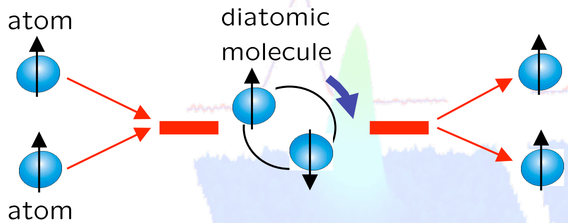
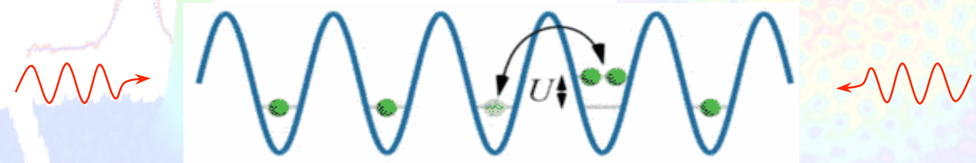
BCS-BEC crossover and phase transition in optical lattices

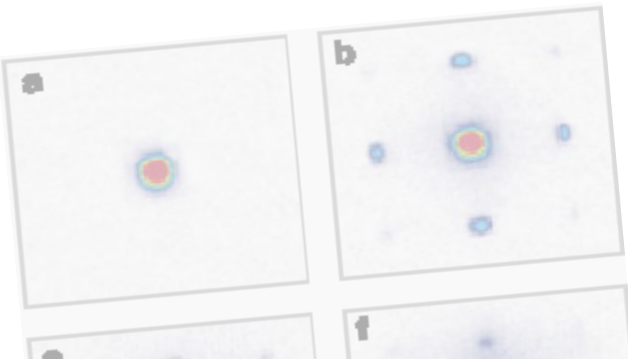


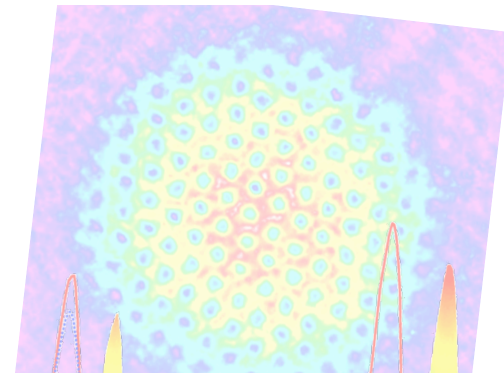


for details see *(many-body)*: *Shen, L.R., Gurarie, PRL '12*
also see *(two-body)*: *von Stecher, Gurarie, L.R., Rey, PRL '11*

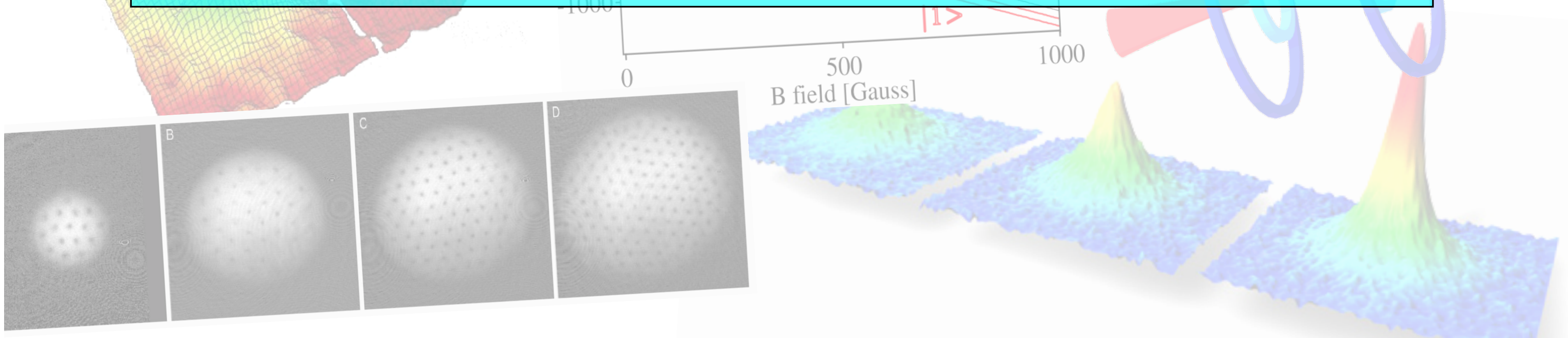
NewSpin3, Mainz, Germany, April 4 2013

Outline



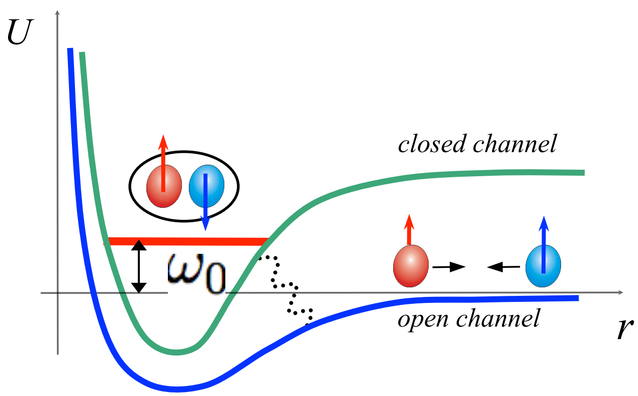


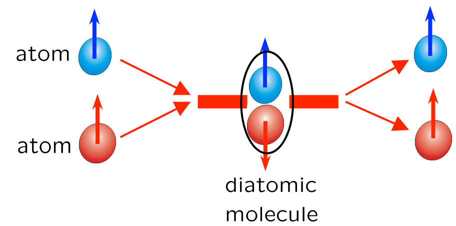
- routes to correlated atomic gases
  - Feshbach resonance
  - optical lattices
- *two-body* Feshbach resonance on a *lattice*
- *many-body* Feshbach resonance on a *lattice*
  - *reentrant* BCS-BEC crossover superfluidity
  - resonant superfluid-insulator transition

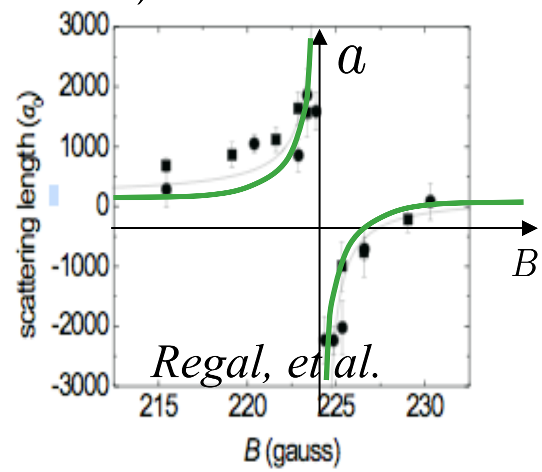


Strong correlations via Feshbach resonance

- tunability (strength and sign) of interactions (sudden and adiabatic)







- s-wave BCS-BEC superfluidity
- p-wave superfluidity *(see e.g., Gurarie and LR, AOP 2007)*
- polarized superfluidity *(see e.g., Sheehy and LR, AOP 2007)*

…quite well understood:

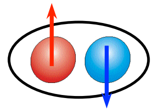
- *quantitatively for narrow (Γ/ε_F <<1) resonance*
- *qualitatively for broad (Γ/ε_F >>1) resonance*

mft, 1/N, ε-expansions $\longrightarrow$ universality

(Veillette, Sheehy, LR '07; Nikolic, Sachdev '07; Nishida, Son '06)

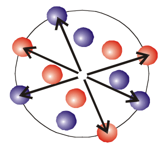
S-wave resonant fermionic superfluidity

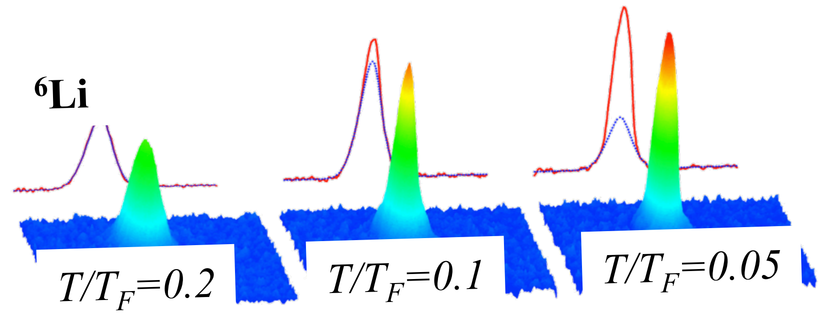
- molecular BEC *(Grimm, Jin '03)*



K_2 , Li_2

- BCS superfluid *(Jin '04 Ketterle '04)*





- BCS-BEC crossover

atom-molecule hybridization

$$\boxed{\mathcal{H}_{2ch} = \psi_\sigma^\dagger\left(\frac{\hat{p}^2}{2m} - \mu\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} - 2\mu + \epsilon_0\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger}$$

fermionic open-channel atoms

bosonic closed-channel molecules

S-wave resonant fermionic superfluidity

$$\boxed{\mathcal{H}_{2ch} = \psi_\sigma^\dagger\left(\frac{\hat{p}^2}{2m} - \mu\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} - 2\mu + \epsilon_0\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger}$$

dimensionless coupling: $\boxed{\gamma \sim \left(\frac{g\sqrt{n}}{\epsilon_F}\right)^2 \sim \frac{g^2}{\epsilon_F^{1/2}} \sim \frac{1}{r_0 n^{1/3}}}$

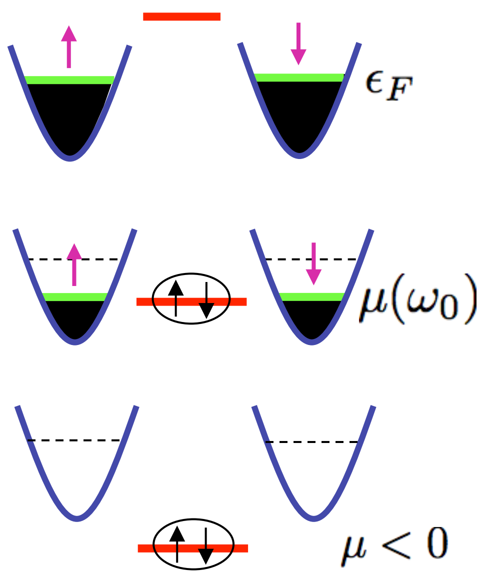
$\gamma_{202G}^{^{40}K} \approx 5,\ \Delta B \sim 1G \sim 100\mu K$

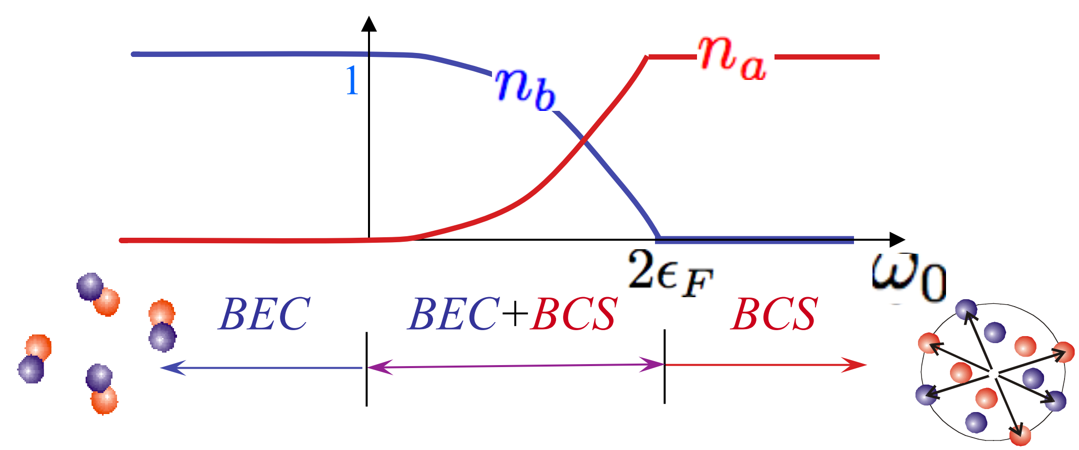
$\gamma_{544G}^{^{6}Li} \approx 0.1,\ \Delta B \sim 0.1G \sim 10\mu K$

