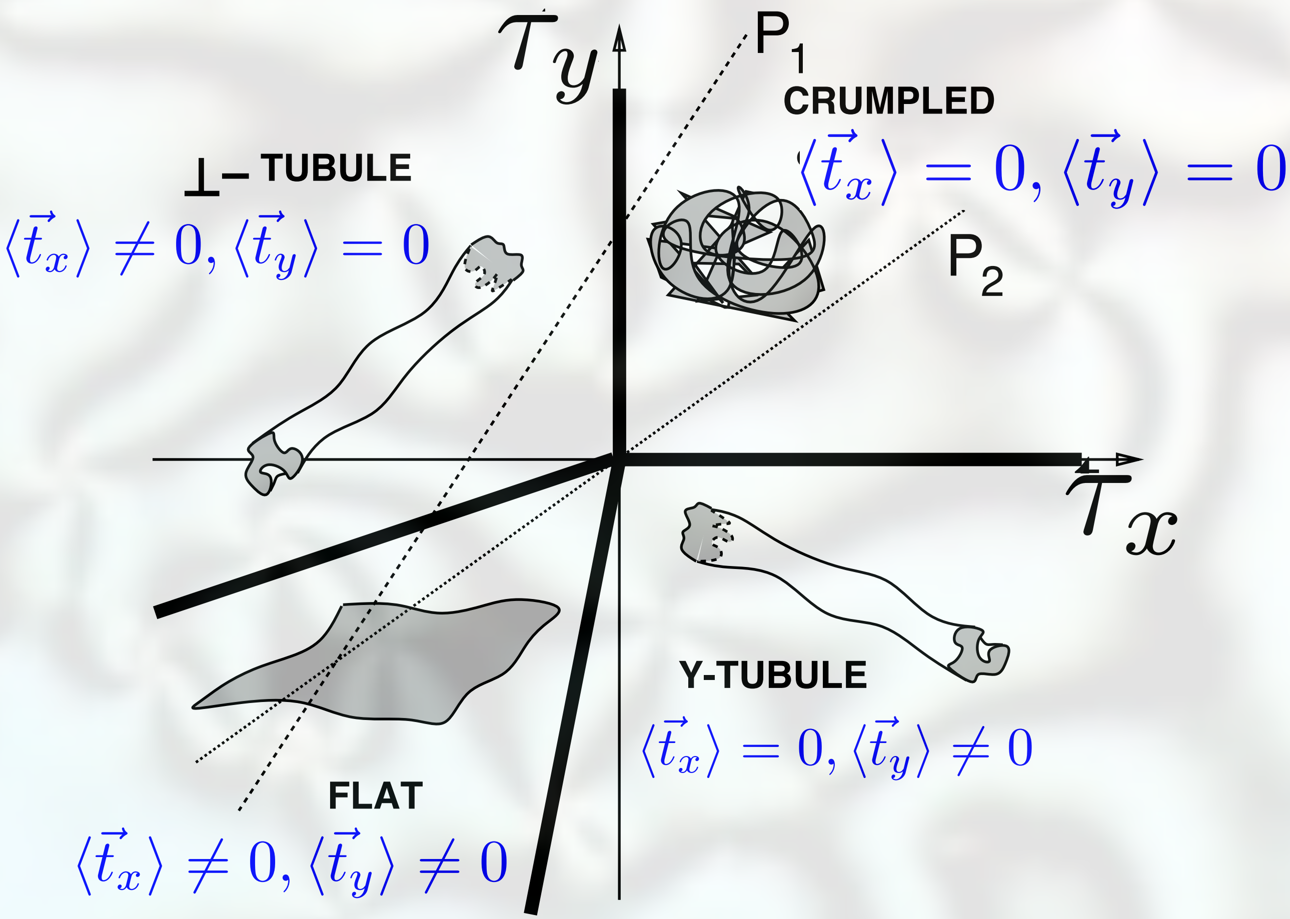
Crumpling transition

L.R., Toner '97, '99

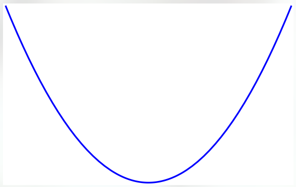


$$f \sim \tau_\alpha (\vec{t}_\alpha)^2 + g(\vec{t}_\alpha)^4$$

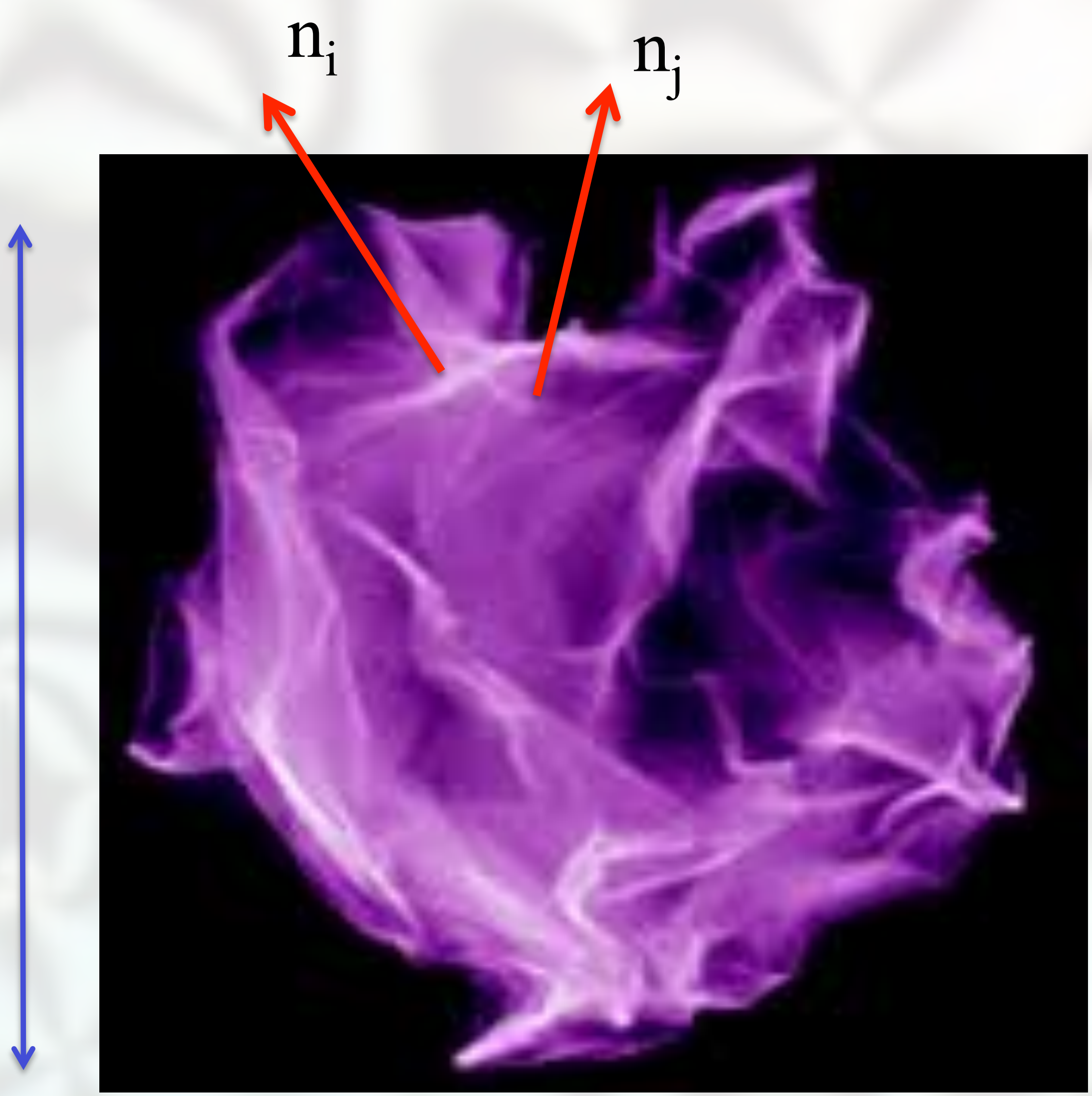




Crumpled phase



$R_G \sim L^{\nu}$



Crumpled phase

$$F_c[\vec{r}] = \tau \int d^D x (\partial_\alpha \vec{r})^2 + v \int d^D x d^D x' \delta^{(N)}(\vec{r}(\mathbf{x}) - \vec{r}(\mathbf{x}'))$$

- short-range order in normals

  $n_i \cdot n_j \approx e^{-|i-j|/\xi}$

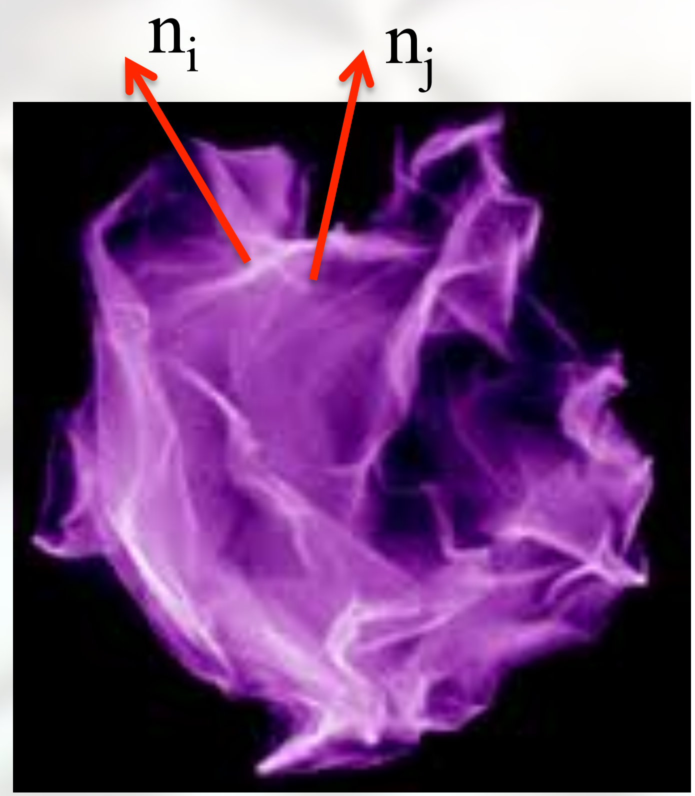
- disordered by $k_B T$

$R_G \sim L^{\nu}$

- analog of PM state of the normals

- fractal $M \sim R_G^{d_F}$ (Flory)

  $d_F = D/\nu \approx D(d+2)/(D+2) = 2.5, \ \nu \approx 0.8$



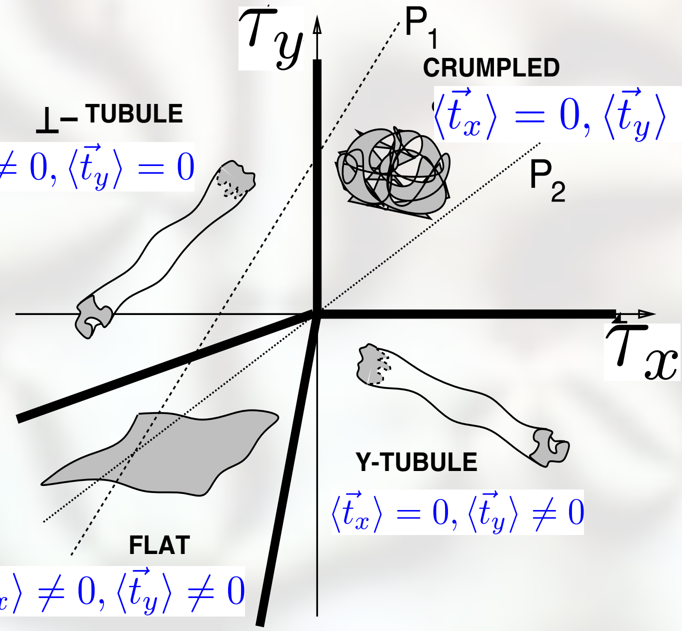
- self-avoiding interaction important: $R_G^0 \sim \sqrt{\ln L} \longrightarrow R_G \sim L^{4/5}$