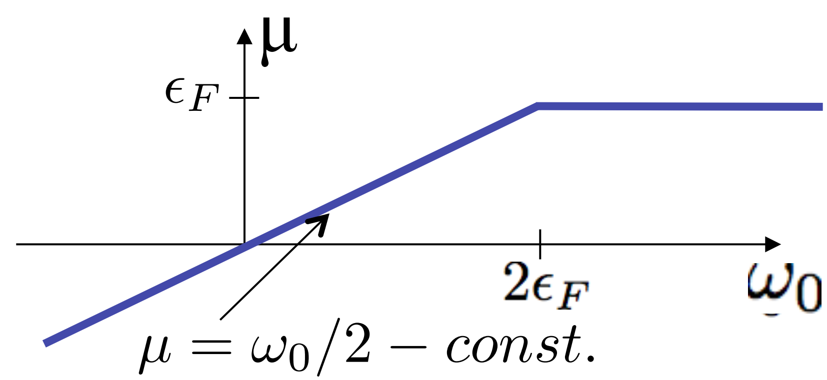
$\epsilon_F \sim 1\mu K$

- **narrow** resonance $\gamma \ll 1 \ \rightarrow\ \mathrm{MFT}: \phi(x) = B$

  Gurarie and L.R., AOP 2007







S-wave resonant fermionic superfluidity

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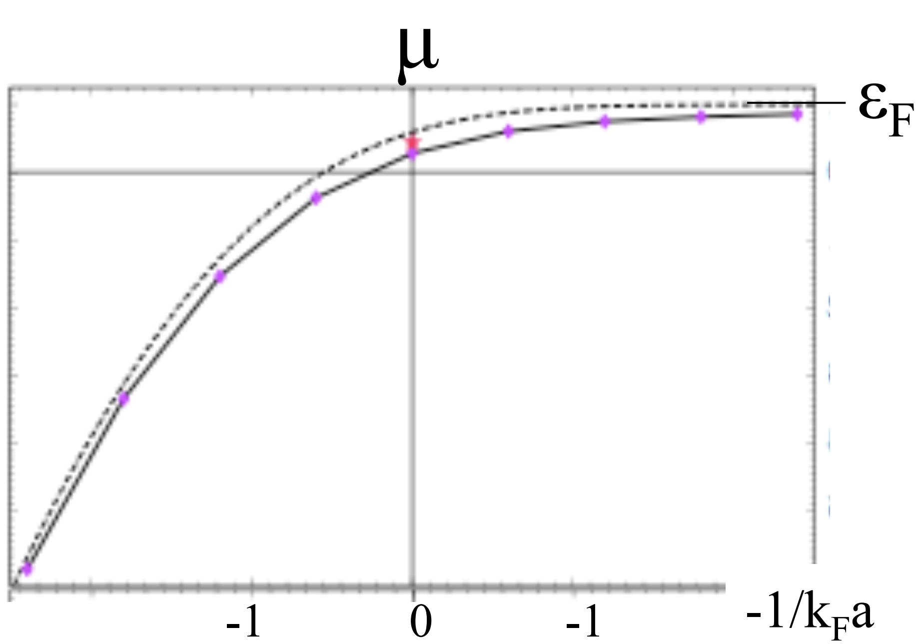
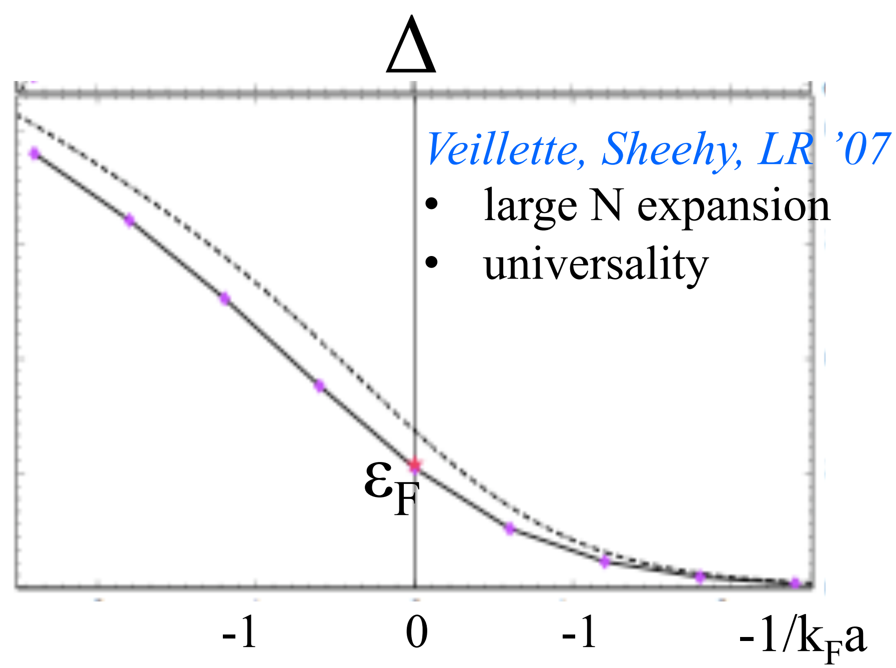
- **narrow** resonance $\quad \gamma \ll 1 \ \rightarrow\ \mathrm{MFT}: \phi(x) = B$

  Gurarie and L.R., AOP 2007

- **broad** resonance $\quad \gamma \gg 1 \Longrightarrow \mathcal{H}_{1ch} = \psi_\sigma^\dagger\left(\frac{p^2}{2m} - \mu\right)\psi_\sigma + \lambda\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$

  strongly coupled ϕ and ψ $\Longrightarrow$ *MFT uncontrolled*

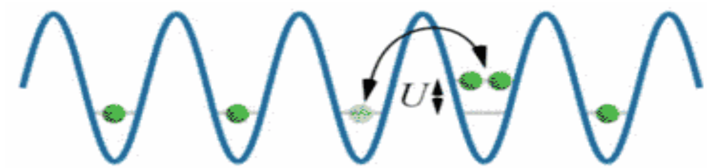




Strong correlations via optical lattices

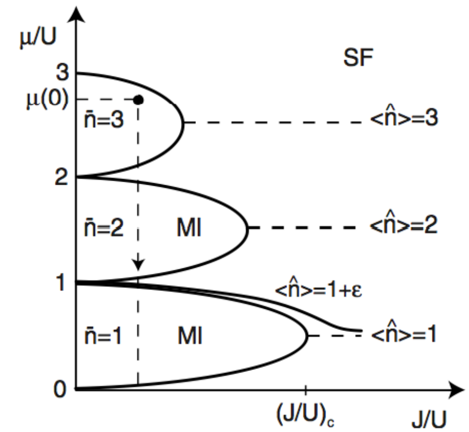
- interfering laser beams *(A. Ashkin'80; I. Bloch '98)*

ac-Stark effect

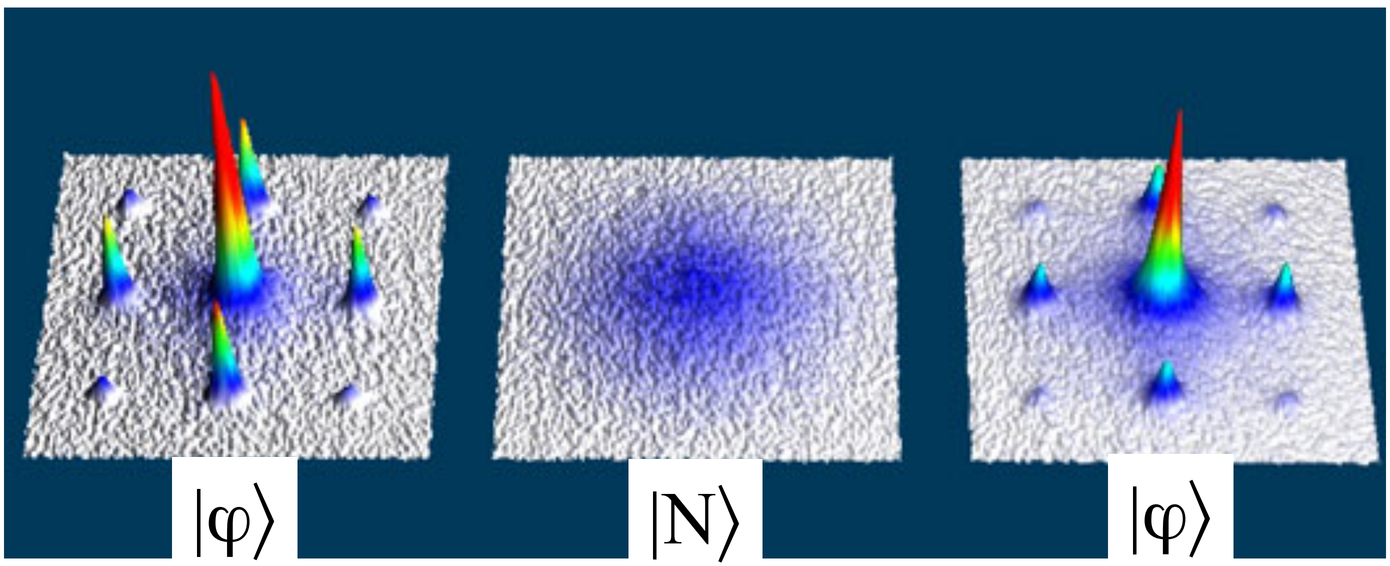


- superfluid-insulator transition of bosons

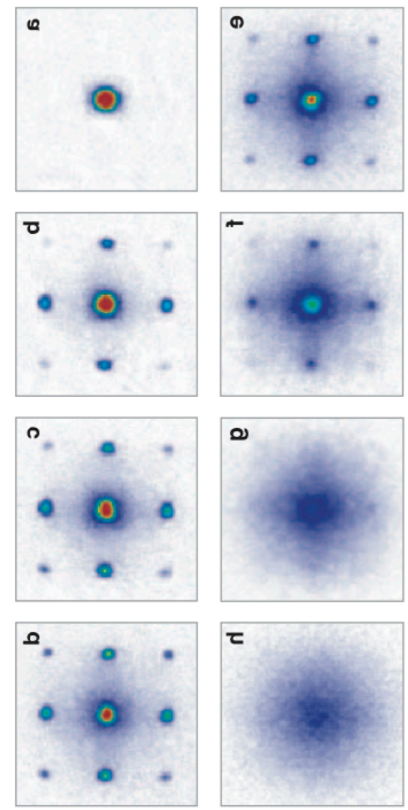
(Doniach'81, Fisher, et al. '89)



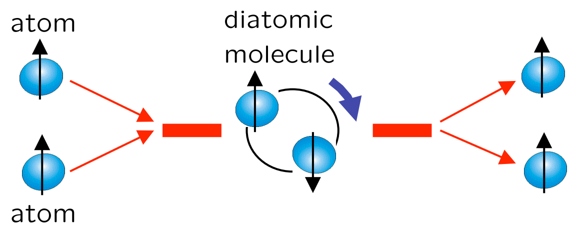
realized in cold atoms *(M. Greiner, et al., '01, Jaksch, et a '98)*

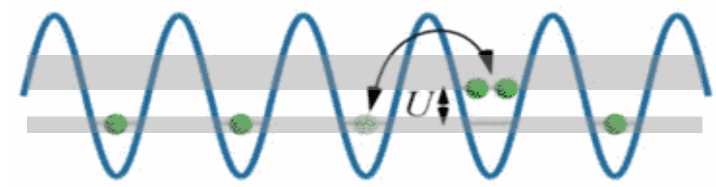


SF MI SF



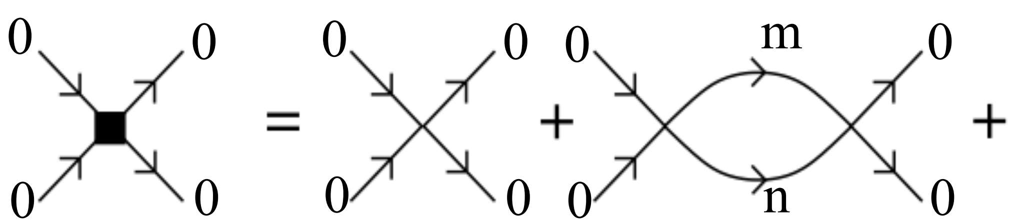
Resonant fermions in a periodic potential





Fedichev, et al., ’04
Zwerger, ’04
Sa de Melo, et al ‘05
Stoof, et al ‘06
Zhai, Ho ’07
Buchler, ’10
Cui, et al., ’10
von Stecher, et al., ‘11