$\vec{t}_\alpha = \partial_\alpha \vec{r}$

Mean-field theory

Kantor, Kardar, Nelson '86
L.R., Toner '97, '99

$f \sim \tau_\alpha (\vec{t}_\alpha)^2 + g(\vec{t}_\alpha)^4$



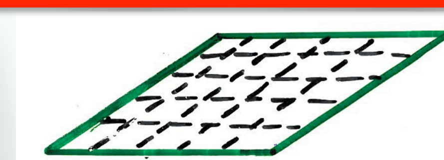
- Crumpled phase ($\tau_x > 0$, $\tau_y > 0$): $\langle \vec{t}_x \rangle = \langle \vec{t}_y \rangle = 0$

$\vec{r}_c = 0$

- Tubule phase ($\tau_x > 0$, $\tau_y < 0$): $\langle \vec{t}_x \rangle = 0, \langle \vec{t}_y \rangle > 0$

$\vec{r}_t = (0, t_y y, 0)$

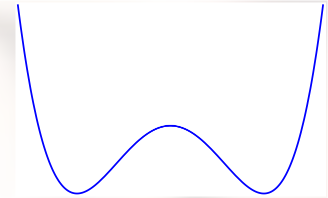
- Flat phase ($\tau_x < 0$, $\tau_y < 0$): $\langle \vec{t}_x \rangle > 0, \langle \vec{t}_y \rangle > 0$



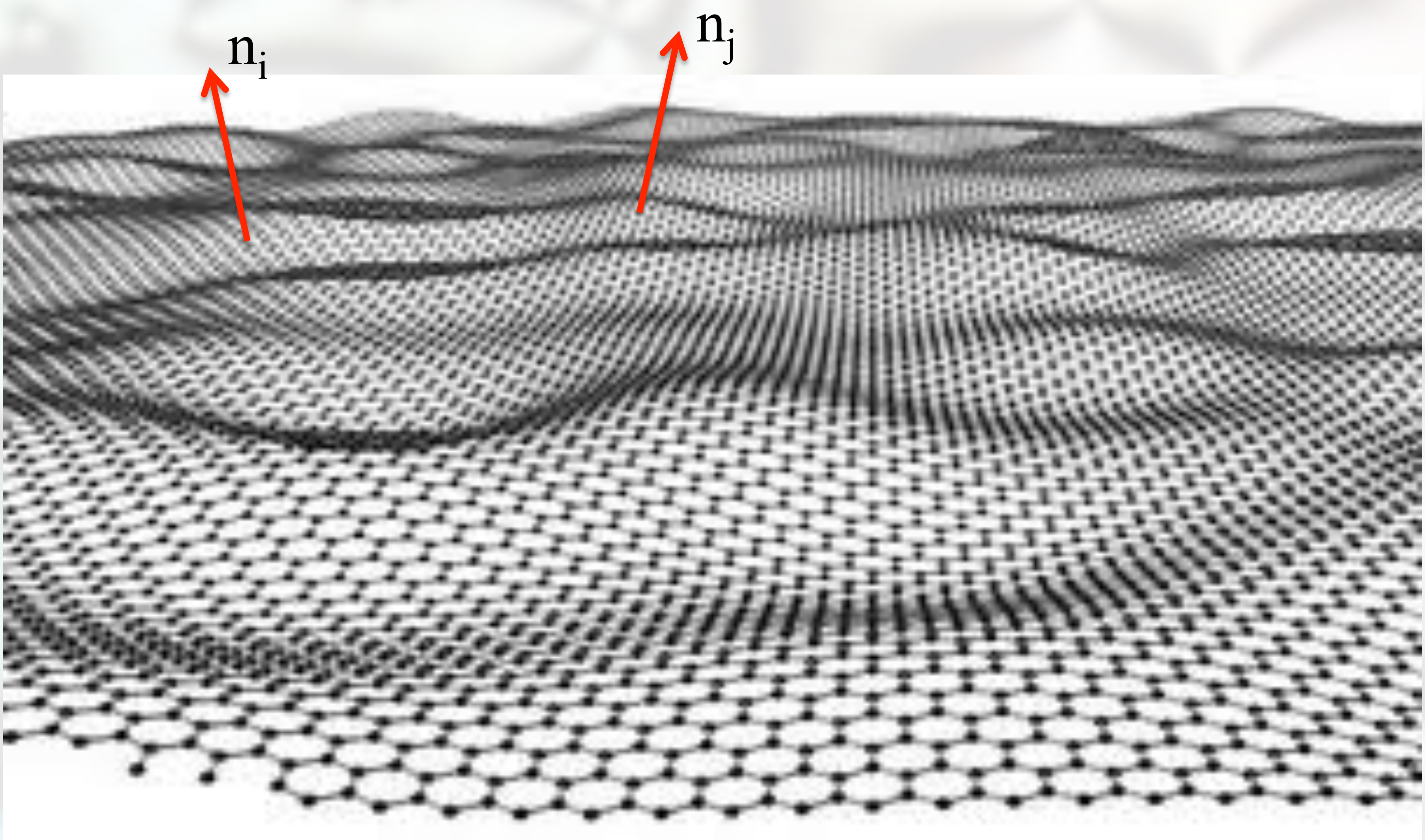
$\vec{r}_t = (t_x x, t_y y, 0)$

$k_B T$, self-avoidance, heterogeneity, nonlinearities: ???

“Flat” phase

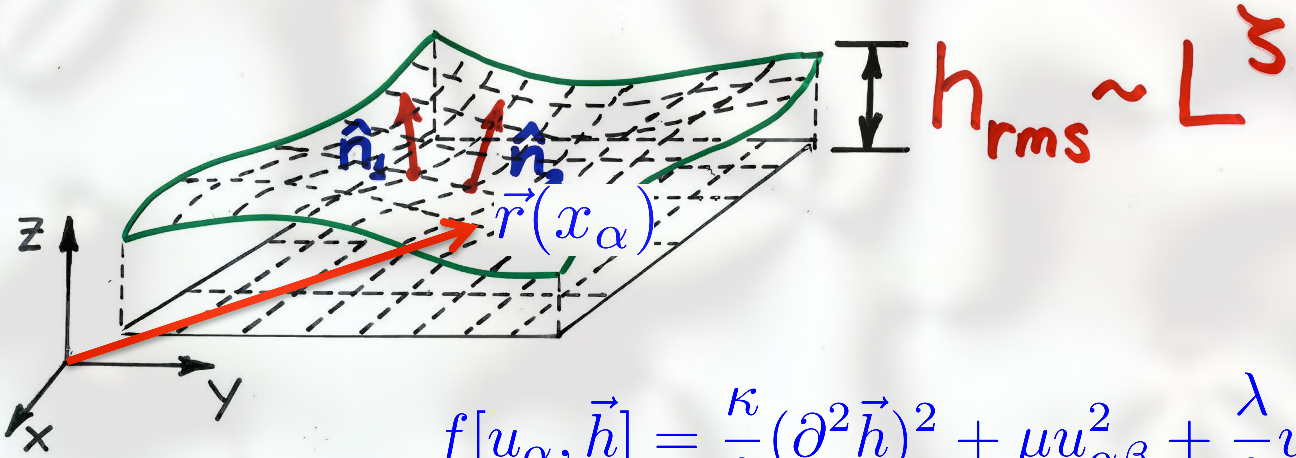


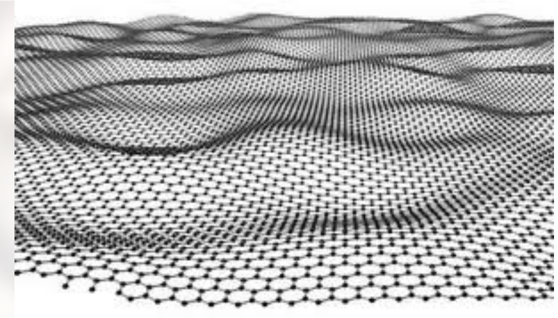
long range 2D orientation order: *impossible ?*



Nelson, Peliti '87

"Flat" phase





Aronovitz, Lubensky '88
David, Guitter '88

$$f[u_\alpha, \vec{h}] = \frac{\kappa}{2}(\partial^2 \vec{h})^2 + \mu u_{\alpha\beta}^2 + \frac{\lambda}{2} u_{\alpha\alpha}^2$$

- long-range order in the normals, breaks O(3) symmetry
  - geometric analog of 2D ferromagnet in the normals
  - "circumvents" Hohenberg-Mermin-Wagner-Coleman theorem, via order-from-disorder
- characterized by power-law roughness, $\zeta \approx 0.59$

LeDoussal, L.R.'92

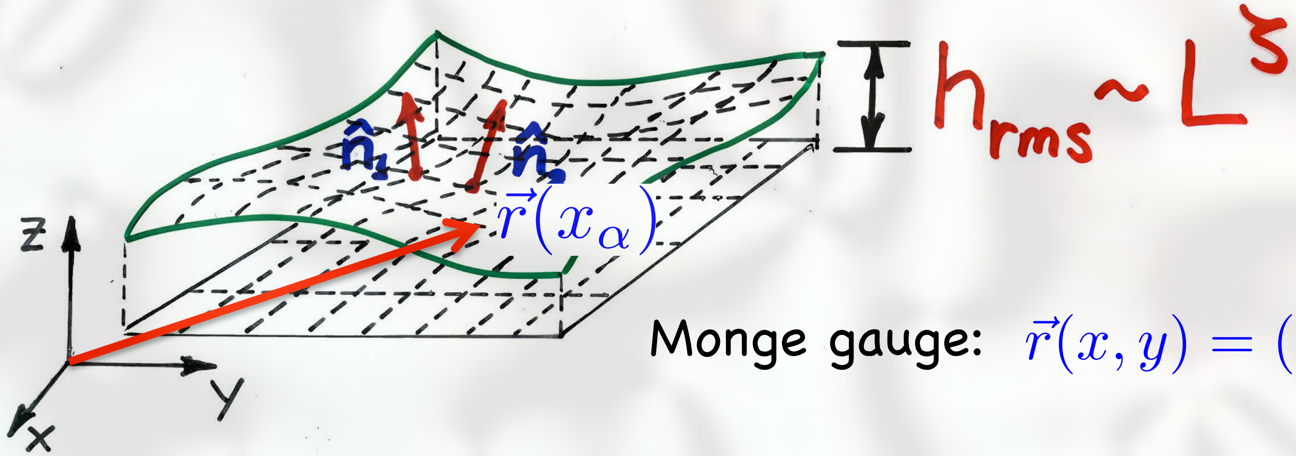
- "critical phase" with universal anomalous elasticity:

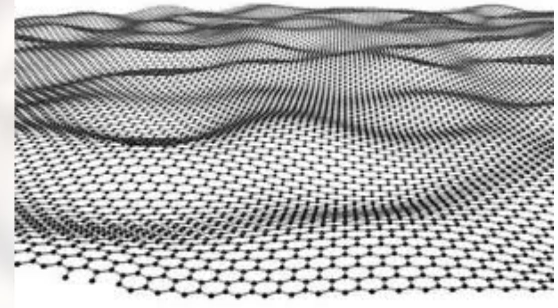
  $\kappa(L) \sim L^{\eta}$, $\mu(L) \sim L^{-\eta_u}$, $\sigma = -1/3$ $\quad (\eta_u = 4 - D - 2\eta)$

  (agrees well with MC simulations by Bowick, Falcioni et al, '96, '97)

Nelson, Peliti '87

"Flat" phase model





Monge gauge: $\vec{r}(x,y) = (x + u_x, y + u_y, h(x,y))$

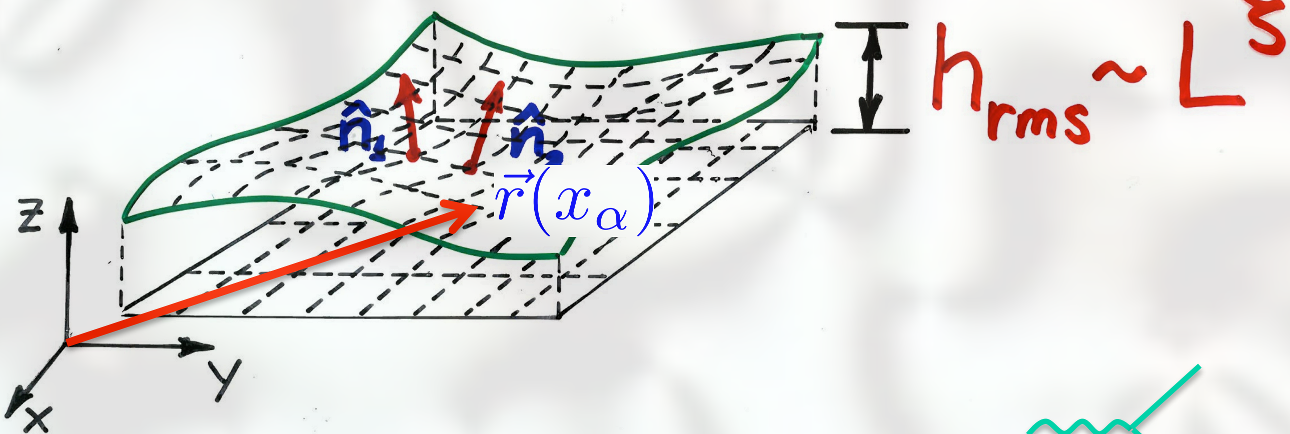
- free-energy density: $f[u_\alpha, \vec{h}] = \frac{\kappa}{2}(\partial^2 \vec{h})^2 + \mu u_{\alpha\beta}^2 + \frac{\lambda}{2} u_{\alpha\alpha}^2$

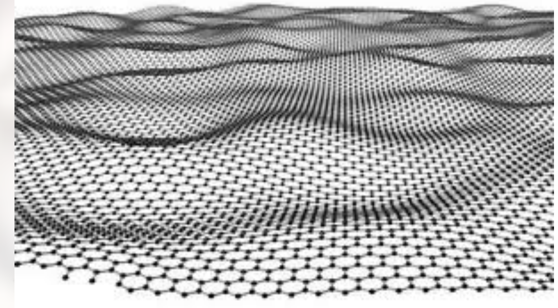
- nonlinear strain: $u_{\alpha\beta} = \frac{1}{2}(\partial_\alpha u_\beta + \partial_\beta u_\alpha + \partial_\alpha \vec{h} \cdot \partial_\beta \vec{h}) = \frac{1}{2}(g_{\alpha\beta} - \delta_{\alpha\beta})$

- integrate out u_α: $f_{\text{eff}}[\vec{h}] = \frac{\kappa}{2}(\partial^2 \vec{h})^2 + \frac{1}{4}\underbrace{(\partial_\alpha \vec{h} \cdot \partial_\beta \vec{h}) K_{\alpha\beta,\gamma\delta} (\partial_\gamma \vec{h} \cdot \partial_\delta \vec{h})}$

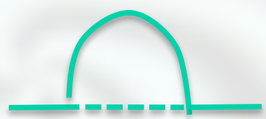
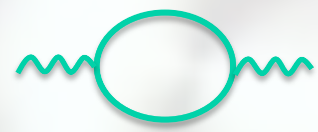
Gaussian curvature interaction: $R \frac{1}{\nabla^4} R$

k_BT + nonlinearities





- PT in elastic nonlinearities: $\partial u \partial h \partial h + \frac{1}{4}(\partial h \partial h)^2$

$\delta\kappa \sim \frac{\mu T}{\kappa^2} L^{4-D} =$  $\qquad \delta\mu, \delta\lambda \sim -\frac{\mu T}{\kappa^2} L^{4-D} = -$ 

- diverges for $L > \xi_{\mathrm{NL}} \equiv \sqrt{\frac{\kappa^2}{\mu T}}$ *≈ 10Å for graphene -> electronic physics*

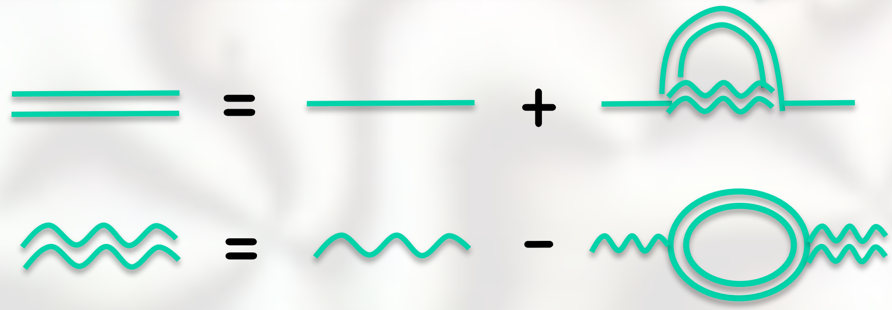
- need a fully nonlinear treatment; physical interpretation?

Nelson, Peliti '87

Anomalous elasticity

LeDoussal, L.R.'92

- SCSA theory:

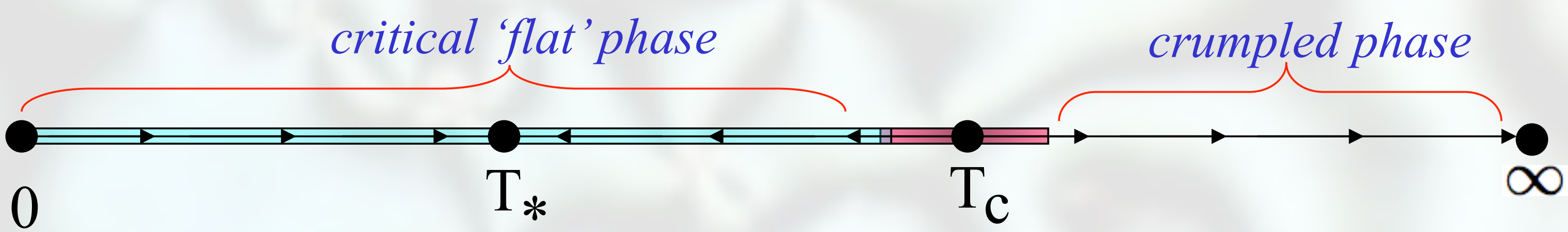


- length-scale dependent moduli: $\kappa(k) \sim k^{-\eta}, \quad \mu(k), \lambda(k) \sim k^{\eta_u}$

- Ward identity O(3) symmetry $(\partial u + \frac{1}{2}\partial h \partial h)$ -> $\eta_u = 2 - 2\eta$

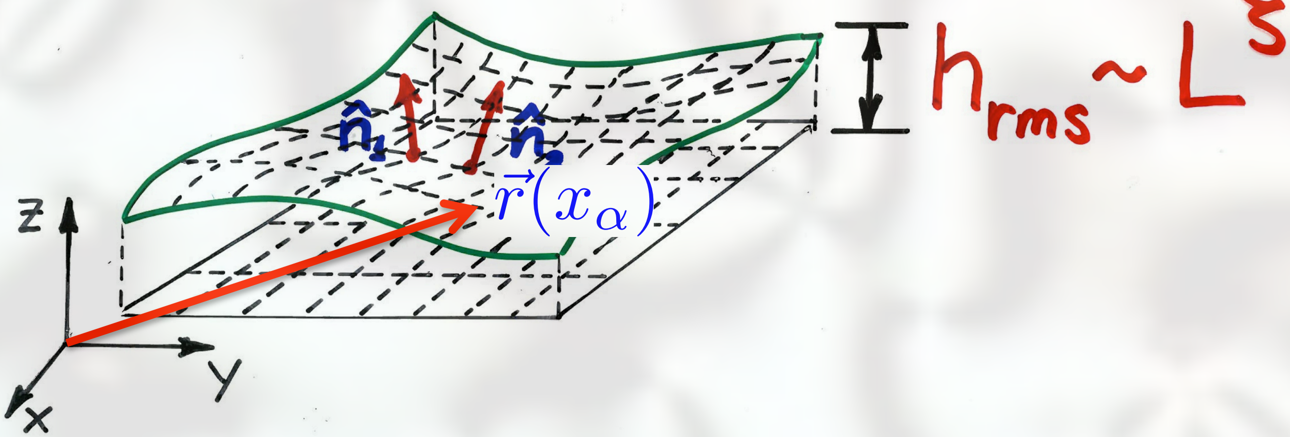
  $\kappa(L) = \mu(L) h_{rms}(L)^2$

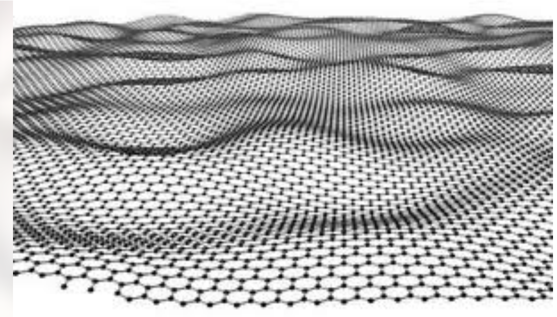
- RG with ε = 4-D, 1/d expansions *(Aronovitz-Lubensky, David-Guitter, '88)*



- SCSA exact: O(ε,d), O(1/d,D), at d=D: $\eta = 0.82, \ \zeta = 0.59, \ \sigma = -1/3$

Order-from-disorder





- *unstable* harmonically for $L > ae^{4\pi\kappa/3k_BT} \equiv \xi_{\text{crump.}}$

$$\langle\theta^2\rangle \approx \frac{k_BT}{\kappa}\int \frac{d^2q}{(2\pi)^2}\frac{q^2}{q^4} \approx \frac{3k_BT}{4\pi\kappa}\ln L/a \longrightarrow \infty$$