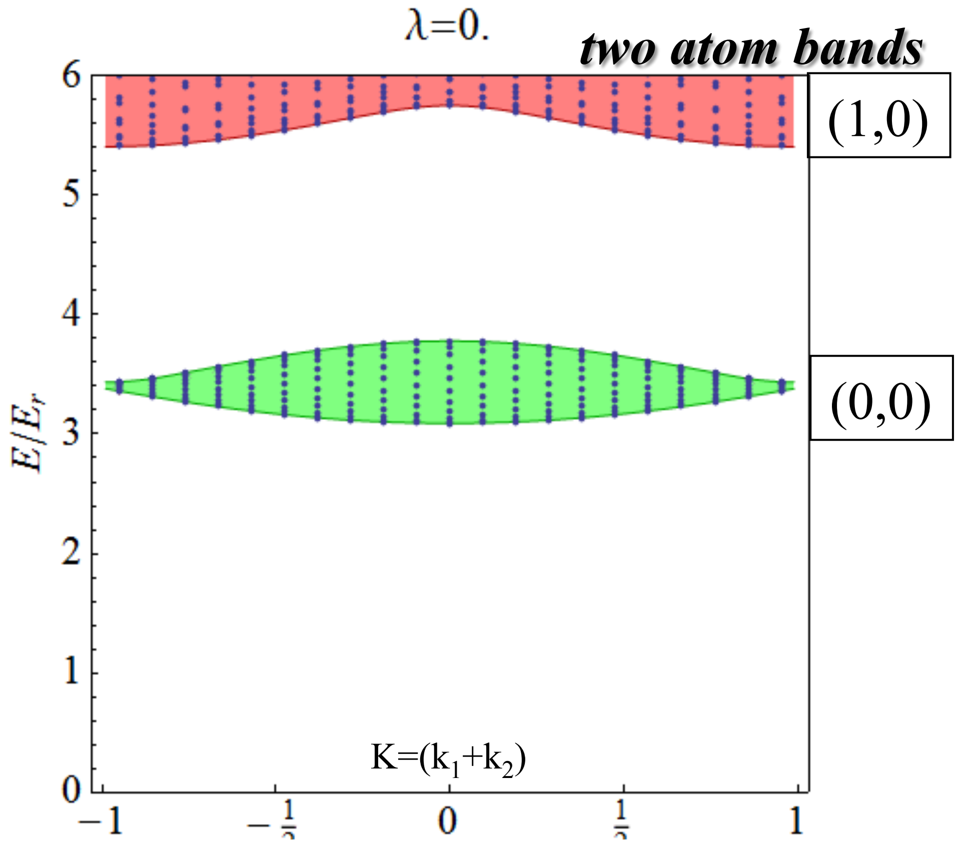
- 2-body problem:



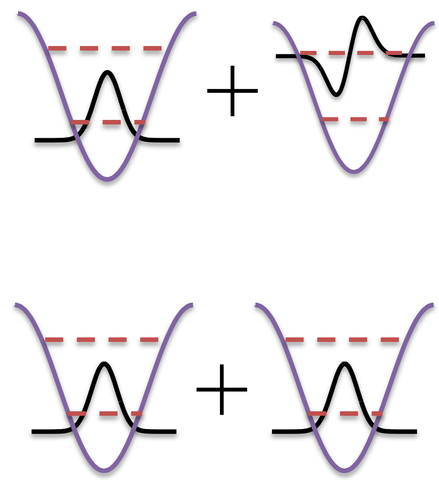
$$T = \lambda \cdot (1 - \lambda \cdot \Pi)^{-1}$$

Band hybridization: periodic potential

two atoms in a periodic potential with attractive interactions λ in 1D for a lattice with $V_o = 4E_r$:



Center of mass momentum in units of π/l



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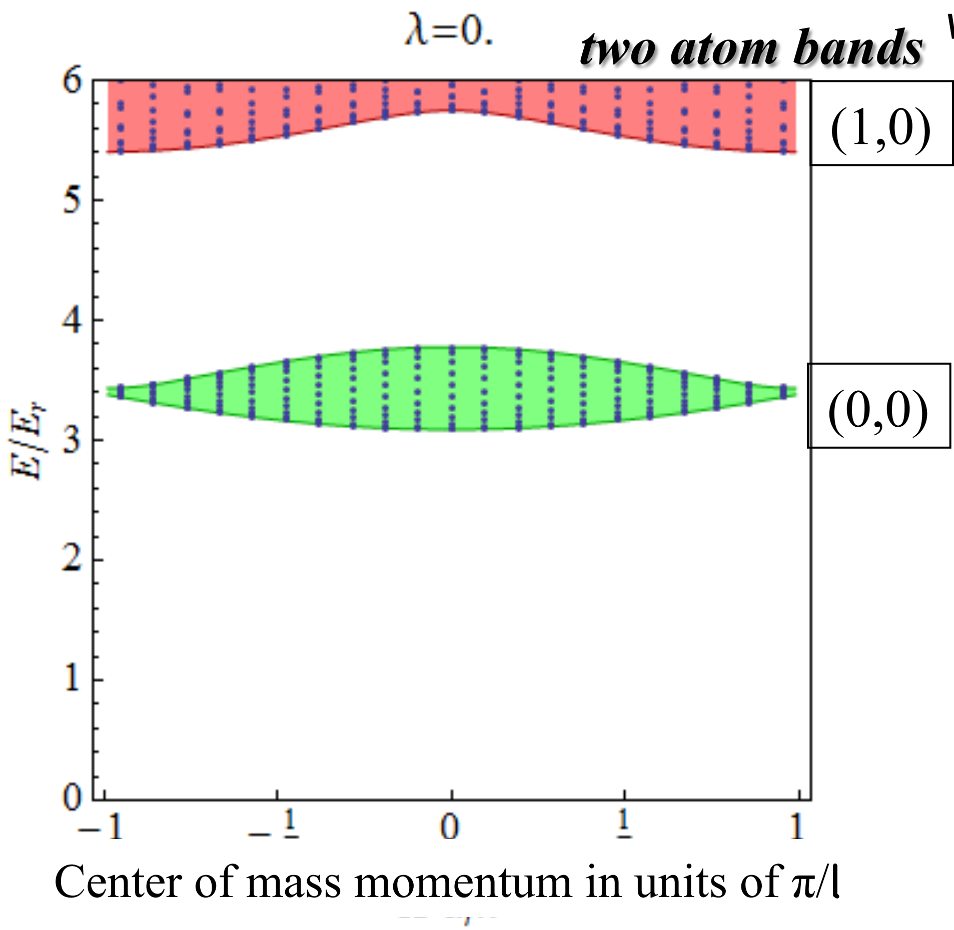
von Stecher, Gurarie, L.R., Rey, PRL '11



- *band hybridization*
- *lattice induced resonances*
- *K-dependent binding*
- *threshold-free pairing at BZ edges*

Band hybridization: periodic potential

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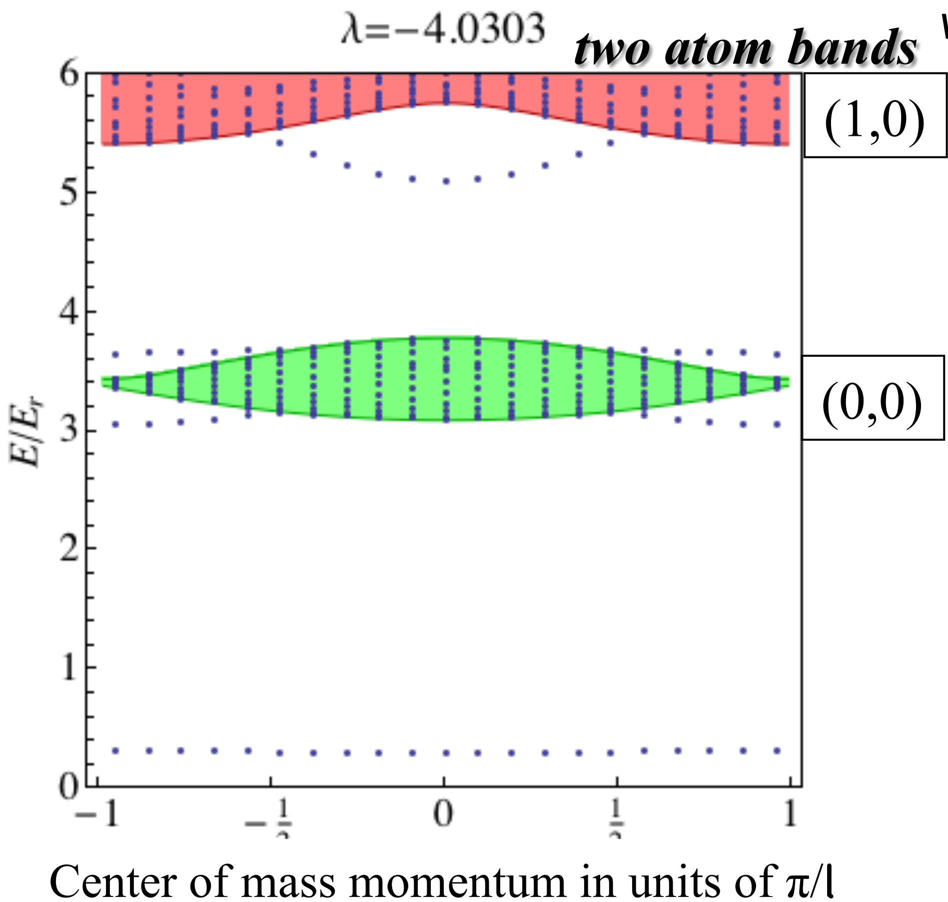
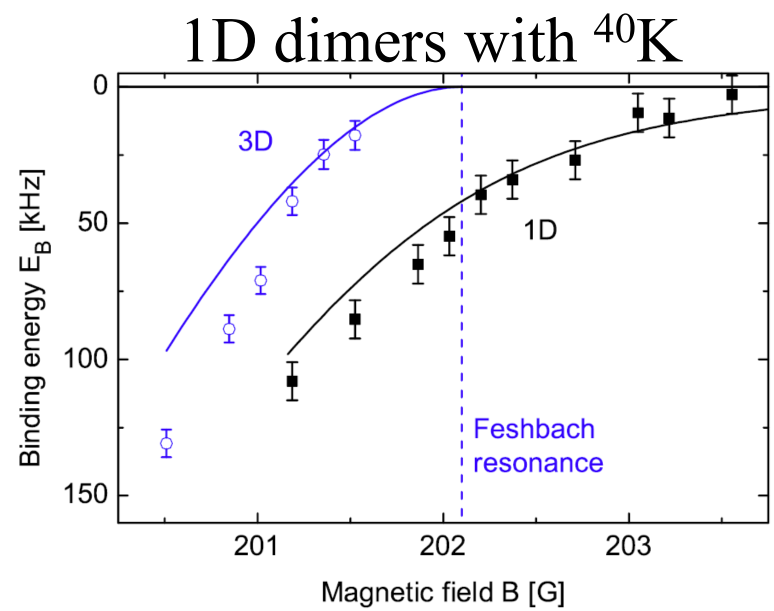
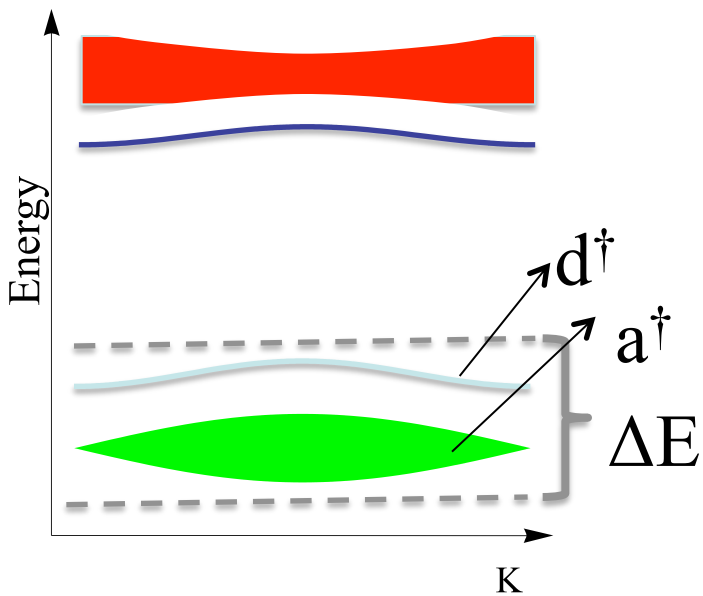


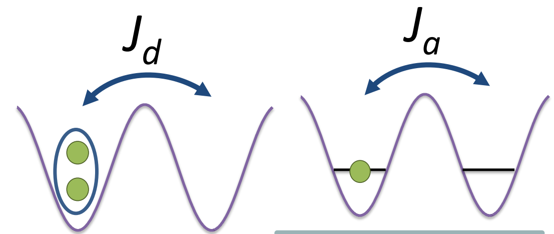


H. Moritz, ...,T. Esslinger PRL 2005

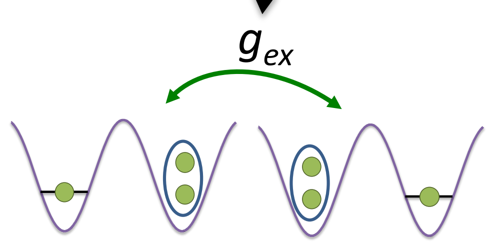
Effective 2-channel model

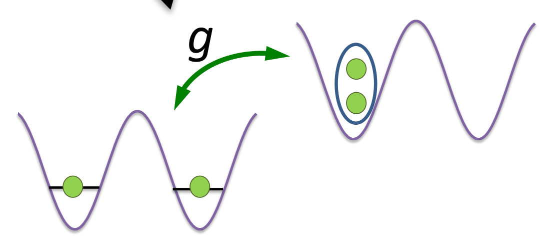
von Stecher, Gurarie, L.R., Rey, PRL '11





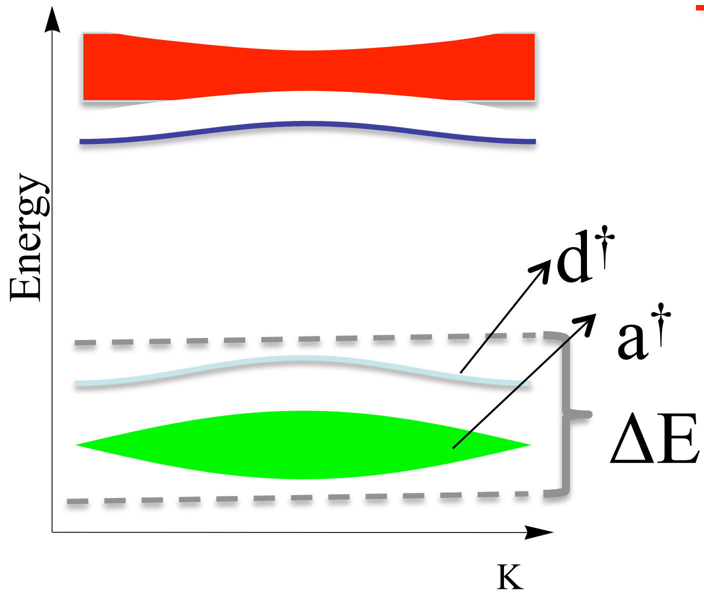
$$H_{eff} = \mathcal{P}_{ad}^{\dagger} \left(\sum_i \epsilon_d d_i^\dagger d_i - \boxed{\sum_{\langle i,j\rangle} J_d d_i^\dagger d_j} + \boxed{\sum_{\langle i,j\rangle} -J_a a_i^\dagger a_j} + \sum_{\langle i,j\rangle} g_{ex,i-j} d_i^\dagger d_j a_j^\dagger a_i + \sum_{\langle i,j\rangle} [g_{i-j} d_i^\dagger a_j a_i + \mathrm{H.c.}] \right) \mathcal{P}_{ad},$$



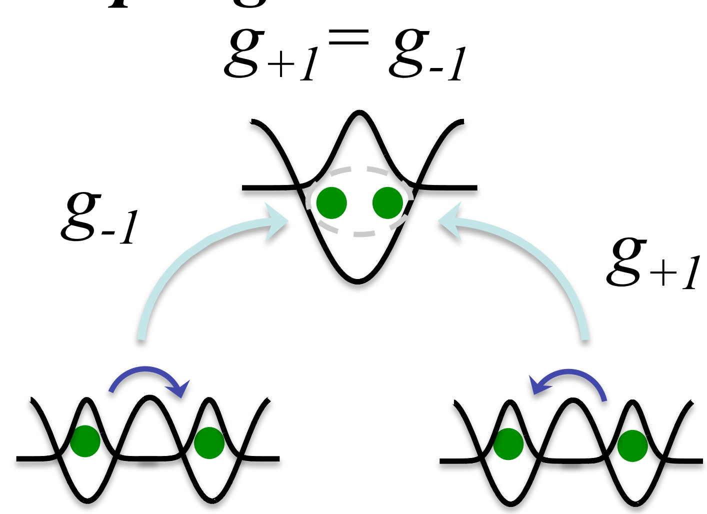


Parity: even

von Stecher, Gurarie, L.R., Rey, PRL '11



S coupling

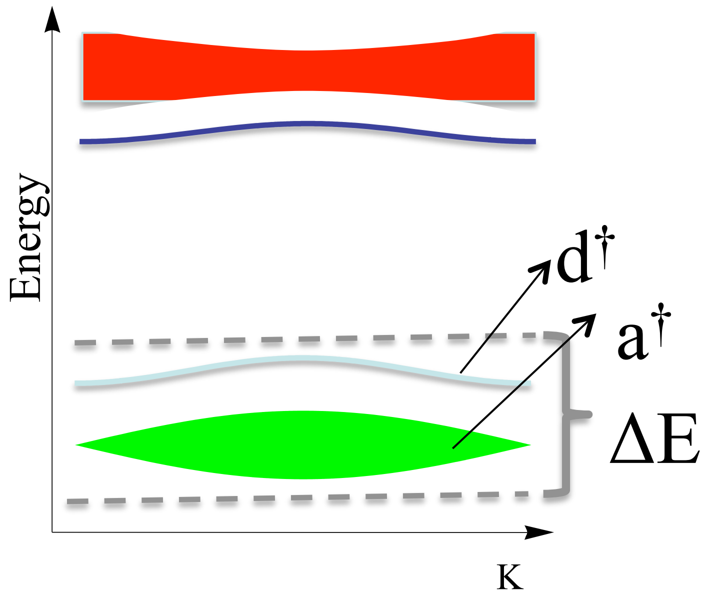


$$g_{ij} \sim g_{1D} \int W^*_{m,i}(r,0) w_{a,i}(r) w_{a,j}(r) dr$$

$$\boxed{g \sum_i d_i^\dagger a_i (a_{i+1} + a_{i-1}) + \mathrm{H.c.}}$$

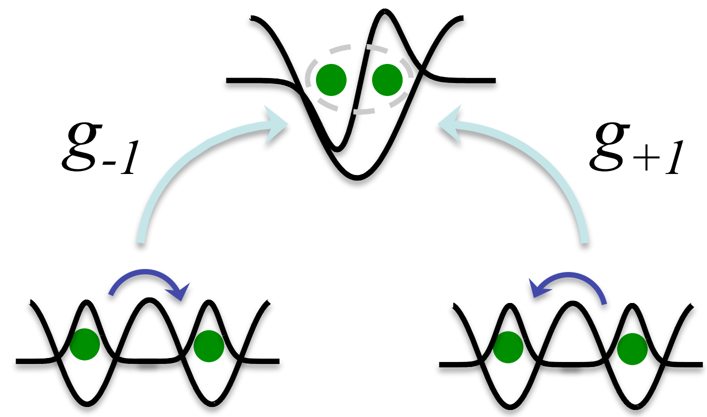
Parity: odd

von Stecher, Gurarie, L.R., Rey, PRL '11



AS coupling

$g_{+1} = -g_{-1}$



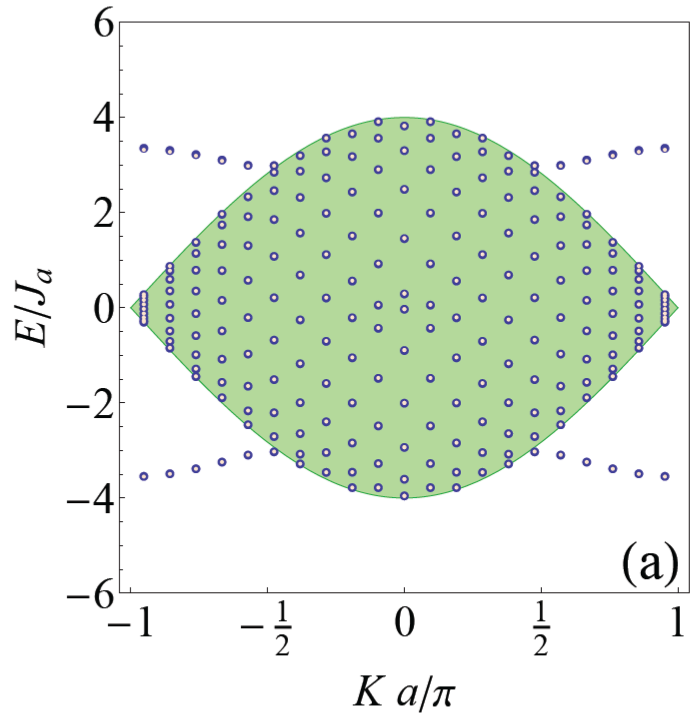
$$g_{ij} \sim g_{1D} \int W^*_{m,i}(r,0) w_{a,i}(r) w_{a,j}(r) dr$$

$$\boxed{g \sum_i d_i^\dagger a_i (a_{i+1} - a_{i-1}) + \mathrm{H.c.}}$$

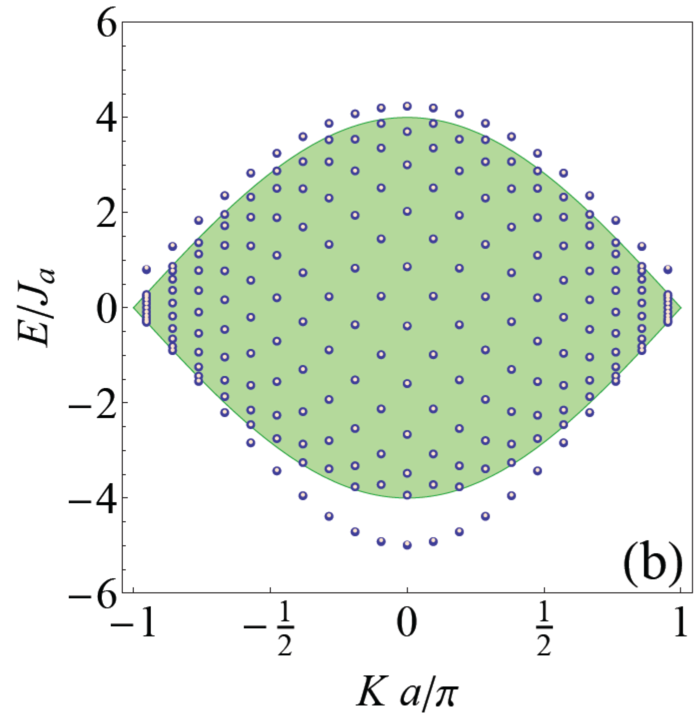
Parity: odd vs even dimers

von Stecher, Gurarie, L.R., Rey, PRL '11

(1,0) molecule: 1st excited



(2,0) molecule: 2nd excited



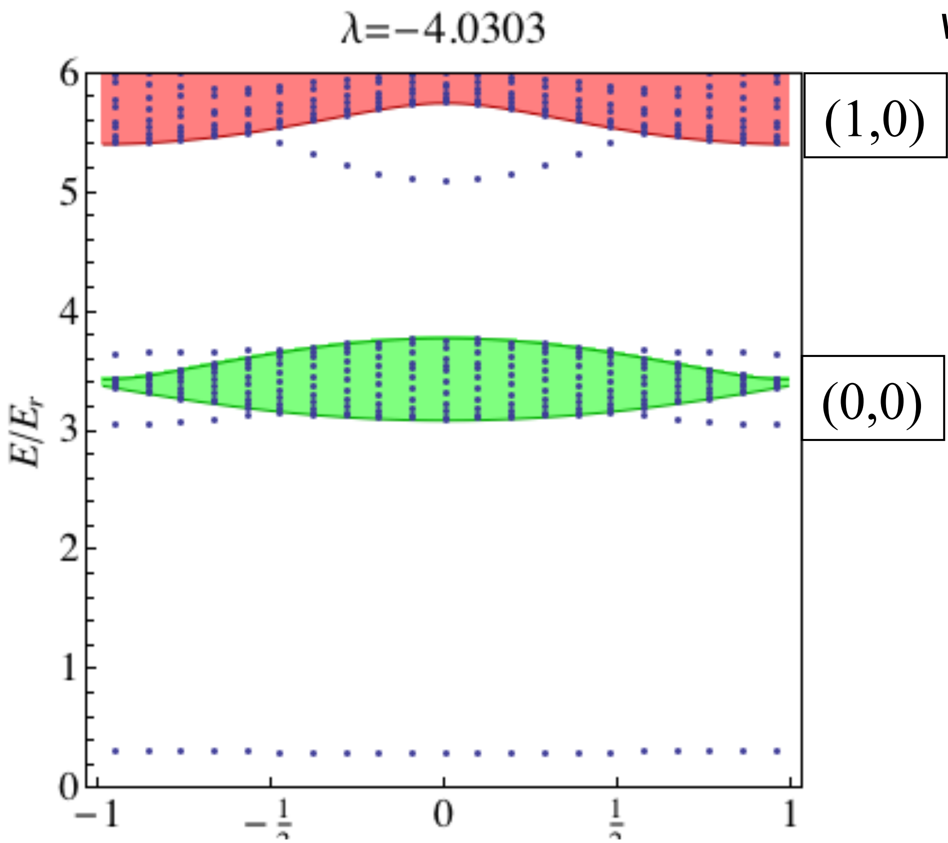
Molecules above and below!