($\sim 10^{10}$ graphene -> crumpling is irrelevant)

- *stabilized* anharmonically by k_BT: $\theta_{\text{rms}} \sim L^{-\eta/2}, h_{\text{rms}} \sim L^{1-\eta/2}$

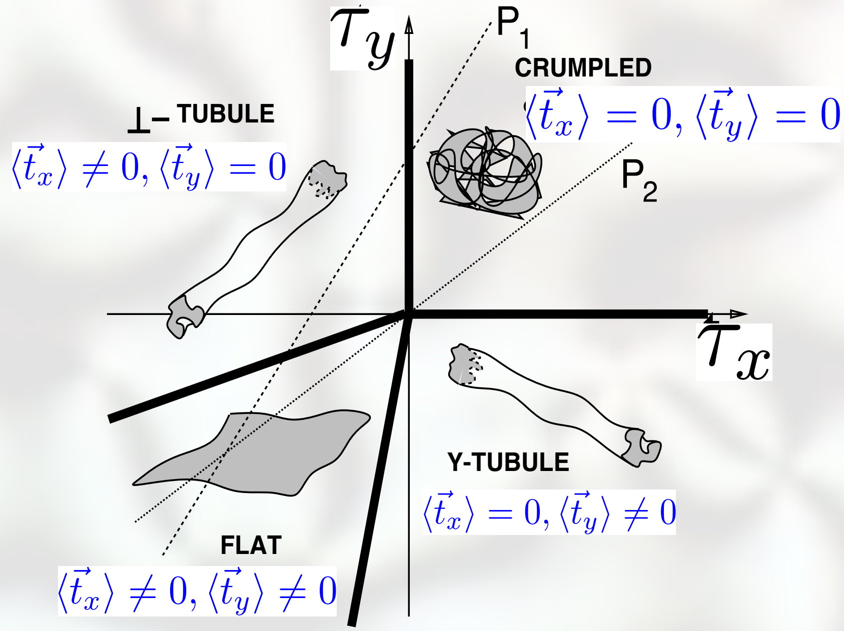
$$\langle\theta^2\rangle \approx k_BT\int \frac{d^2q}{(2\pi)^2}\frac{q^2}{\kappa(q)q^4} \sim TL^{-\eta} \longrightarrow 0$$

$\vec{t}_\alpha = \partial_\alpha \vec{r}$

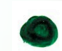
Mean-field theory

Kantor, Kardar, Nelson '86
L.R., Toner '97, '99

$f \sim \tau_\alpha (\vec{t}_\alpha)^2 + g(\vec{t}_\alpha)^4$



- Crumpled phase ($\tau_x > 0$, $\tau_y > 0$): $\langle \vec{t}_x \rangle = \langle \vec{t}_y \rangle = 0$

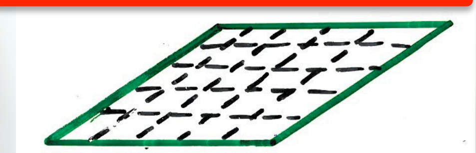


$\vec{r}_c = 0$

- Tubule phase ($\tau_x > 0$, $\tau_y < 0$): $\langle \vec{t}_x \rangle = 0, \langle \vec{t}_y \rangle > 0$

$\vec{r}_t = (0, t_y y, 0)$

- Flat phase ($\tau_x < 0$, $\tau_y < 0$): $\langle \vec{t}_x \rangle > 0, \langle \vec{t}_y \rangle > 0$



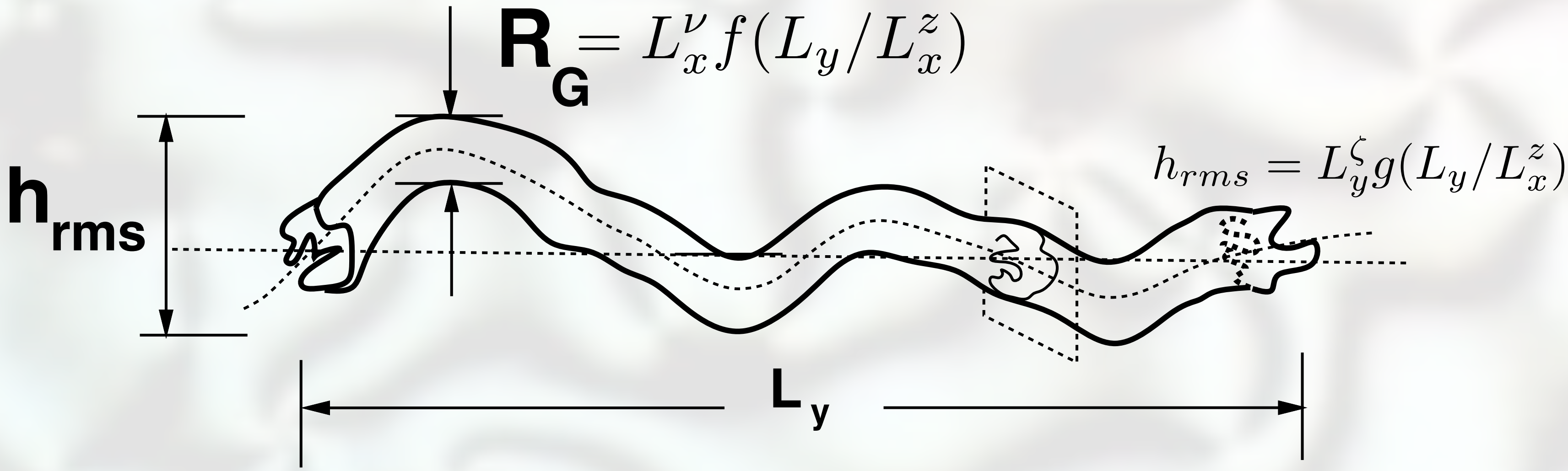
$\vec{r}_t = (t_x x, t_y y, 0)$

$k_B T$, self-avoidance, heterogeneity, nonlinearities: ???

Tubule phase

L.R., Toner '97, '99

$$f_y = \frac{\kappa}{2}(\partial_y^2 \vec{h})^2 + \frac{t}{2}(\partial_\alpha^\perp \vec{h})^2 + \frac{g_\perp}{2}(\partial_\alpha^\perp u)^2 + \frac{g_y}{2}\left(\partial_y u + \frac{1}{2}(\partial_y \vec{h})^2\right)^2 + f_{\mathrm{SA}}$$



- long-range orientational order in 1d, breaks O(3) symmetry:

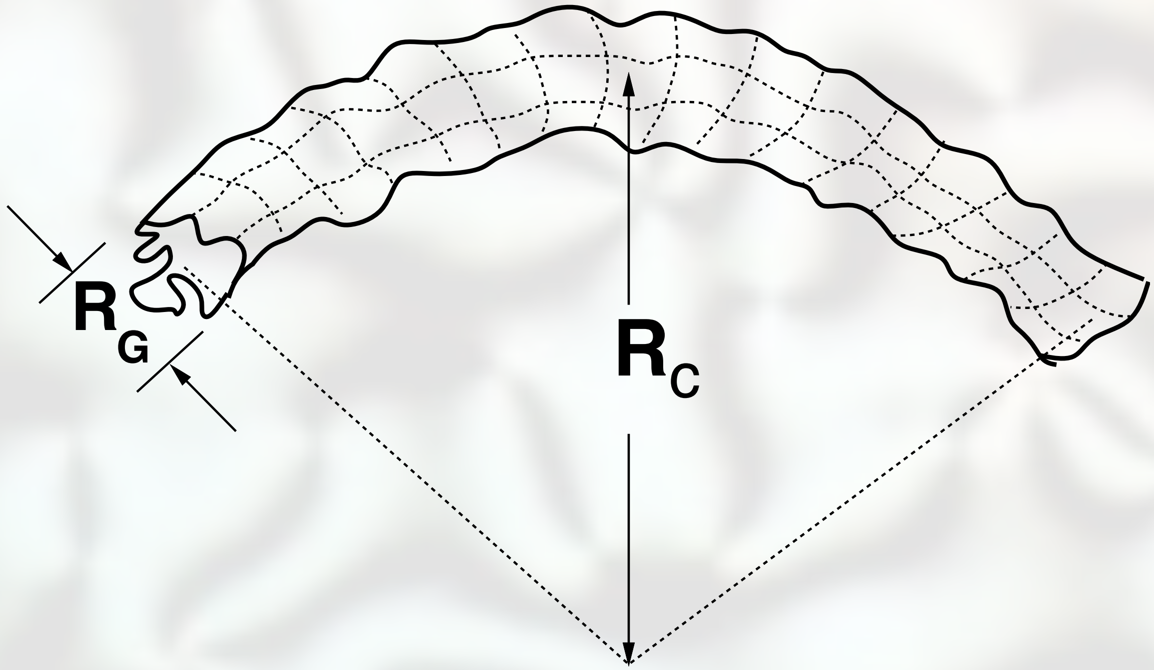
$$\langle \theta^2 \rangle \sim L^{-\eta} \ll 1 \longrightarrow \text{stable to } \; k_B T > 0$$

- nontrivial anomalous fixed point (with SA):

$$h_{rms} \sim L^{1/4}, R_G \sim L^{3/4}, \kappa(L) \sim L^{3/2}$$

Tubule anomalous elasticity

L.R., Toner '97, '99



$$h_{rms} = L_y^{\zeta} g(L_y/L_x^z)$$

$$R_G = L_x^{\nu} f(L_y/L_x^z)$$

- Length-scale dependent bending rigidity:

$$\kappa(L) \sim \mu(L) R_G(L)^2$$

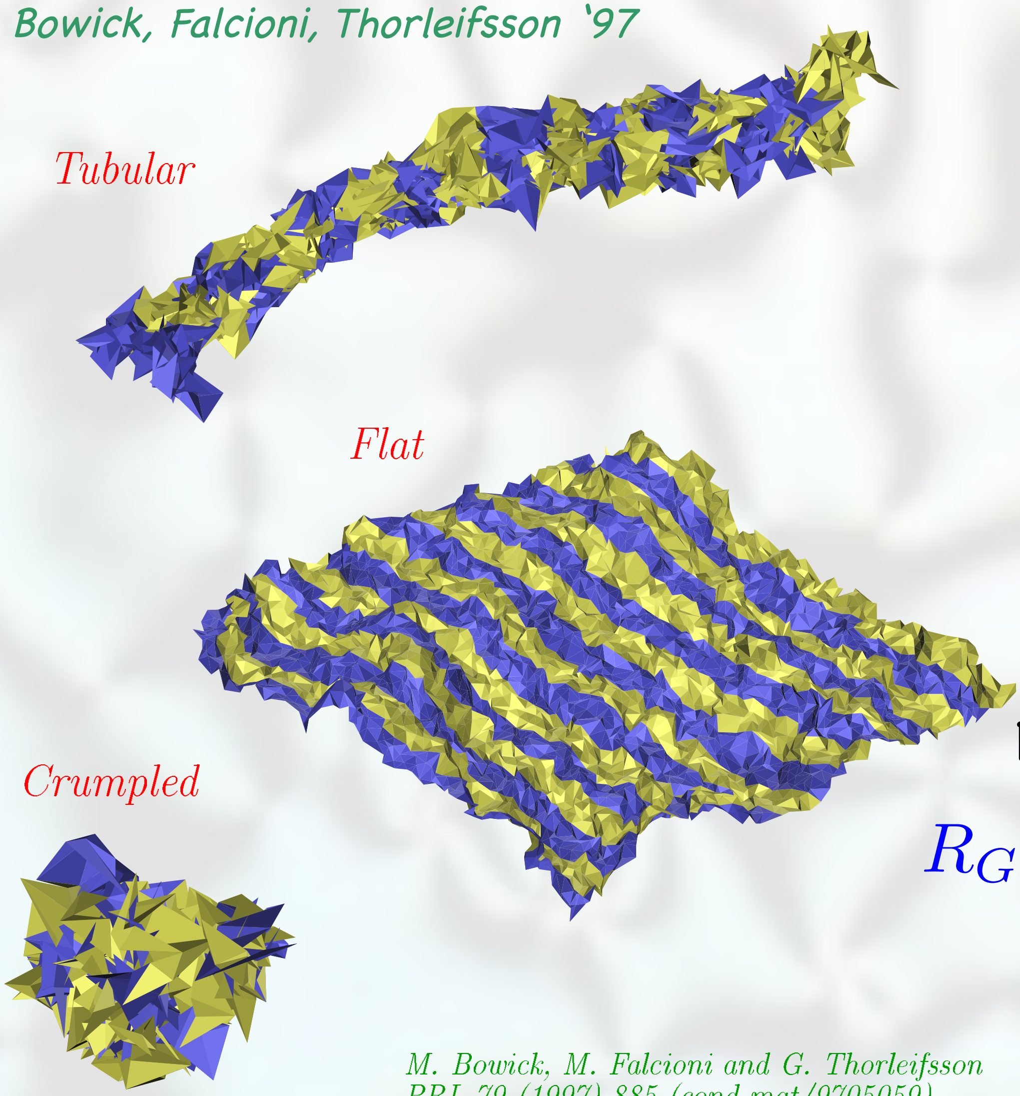
-> $2\nu = z(\eta_\kappa + \eta_\mu)$

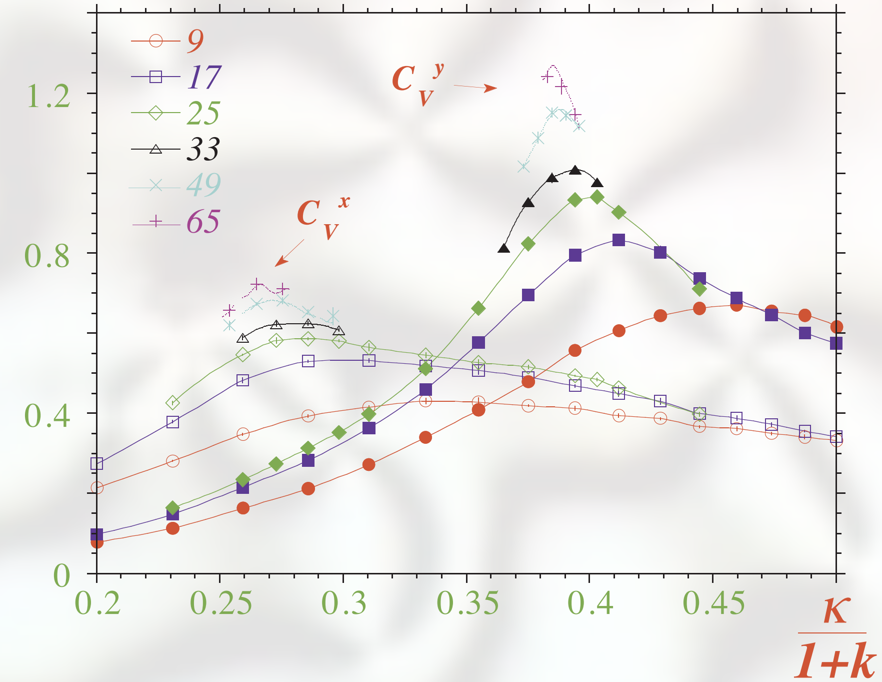
phantom

Monte-Carlo simulations

$$h_{rms} = L_y^{\zeta} g(L_y/L_x^z)$$
$$R_G = L_x^{\nu} f(L_y/L_x^z)$$

Bowick, Falcioni, Thorleifsson '97





Excellent agreement with R.T.:

$$R_G \sim L^{1/4},\ h_{\rm rms} \sim L\ \ (z = 1/2)$$

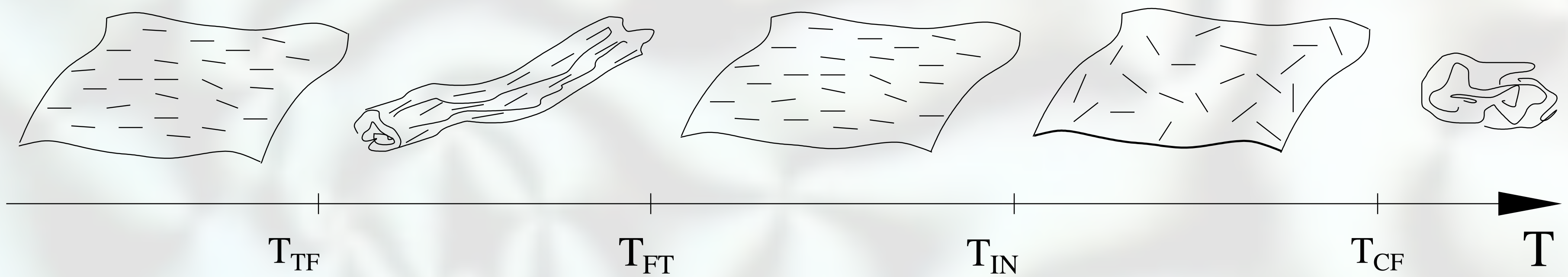
zero (ribbon) mode

M. Bowick, M. Falcioni and G. Thorleifsson
PRL 79 (1997) 885 (cond-mat/9705059)

Tunable spontaneous anisotropy

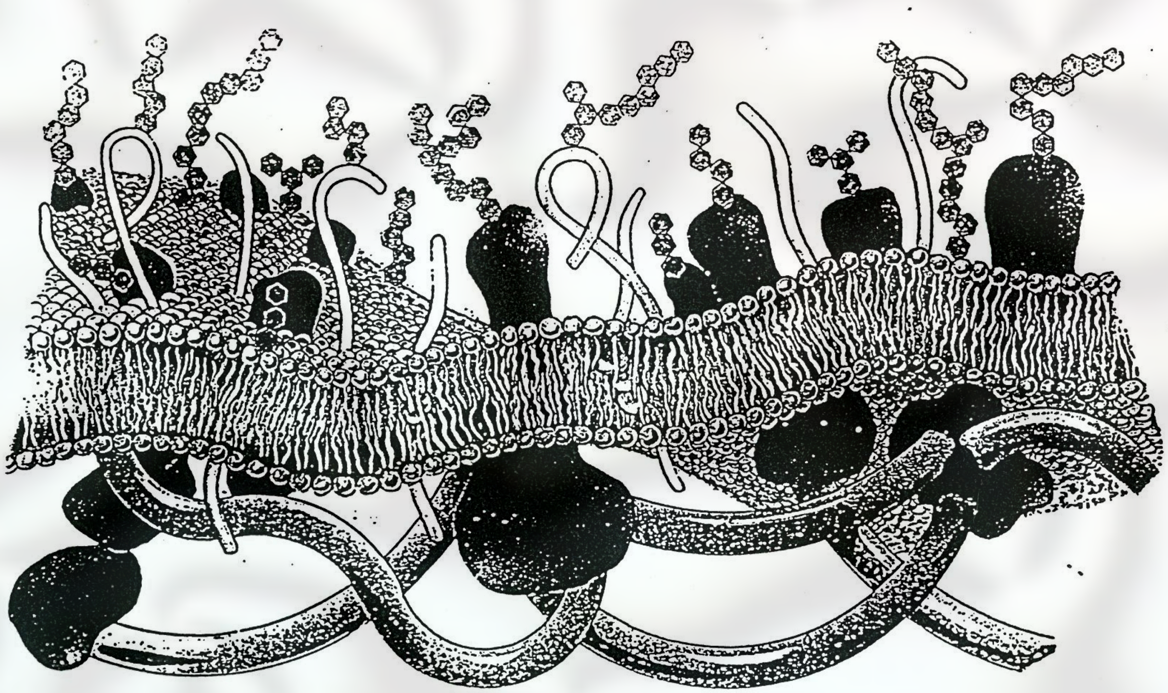
X. Xing, L.R. '04

- *spontaneous* in-plane nematic order (e.g., nematic elastomer membrane) -> reentrant flat phase:



Local heterogeneity

L.R., Nelson '91, '92
Bensimon, et al '91
L.R., LeDoussal '91, '92
Morse, Lubensky '92

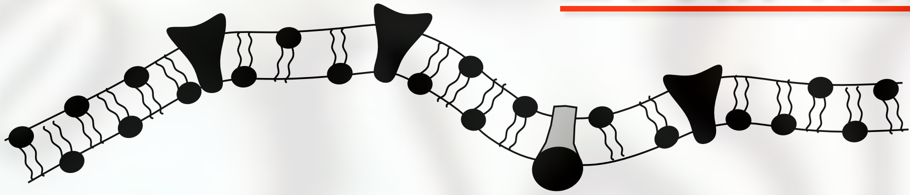


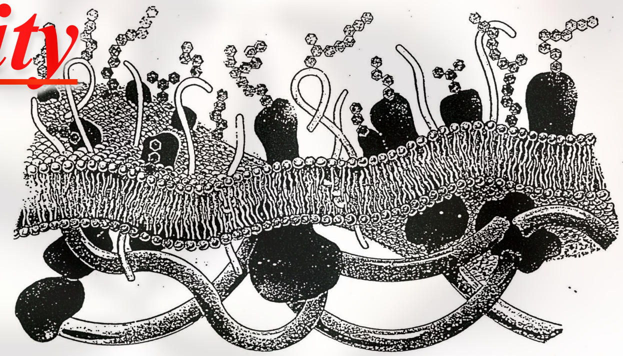
- proteins, nano-pores, holes, network defects, ...

  -> random distribution of interstitials, dislocation, disclinations, grain-boundaries, ...