21 sites and $V_0=20E_r$

Band hybridization: periodic potential

two atoms in a periodic potential with attractive interactions λ in 1D for a lattice with $V_o = 4E_r$:

von Stecher, Gurarie, L.R., Rey, PRL '11



Center of mass momentum in units of π/l

- *band hybridization*
- *lattice induced resonances*
- *K-dependent binding*
- *threshold-free pairing at BZ edges*

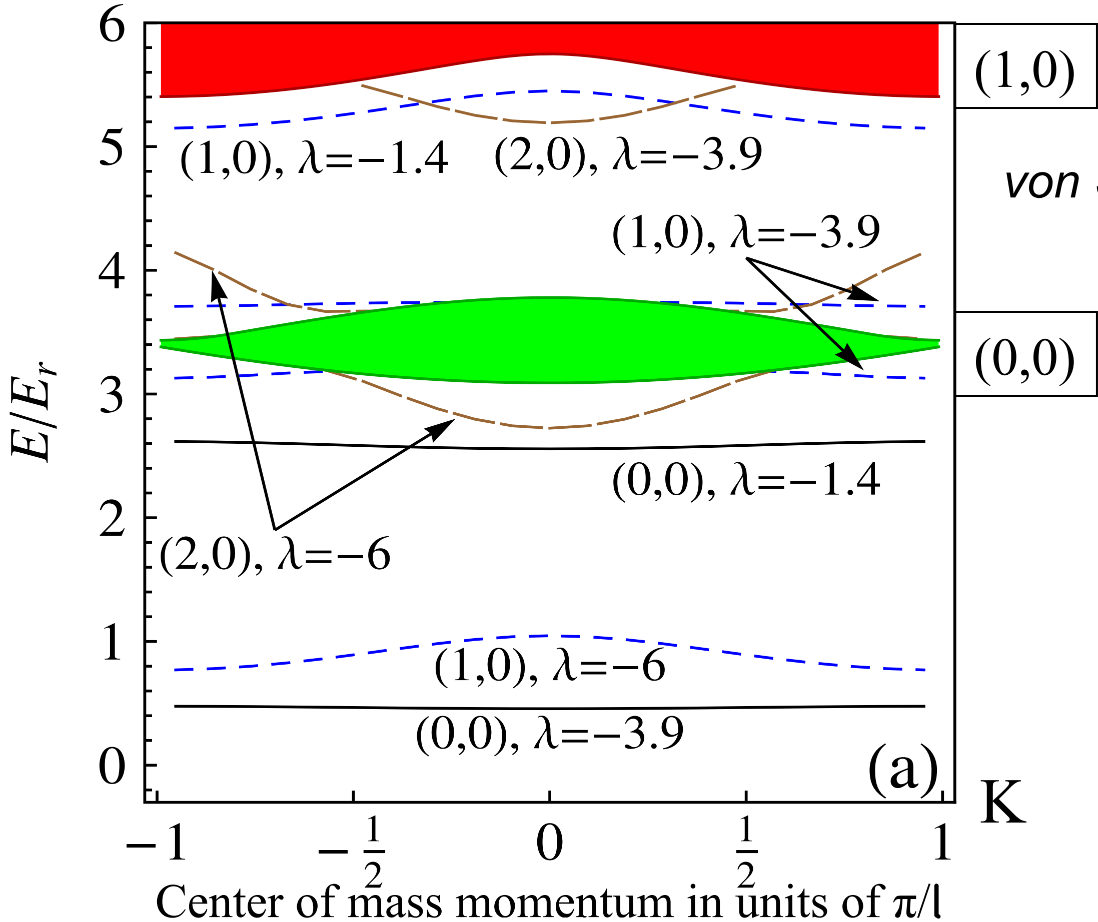
Is it possible to reduce to single band resonant lattice model?

yes: $U = \lambda / l_w^3 + \Pi_{n>0}$ ⟵ *need to convert λ to a_s*

Band hybridization: periodic potential

two atoms in a periodic potential with attractive interactions λ in 1D:

two particle bands (n,m) and bound states



von Stecher, Gurarie, L.R., Rey, PRL '11

- *band hybridization*
- *lattice induced resonances*
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yes: $U = \lambda / l_w^{\,3} + \Pi_{n>0}$ ⟵ *need to convert λ to a_s*

Reduction to single-band resonant model

- band width << band gaps ($E_{recoil} << E_{gap}$ $\longleftrightarrow$ $\ell \gg \ell_w$):

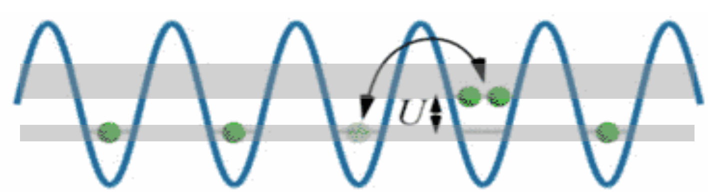
$$E_{gap} = \frac{\hbar^2}{2m\ell_w^2}$$

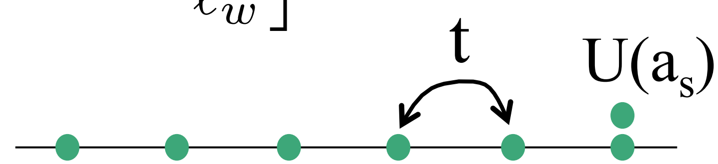
$$E_{recoil} = \frac{\hbar^2}{2m\ell^2}$$

$$U = \frac{4\pi a}{\ell_w^3 m}\left[1 - \frac{a}{\ell_w} f\left(\frac{E_{recoil}}{E_{gap}}, \frac{U_0}{E_{gap}}\right)\right]^{-1}$$

- interaction << band gaps ($U_0 << E_{gap}$) $\longrightarrow$ no band hybridization:

$$U = \frac{4\pi a}{\ell_w^3 m}\left[1 - const.\frac{a}{\ell_w}\right]^{-1}$$





$$H_{2ch} = \sum_{\langle \mathbf{r},\mathbf{r}'\rangle}\left(-tc^\dagger_{\mathbf{r},\sigma}c_{\mathbf{r}',\sigma} - t_b b^\dagger_{\mathbf{r}} b_{\mathbf{r}'}\right) + \sum_{\mathbf{r}}\left(\nu_0 b^\dagger_{\mathbf{r}} b_{\mathbf{r}} + g b^\dagger_{\mathbf{r}} c_{\mathbf{r},\uparrow}c_{\mathbf{r},\downarrow} + h.c.\right)$$

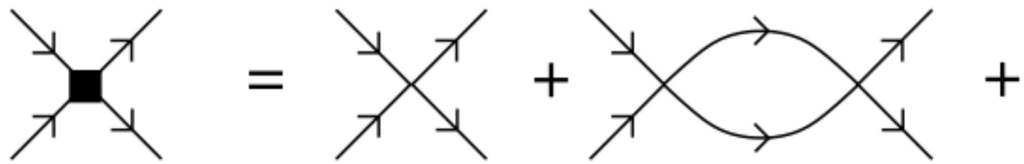
$for\ g \to \infty,\ \nu_0 \to \infty, with\ U \equiv -g^2/\nu_0\ fixed$

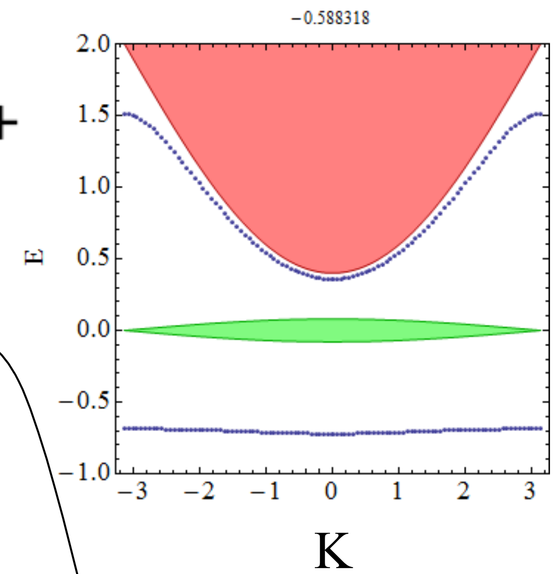
$$H_{1ch} = -t\sum_{\langle \mathbf{r},\mathbf{r}'\rangle} c^\dagger_{\mathbf{r},\sigma}c_{\mathbf{r}',\sigma} + U\sum_{\mathbf{r}} c^\dagger_{\mathbf{r},\uparrow}c^\dagger_{\mathbf{r},\downarrow}c_{\mathbf{r},\downarrow}c_{\mathbf{r},\uparrow}$$

Single-band resonant lattice model: **2-body**

$$H_{2ch} = \sum_{\langle \mathbf{r},\mathbf{r}'\rangle} \left(-t c^\dagger_{\mathbf{r},\sigma} c_{\mathbf{r}',\sigma} - t_b b^\dagger_{\mathbf{r}} b_{\mathbf{r}'}\right) + \sum_{\mathbf{r}} \left(\nu_0 b^\dagger_{\mathbf{r}} b_{\mathbf{r}} + g b^\dagger_{\mathbf{r}} c_{\mathbf{r},\uparrow} c_{\mathbf{r},\downarrow} + h.c.\right)$$

- T-matrix:

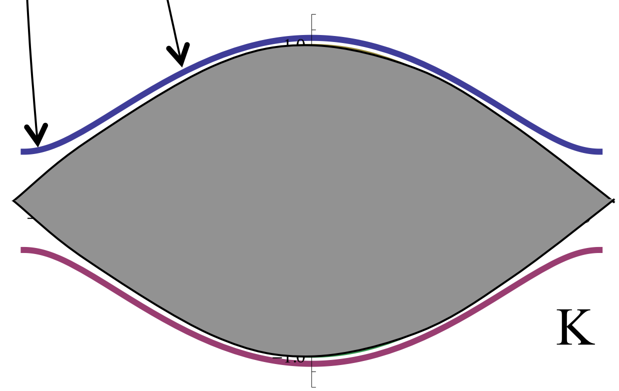




- lattice induced resonances *(F. Zhou)*
- repulsively bound pairs *(A. Rosch)*
- bound states at BZ boundary for arbitrary U

$$H_{1ch} = -t \sum_{\langle \mathbf{r},\mathbf{r}'\rangle} c^\dagger_{\mathbf{r},\sigma} c_{\mathbf{r}',\sigma} + U \sum_{\mathbf{r}} c^\dagger_{\mathbf{r},\uparrow} c^\dagger_{\mathbf{r},\downarrow} c_{\mathbf{r},\downarrow} c_{\mathbf{r},\uparrow}$$

$$\Pi(E, \mathbf{k}_1 + \mathbf{k}_2) = \int_{\mathbf{q}\in BZ} \frac{1}{E - \epsilon_{\mathbf{k}_1+\mathbf{q}} - \epsilon_{\mathbf{k}_2-\mathbf{q}}} = \frac{-1}{\sqrt{E^2 - 16t^2\cos^2\left(\frac{k_1+k_2}{2}\right)}}$$



$$E(K) = \sqrt{U^2 + 16t^2\cos^2\left(\frac{K}{2}\right)}, \quad \text{for } U > 0,$$

1D

$$= -\sqrt{U^2 + 16t^2\cos^2\left(\frac{K}{2}\right)}, \quad \text{for } U < 0$$

$$-2t\cos\left(\frac{K}{2}+k\right) - 2t\cos\left(\frac{K}{2}-k\right) = -4t\cos\left(\frac{K}{2}\right)\cos(k)$$

Single-band resonant lattice model: many-body

- two-channel model:

$$H_{2ch} = -t \sum_{\langle \mathbf{r},\mathbf{r}' \rangle,\sigma} c^\dagger_{\mathbf{r},\sigma} c_{\mathbf{r}',\sigma} - \mu \sum_{\mathbf{r},\sigma} c^\dagger_{\mathbf{r},\sigma} c_{\mathbf{r},\sigma} - t_b \sum_{\langle \mathbf{r},\mathbf{r}' \rangle} b^\dagger_{\mathbf{r}} b_{\mathbf{r}'}$$

$$+(\nu_0 - 2\mu) \sum_{\mathbf{r}} b^\dagger_{\mathbf{r}} b_{\mathbf{r}} + g \sum_{\mathbf{r}} \left(b^\dagger_{\mathbf{r}} c_{\mathbf{r},\uparrow} c_{\mathbf{r},\downarrow} + h.c. \right) .$$

- broad resonance, $\gamma >> 1$: $g \to \infty,\ \nu_0 \to \infty, with\ g^2/\nu_0\ fixed$