L.R., LeDoussal '91, '92

Local heterogeneity





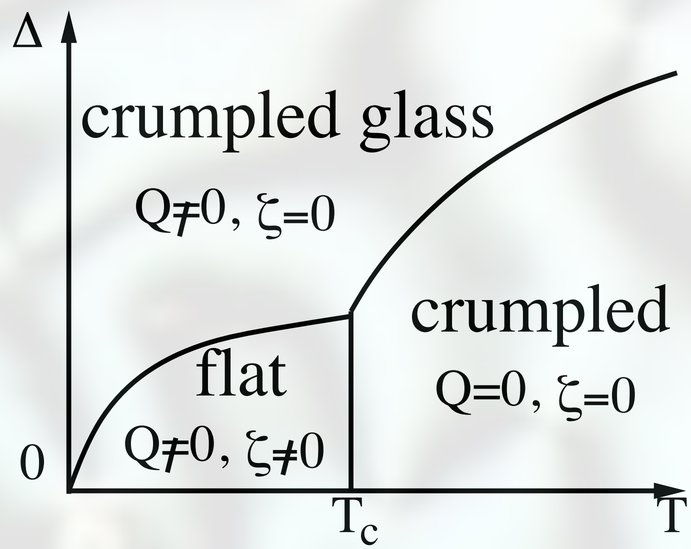
- random stresses, preferred curvature:

$$f = \frac{\kappa}{2}(\partial^2 h - c(\mathbf{x}))^2 + \mu u_{\alpha\beta}^2 + \frac{\lambda}{2} u_{\alpha\alpha}^2 - u_{\alpha\beta}\sigma_{\alpha\beta}(\mathbf{x})$$

- ***"flat glass"*** ground state, anomalous elasticity:

$$\eta = 0.45$$
$$\zeta = 0.775$$

- ***"crumpled glass"*** ground state



Open questions and implications

- systematic quantitative measurements e.g., graphene

- realization of the crumpling transition

- sheets with tunable anisotropy

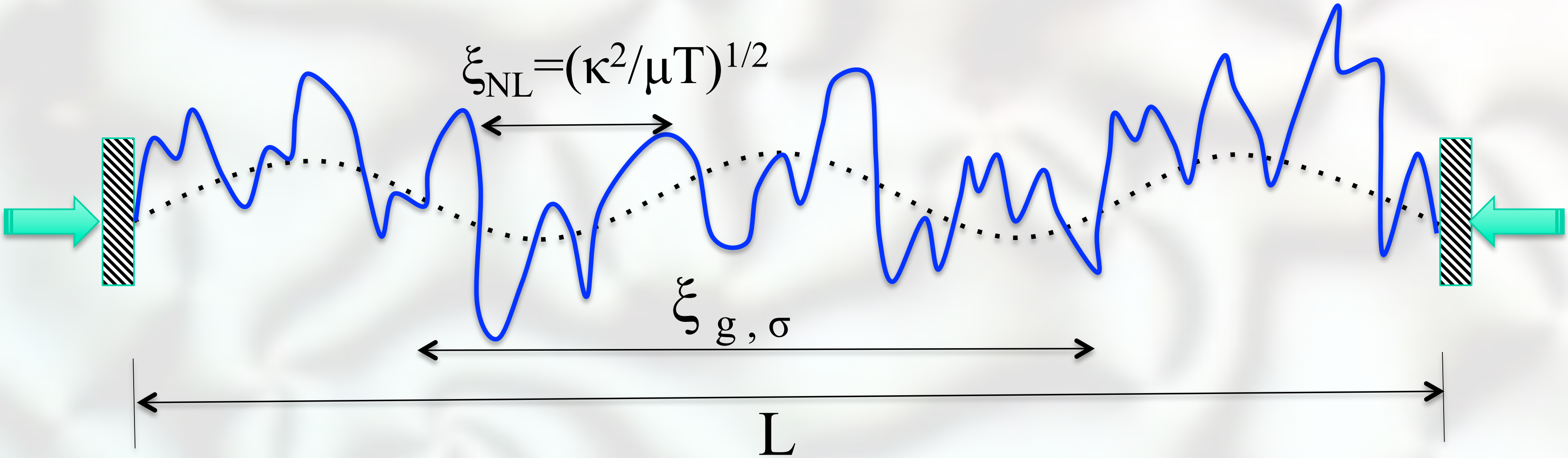
- nature of glassy phases

- statistical mechanics of membranes with nontrivial background strain and topology *(see e.g., vesicles: Nelson, et al.)*

- *redoing deformation analysis (Euler, Lame, crumpling,...) for free energy*

Buckling of "flat" phase

- want: $e^{-F/T} = \mathrm{Tr}_{h,u} e^{-H[h,u]/T}$
- "poor man's" scaling theory -> nonlinear elasticity = no linear response



$$H = \frac{1}{2}\int_x \left[\kappa_R(\nabla^2 h)^2 + \rho g h^2 - \sigma(\nabla h)^2 + \ldots\right]$$

$$\xi_L = L \qquad \xi_g = \left(\frac{\kappa}{\rho g}\right)^{1/4} \to \left(\frac{\kappa}{\rho g}\right)^{1/(4-\eta)}$$

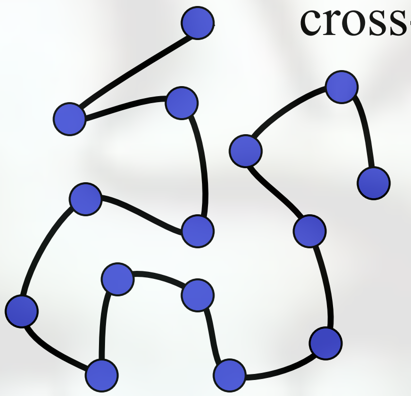
$$P_c^{(L)} = \frac{\kappa}{L^2} \to \frac{\kappa}{L^{2-\eta}} \qquad P_c^{(g)} = (\rho g \kappa)^{1/2} \to (\rho g \kappa)^{(2-\eta)/(4-\eta)}$$

Nematic Elastomer

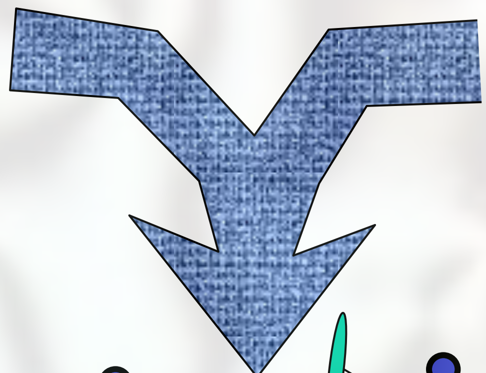
Terentjev
Finkelmann
Ratna

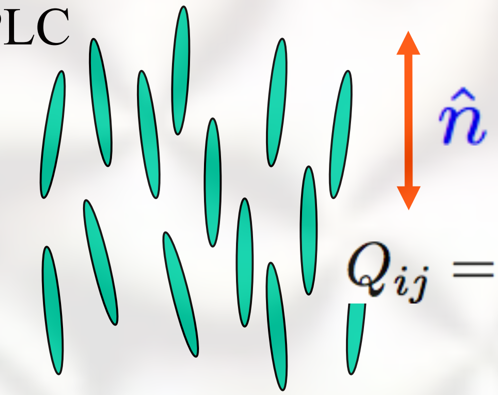
cross-linked polymer liquid crystal PLC

u_{ij}



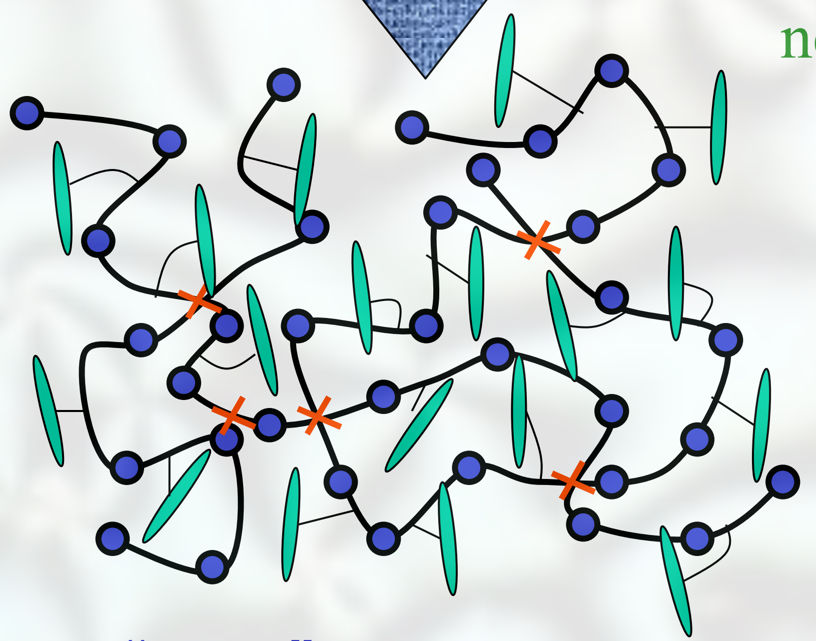
polymer





$Q_{ij} = n_i n_j$

nematic liquid-crystal



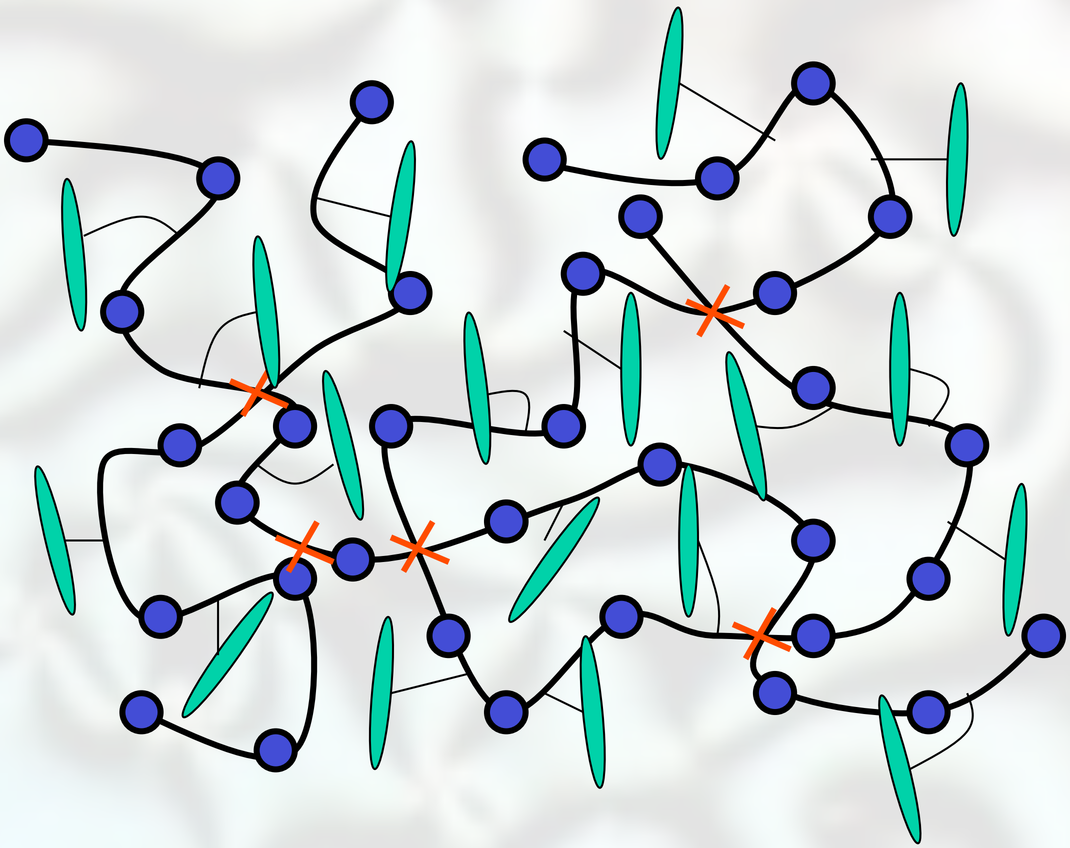
"Solid" Liquid-Crystal

exhibits most conventional liquid-crystal phases (I, N, Sm-A, Sm-C,…)