⟹ one-channel model:

$$H_{1ch} = -t \sum_{\langle \mathbf{r},\mathbf{r}' \rangle} c^\dagger_{\mathbf{r},\sigma} c_{\mathbf{r}',\sigma} + \sum_{\mathbf{r}} (-\mu n_{\mathbf{r}} + U n_{\mathbf{r}\uparrow} n_{\mathbf{r}\downarrow})$$

Resonant tight binding model: superfluidity

- narrow resonance ($\gamma << 1$):

closed-channel molecules

open-channel atoms

- *two*-channel:
  - number

$$n = 2|B|^2 + \int_{k\in BZ}\left(1 - \frac{\xi_k}{\sqrt{\xi_k^2 + g^2|B|^2}}\right)$$

  - gap

$$\nu_0 - 2\mu = \frac{g^2}{2}\int_{k\in BZ}\frac{1}{\sqrt{\xi_k^2 + g^2|B|^2}}$$

$$\epsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y + \cos k_z) \qquad \xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$$

- broad resonance (mft): $g \to \infty,\ \nu_0 \to \infty, with\ g^2/\nu_0, \Delta = gB\ fixed$

⟹ *one*-channel model:

  - number

$$n = \int_{k\in BZ}\left(1 - \frac{\xi_k}{\sqrt{\xi_k^2 + \Delta^2}}\right)$$

  - gap

$$\frac{1}{U} = \frac{1}{2}\int_{k\in BZ}\frac{1}{\sqrt{\xi_k^2 + \Delta^2}}$$

Resonant tight binding model: superfluidity

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- one-channel model:
  - number $$n = \int_{k \in BZ} \left(1 - \frac{\xi_k}{\sqrt{\xi_k^2 + \Delta^2}}\right)$$
  - gap $$\frac{1}{U} = \frac{1}{2} \int_{k \in BZ} \frac{1}{\sqrt{\xi_k^2 + \Delta^2}}$$

$\Longrightarrow$ Δ(U,n), μ(U,n)

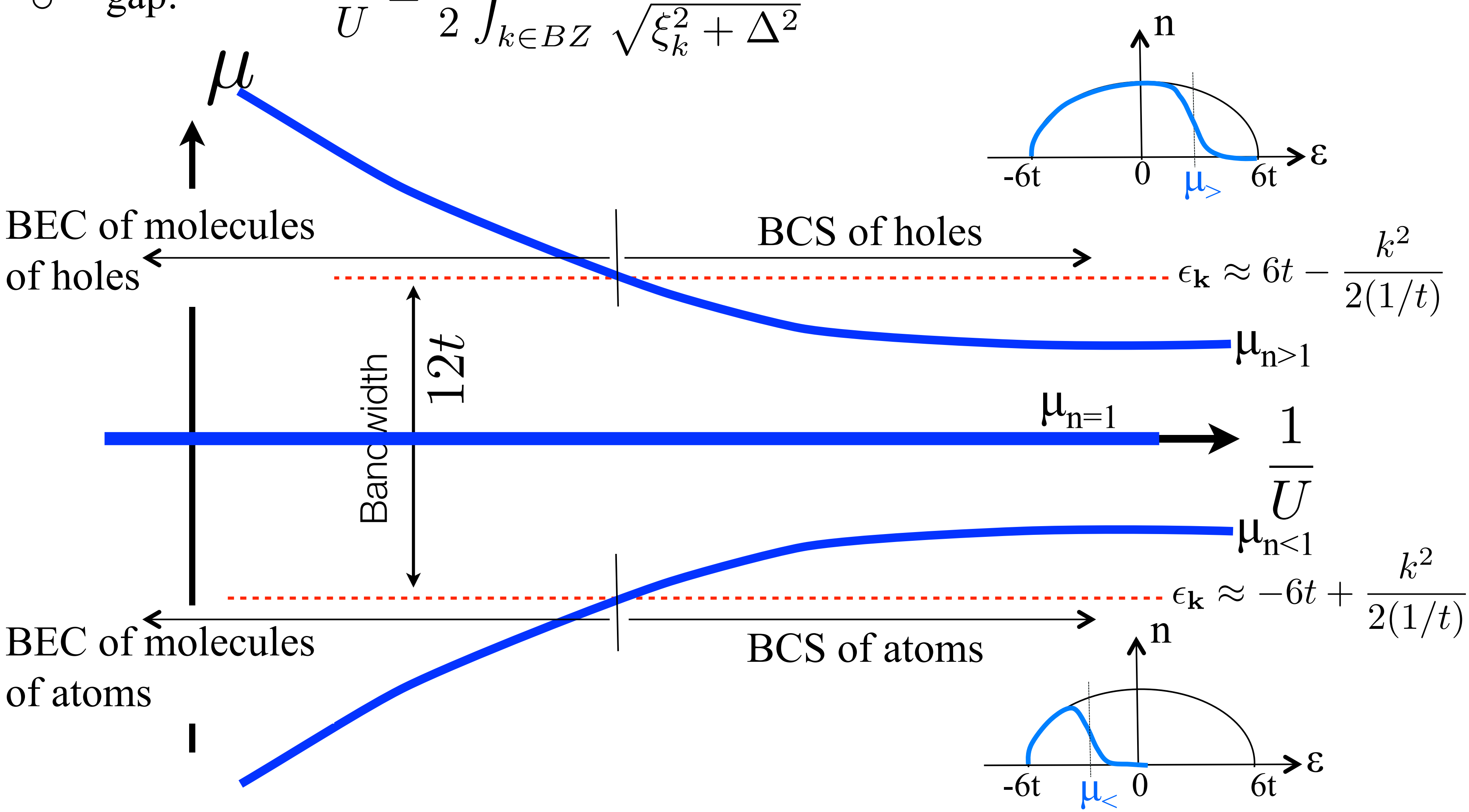
- particle-hole symmetry: $c^\dagger \to c, \; n \to 2 - n, \quad \epsilon_k \to -\epsilon_k, \; \mu = -\mu$
  - μ < 0, for n < 1 $\Longrightarrow$ *pairing of atoms*
  - μ = 0, for n = 1 $\Longrightarrow$ *no BCS-BEC crossover*
  - μ > 0, for n > 1 $\Longrightarrow$ *pairing of holes*

BCS-BEC crossover: one-channel model

- number: $$n = \int_{k \in BZ} \left(1 - \frac{\xi_k}{\sqrt{\xi_k^2 + \Delta^2}}\right)$$
- gap: $$\frac{1}{U} = \frac{1}{2} \int_{k \in BZ} \frac{1}{\sqrt{\xi_k^2 + \Delta^2}}$$

see, Sa de Melo, et al., PRB, '05

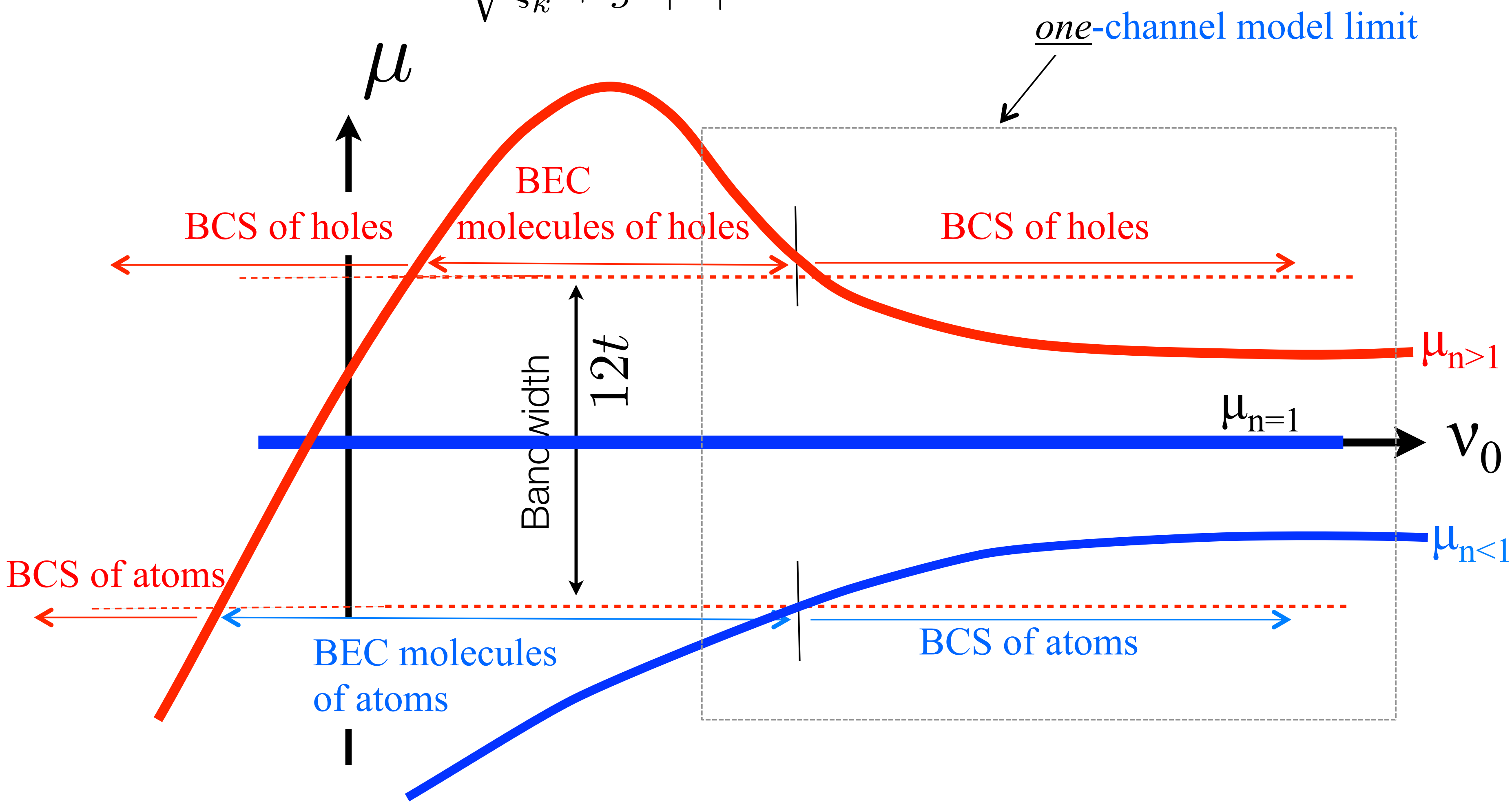
$\Longrightarrow$ Δ(U,n), μ(U,n)



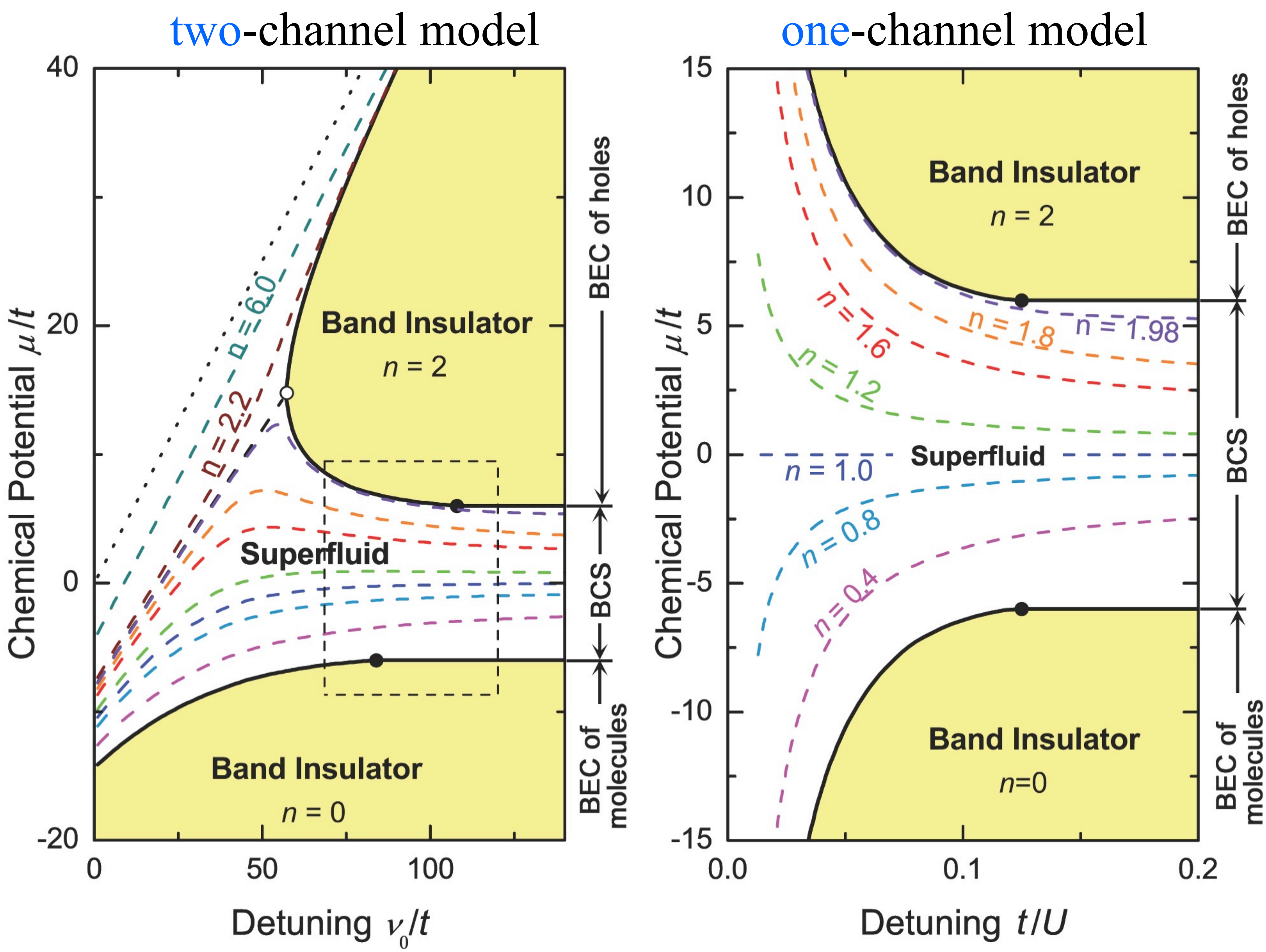
BCS-BEC crossover: two-channel model

$$n = 2\left|B\right|^2 + \int_{k\in BZ}\left(1 - \frac{\xi_k}{\sqrt{\xi_k^2 + g^2\left|B\right|^2}}\right)$$

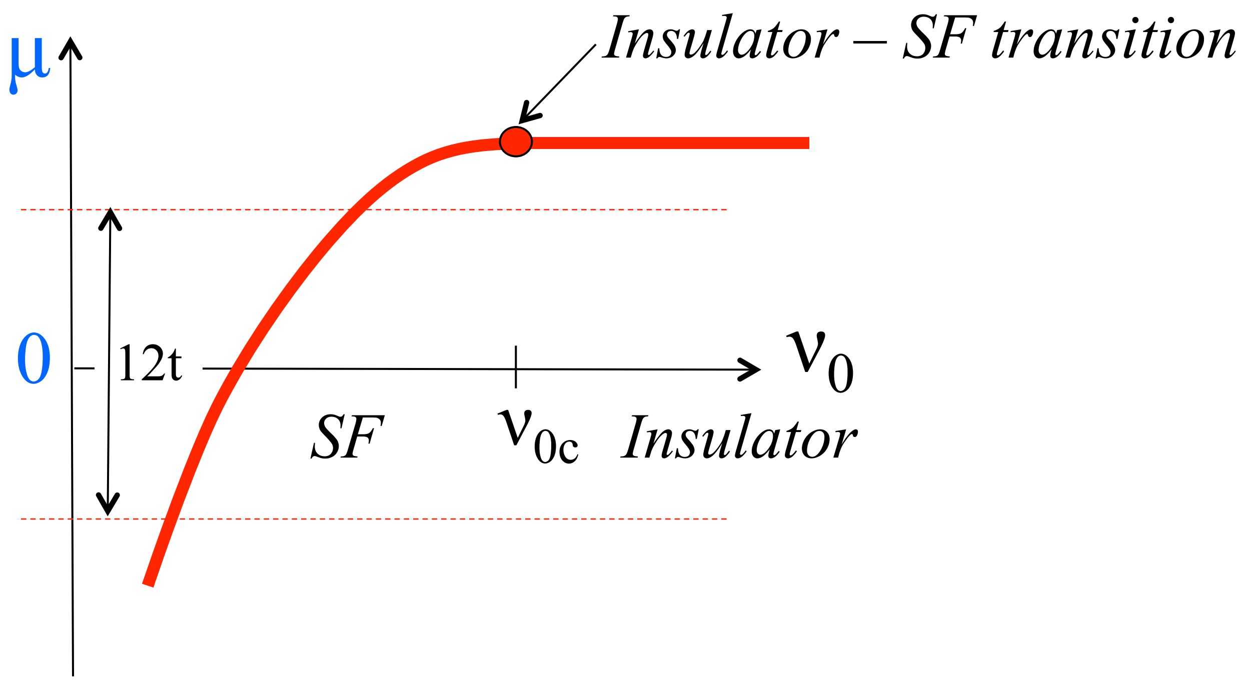
$$\nu_0 - 2\mu = \frac{g^2}{2}\int_{k\in BZ}\frac{1}{\sqrt{\xi_k^2 + g^2\left|B\right|^2}} \quad \Longrightarrow \quad \Delta(U,n),\ \mu(U,n)$$

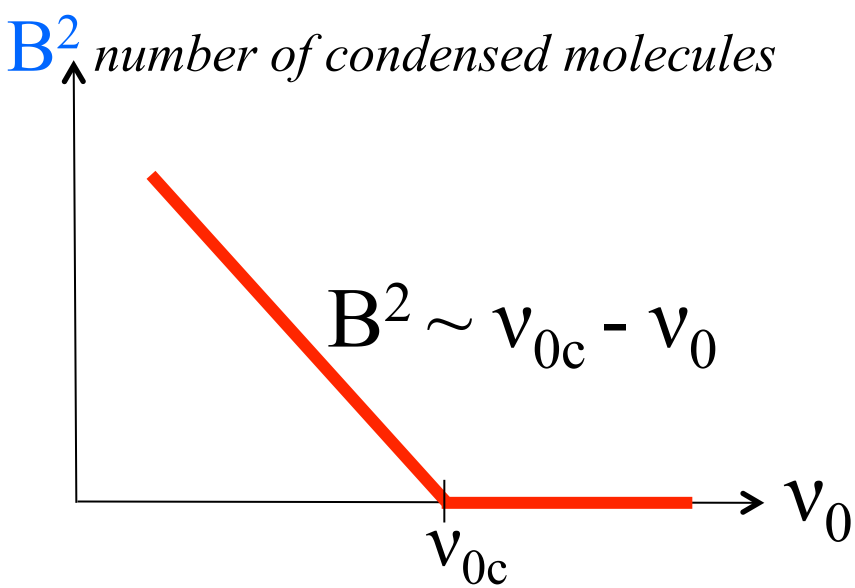


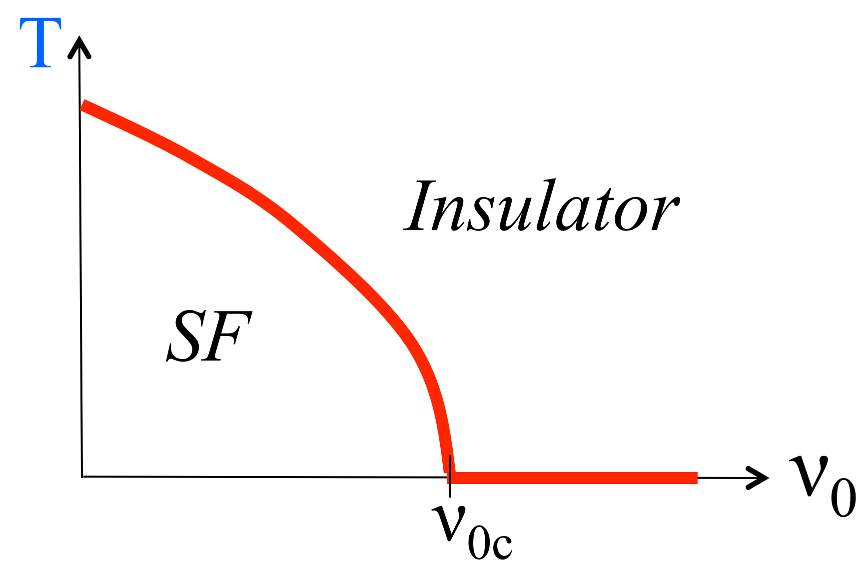
BCS-BEC phase diagram



SF-Insulator transition: n=2





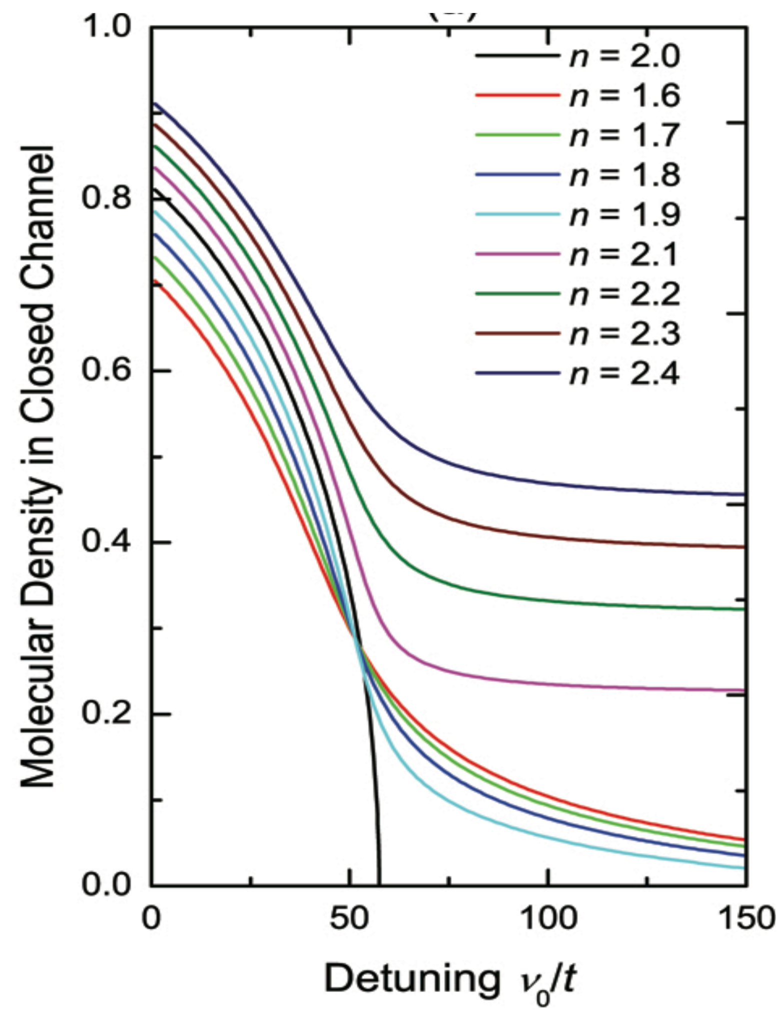


Experimental implications

• reentrant BCS-BEC crossover for s-wave Feshbach resonance

• nonmonotonic dependence of thermodynamics, Cooper pair size, …

• **SF-Insulator transition at n=2:**



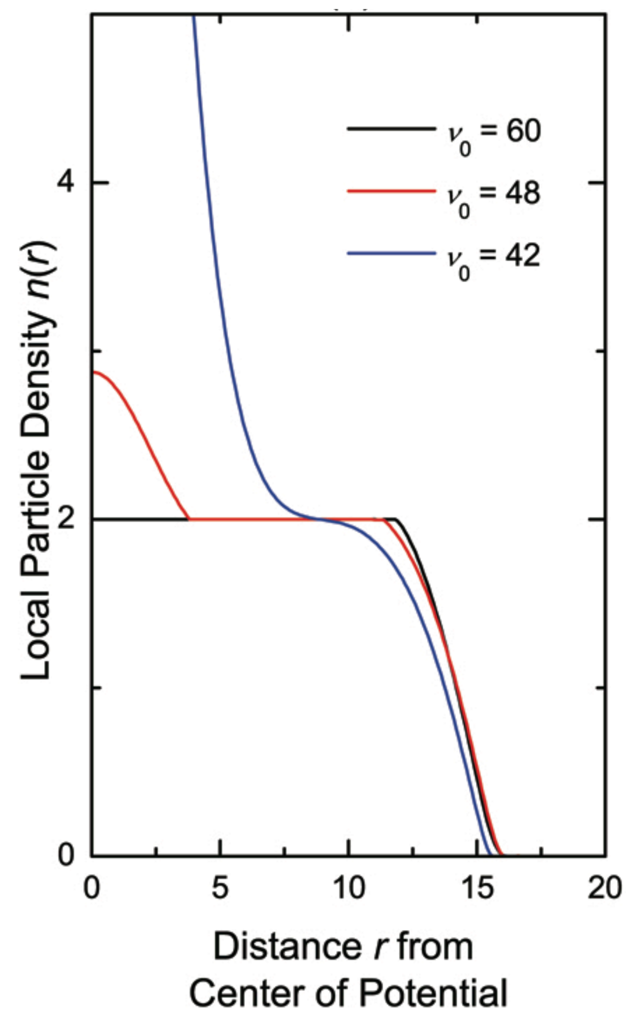
Experimental implications

• reentrant BCS-BEC crossover for s-wave Feshbach resonance

• nonmonotonic dependence of thermodynamics, Cooper pair size, …

• superfluid-insulator transition at n=2

• **“wedding cake” density profile:**



Experimental implications

- reentrant BCS-BEC crossover for s-wave Feshbach resonance
- nonmonotonic dependence of thermodynamics, Cooper pair size, …
- superfluid-insulator transition at n=2
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- **vanishing compressibility:** $\kappa \sim |n - 2|^{2/3}$