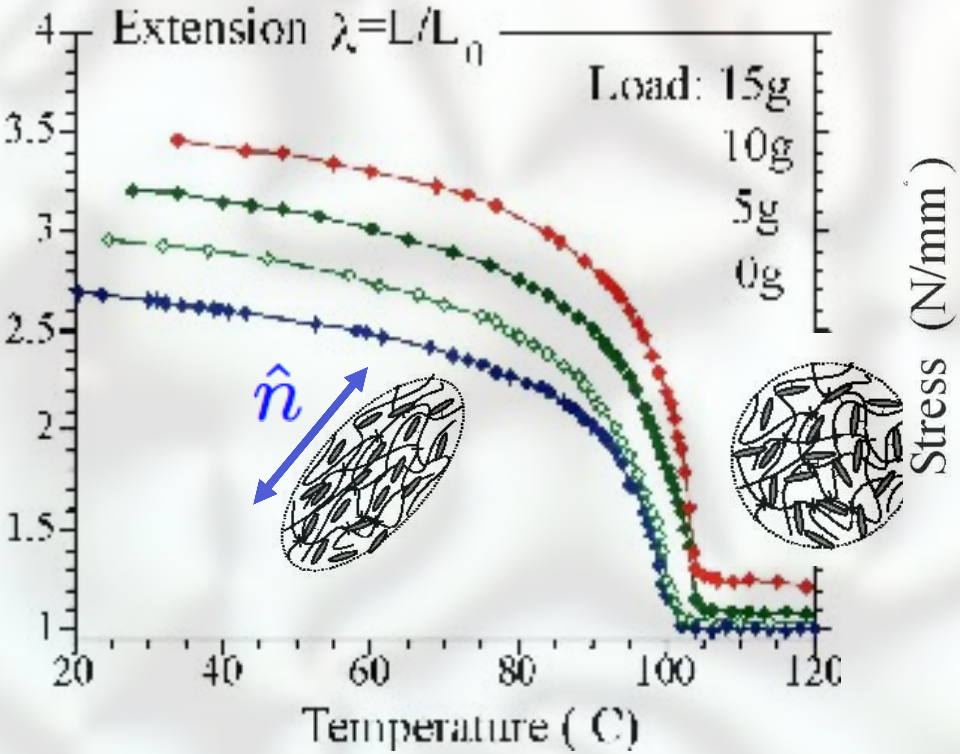
Nematic Elastomer

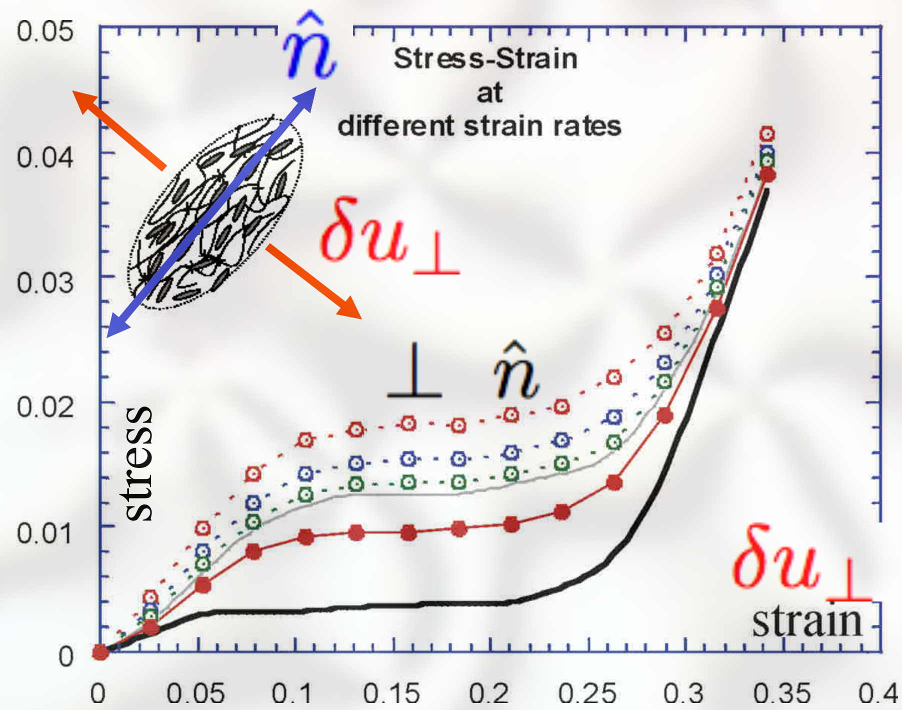
Terentjev
Finkelmann
Ratna

"Solid" Liquid-Crystal



Thermal response and stress-strain relation



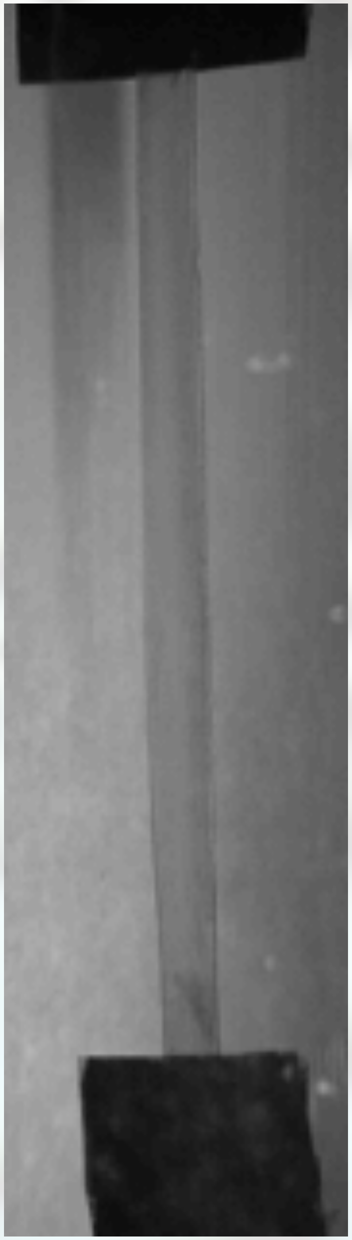


Properties: • *spontaneous distortion (~ 400%) at T_{IN} , thermoelastic*
• *"soft" elasticity*
• *giant electrostriction*

Applications: • *plastic displays*
• *switches*
• *actuators*
• *artificial muscle*

Terentjev, et al

Nematic elastomer as heat engine



n

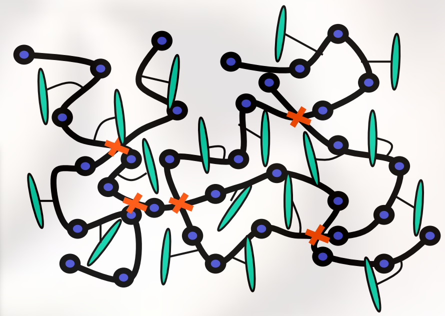
- monodomain nematic LCE
- 5cm x 5mm x 0.3mm
- lifts 30g wt. on heating, lowers it on cooling
- large strain (>400%)

$$\eta \simeq 10^5\, Pa$$

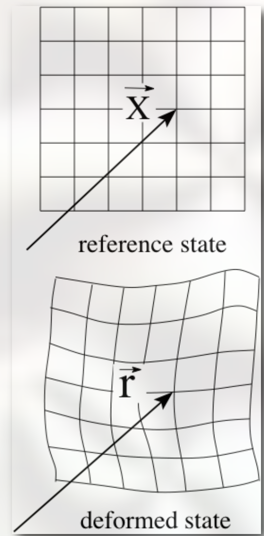
(P. M-Palffy)

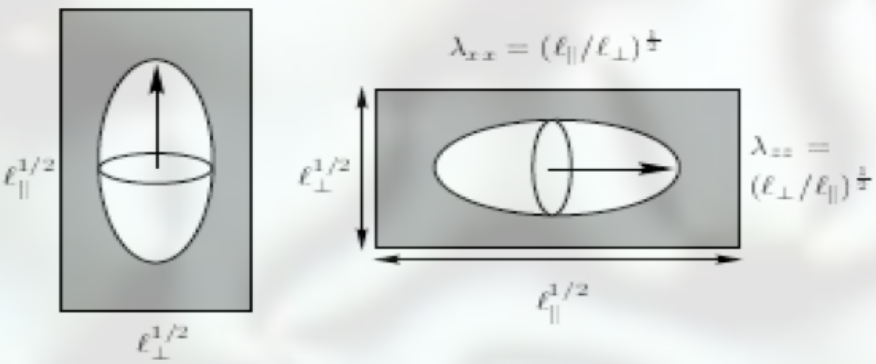
H. Finkelmann, Shahinpoor, et al

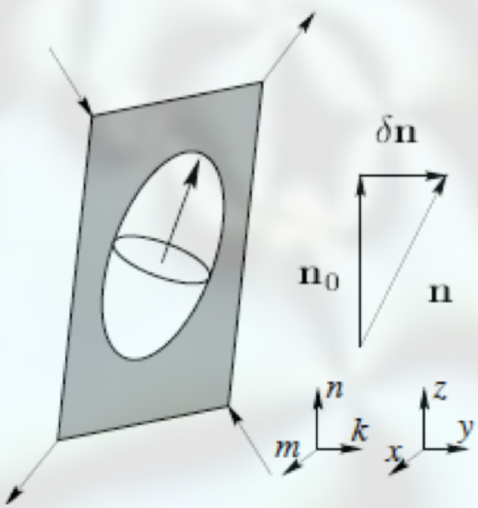
Visualization of soft deformation

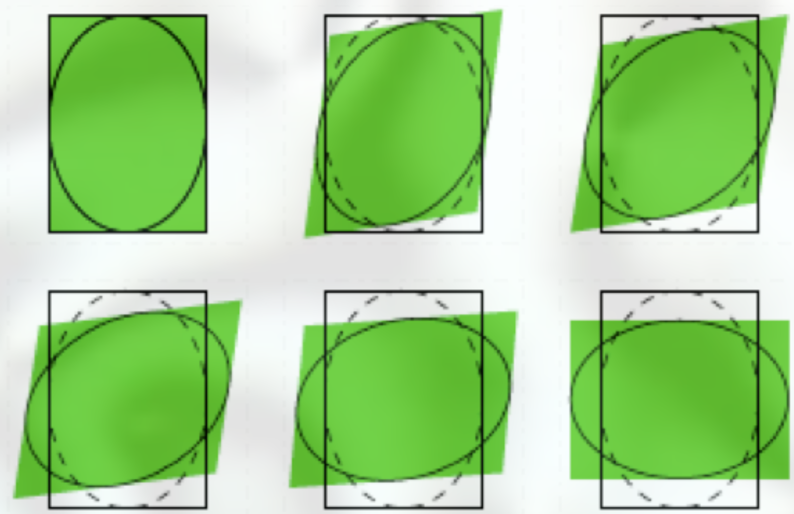


a "liquid" solid <--> a "solid" liquid crystal









Warner, Terentjev '90
Olmsted '94

Elastic theory of NE

Xing + L.R., PRL, EPL, AOP
Lubensky + Stenull, EPL (2003)

- Construct rotationally invariant elastic theory of deformations about $\underline{\underline{u}}_0$
- Study fluctuations and heterogeneities about $\underline{\underline{u}}_0$

Must incorporate underlying rotational invariance of the nematic state

→ *some distortions cost no energy:* ***"soft" uniaxial solid***

$$f[\vec{R}(\mathbf{x})] = f[O_T \vec{R}(O_R \mathbf{x})]$$

$$u' \approx \frac{(r-1)}{2\sqrt{r}} \begin{pmatrix} 0 & \theta \\ \theta & 0 \end{pmatrix} = (\Lambda_0^T)^{-1}\, \delta u\, \Lambda_0^{-1}$$

- Vanishing energy cost for: $\delta \underline{\underline{u}} = \underline{\underline{O}} \cdot \underline{\underline{u}}_0 \cdot \underline{\underline{O}}^T - \underline{\underline{u}}_0$

- *Harmonic* elasticity about nematic state: $\underline{\underline{\varepsilon}} = \underline{\underline{u}} - \underline{\underline{u}}_0$

$$\mathcal{H}^0_{NE} = \mu_{zi}\varepsilon^2_{zi} + B_z \varepsilon^2_{zz} + \mu_\perp \varepsilon^2_{ij} + \lambda \varepsilon^2_{ii} + \lambda_{zi}\varepsilon_{zz}\varepsilon_{ii}$$

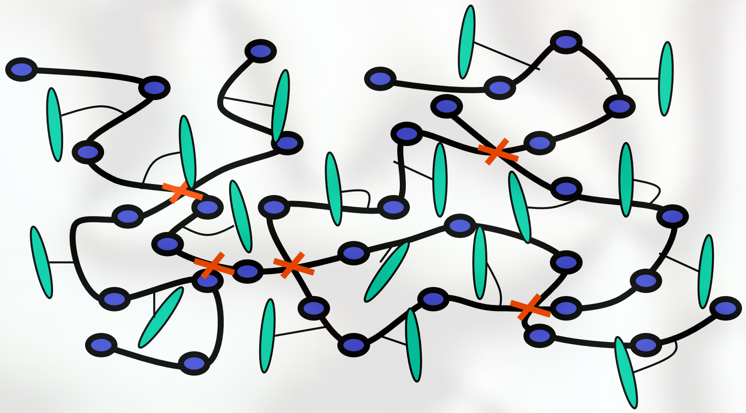
$\mu_{zi} \to 0$, *required by rotational invariance*

- *Nonlinear* elasticity about nematic state:

$$\mathcal{H}_{NE} = B_z w^2_{zz} + \mu_\perp w^2_{ij} + \lambda w^2_{ii} + \lambda_{zi} w_{zz} w_{ii}$$

$$w_{zz} = \partial_z u_z + \frac{1}{2}(\nabla u_z)^2 \qquad w_{ij} = \frac{1}{2}(\partial_{(i}u_{j)} - \partial_i u_z \partial_j u_z)$$

Fluctuations and heterogeneity



- Thermal fluctuations: $\mathcal{Z} = \mathrm{Trace}_u[e^{-\beta\mathcal{H}[u]}]$

- Heterogeneity ⇒ random torques and stresses:

nematic elastomers are only statistically homogeneous and isotropic

$$\mathcal{H}^{real}_{NE} = \mathcal{H}_{NE}[\underline{\underline{u}}] - \underbrace{\underline{\underline{u}} \cdot \underline{\underline{\sigma}}(\mathbf{r}) - (\hat{n} \cdot \vec{g}(\mathbf{r}))^2}_{\text{encodes heterogeneity}}$$

Elastic "softness" leads to strong qualitative effects of thermal fluctuations and network heterogeneity

Predictions

Xing + L.R., PRL (2003)

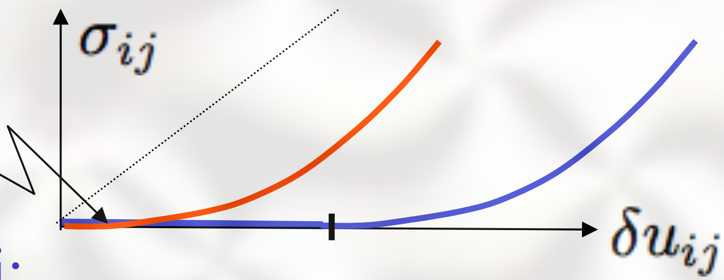
- *Universal* elasticity: $\overline{\langle|\delta u(q)|^2\rangle} \sim q_\perp^{-4+\eta}$, for $r_\perp > \xi_\perp \sim K^2/\Delta$

- Non-Hookean elasticity: $\sigma_{zz} \sim (u_{zz})^\delta \; , \;\; \delta > 1$

(cf. non-Fermi liquid)

vanishing slope

no linear response

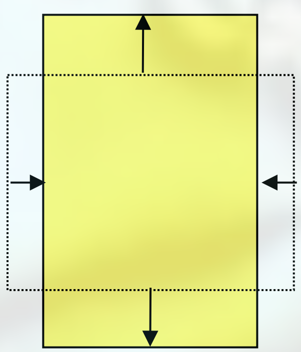


- Length-scale dependent elastic moduli:

$$K_{\text{eff}}(L) \sim L^\eta, \quad \mu_{\text{eff}}(L) \sim L^{-\eta_\mu}, \quad B_{\text{eff}}(L) \sim B_0$$

- Macroscopically incompressible: $\kappa_{\text{eff}} \sim \mu_{\text{eff}}(L)/B_{\text{eff}}(L) \to 0$

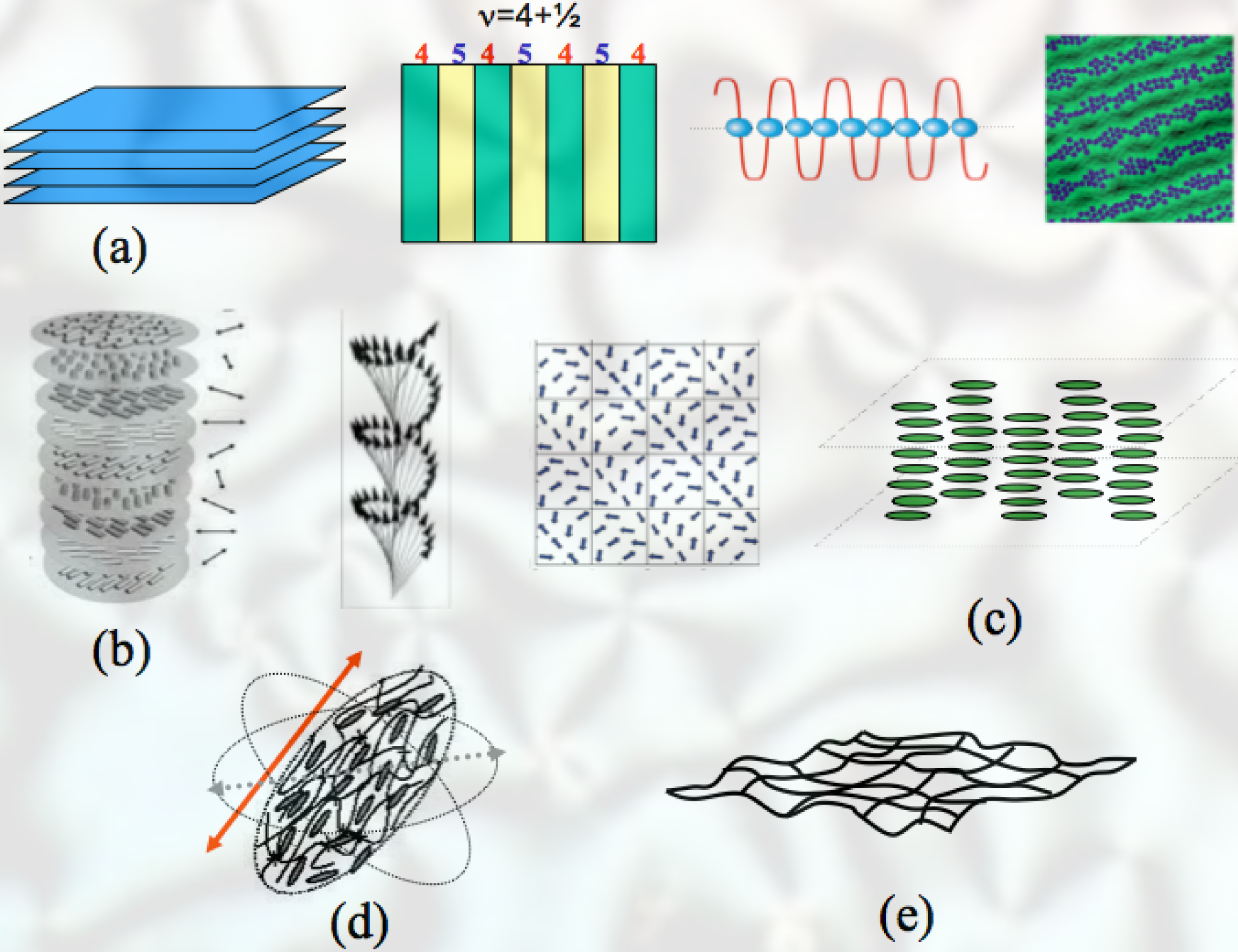
- Universal Poisson ratios:



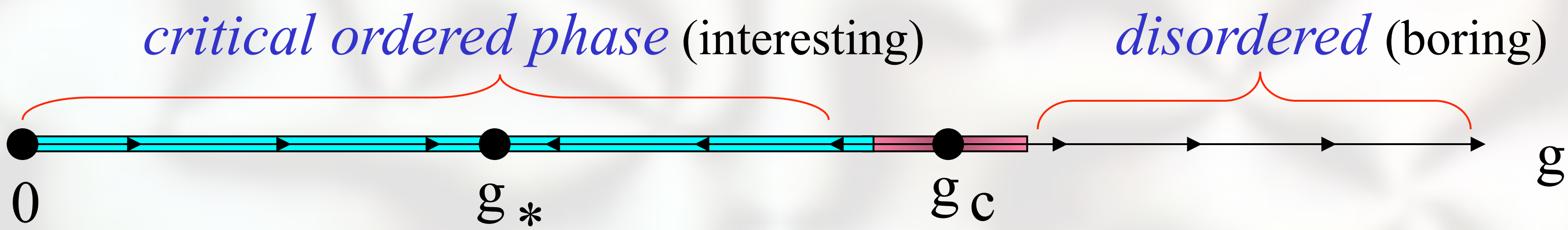
$$u_{xx} > 0 \Longrightarrow \begin{cases} u_{yy} = \frac{5}{7} u_{xx} \\ u_{zz} = -\frac{12}{7} u_{xx} \end{cases}$$

$$u_{zz} > 0 \Longrightarrow u_{xx} = u_{yy} = -\frac{1}{2} u_{zz}$$

Critical soft matter



Properties of critical phases



$$H = (\nabla\phi)^2 + \phi^4$$

- *spontaneously broken continuous symmetry*
- *nontrivial fixed point of strongly interacting Goldstone modes (c.f. nonlinear O(N) sigma-model)*
- *universal power-law correlation functions and amplitude ratios (throughout the phase)*
- *no fine-tuning to a critical point required*
- *quantum analogs? road to 3d "Luttinger liquids"?*