Experimental implications

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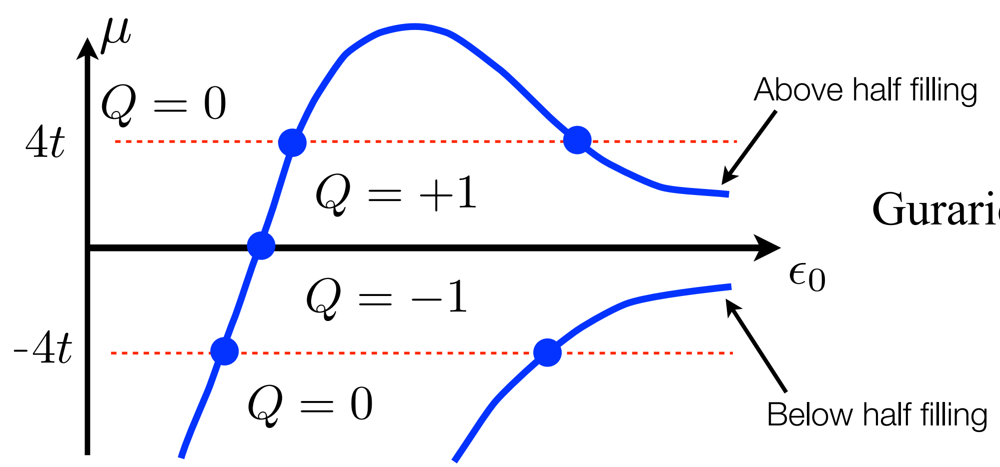
- superfluid-insulator transition at n=2
- “wedding cake” density profile
- vanishing compressibility: $\kappa \sim |n - 2|^{2/3}$
- **lattice unitary universality $U_* \approx 8t$:** $\mu = t\, f(n)$

with $f(n) = -f(2-n)$

$\sim -6 + \xi\, (3\pi^2 n)^{2/3}$

Experimental implications

- nonmonotonic dependence of thermodynamics, Cooper pair size, …
- superfluid-insulator transition at n=2
- "wedding cake" density profile
- vanishing compressibility: $\kappa \sim |n - 2|^{2/3}$
- lattice unitary universality $U_* \approx 8t$: $\mu = t\, f(n)$
- **multiple *topological* phase transitions for p-wave Feshbach resonance:**

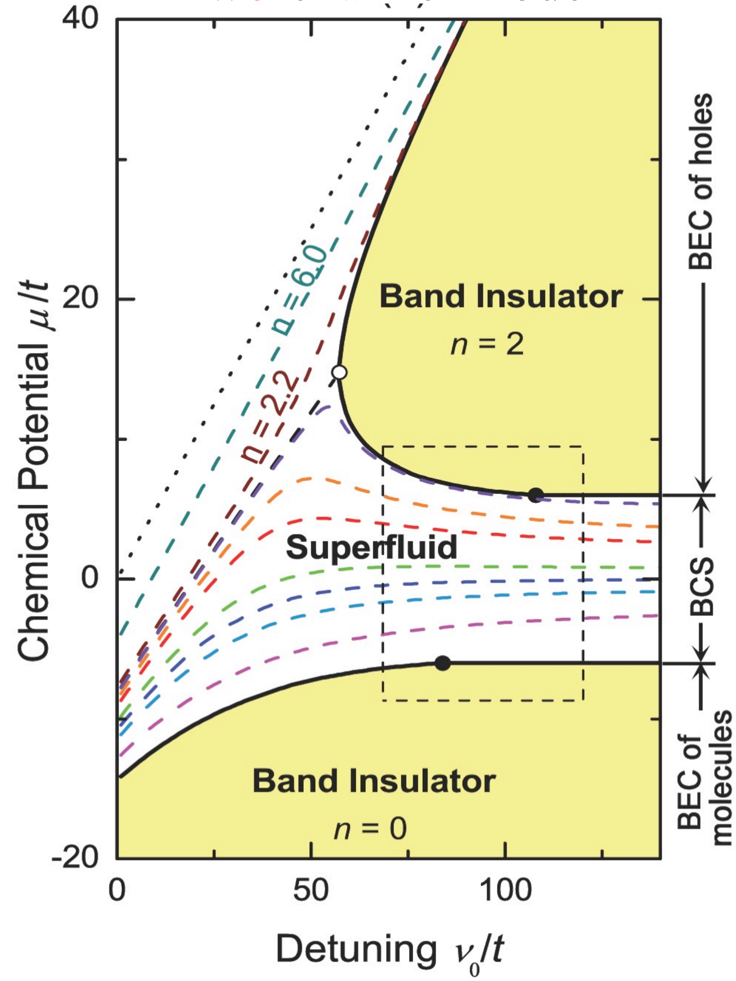


Gurarie + L.R., AOP 2007

Summary and outlook

• non-monotonic BCS-BEC crossover and SF-Insulator transition at n=2:

two-channel model



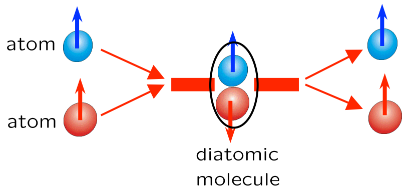
• multi-band extensions?

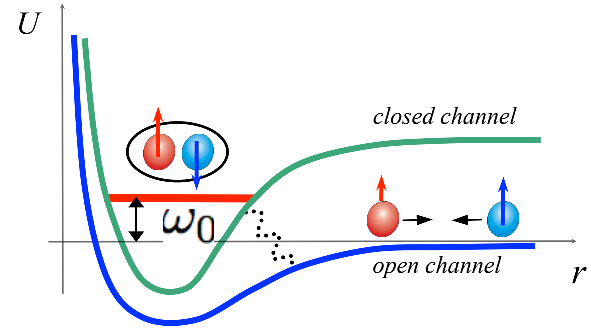
• molecule-molecule interaction ⟹ molecular Mott insulator?

• Bloch oscillations?

S-wave Feshbach resonant scattering

- tunability (strength and sign) of interactions (sudden and adiabatic)

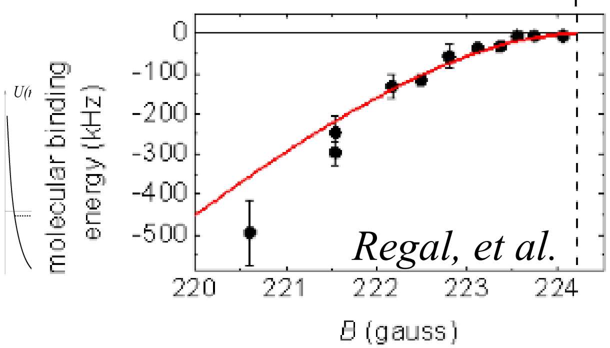


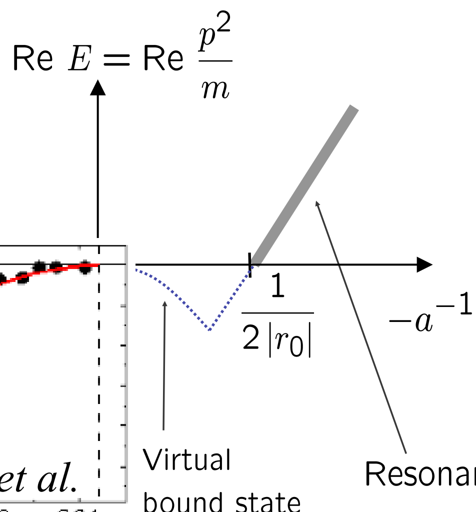


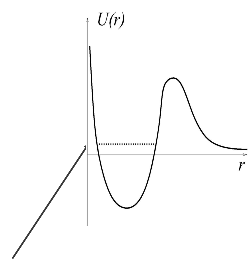
$$\boxed{\mathcal{H}_{2ch} = \psi_\sigma^\dagger \frac{\hat{p}^2}{2m}\psi_\sigma + \phi^\dagger\Big(\frac{\hat{p}^2}{4m} + \epsilon_0\Big)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger}$$

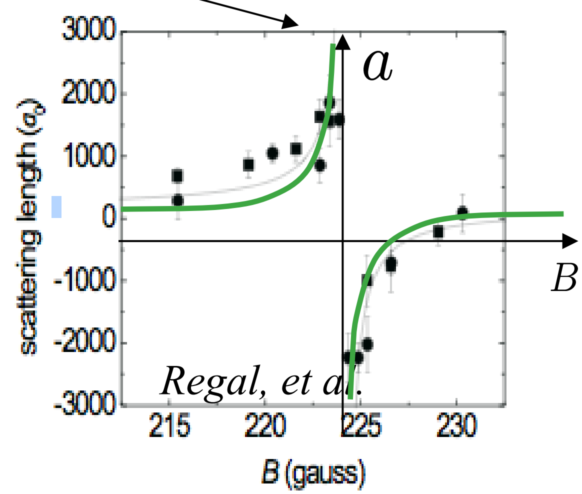
$$\longrightarrow \quad f_s(p) = \frac{1}{-a^{-1} + \frac{r_0}{2}p^2 - ip}, \quad \text{with } a \sim -\frac{g^2}{\omega_0} \sim a_{bg}\frac{\Delta B}{B_0 - B}, \quad r_0 \sim -\frac{1}{g^2}$$

$$f_s(p) = \frac{1}{F(p^2) - ip}$$



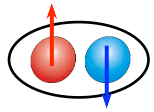




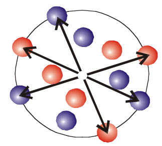


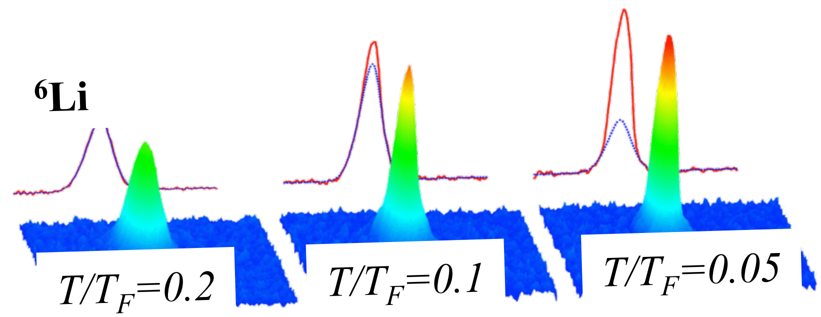
S-wave resonant fermionic superfluidity

- molecular BEC *(Grimm, Jin '03)*



- BCS superfluid *(Jin '04 Ketterle '04)*





- BCS-BEC crossover

atom-molecule hybridization

$$\boxed{\mathcal{H}_{2ch} = \psi_\sigma^\dagger\Big(\frac{\hat{p}^2}{2m} - \mu\Big)\psi_\sigma + \phi^\dagger\Big(\frac{\hat{p}^2}{4m} - 2\mu + \epsilon_0\Big)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger}$$

fermionic open-channel atoms

bosonic closed-channel molecules

S-wave resonant fermionic superfluidity

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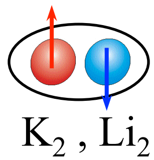

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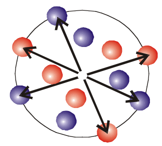

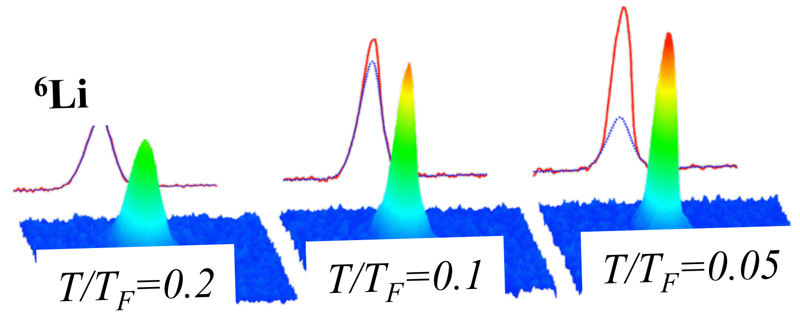

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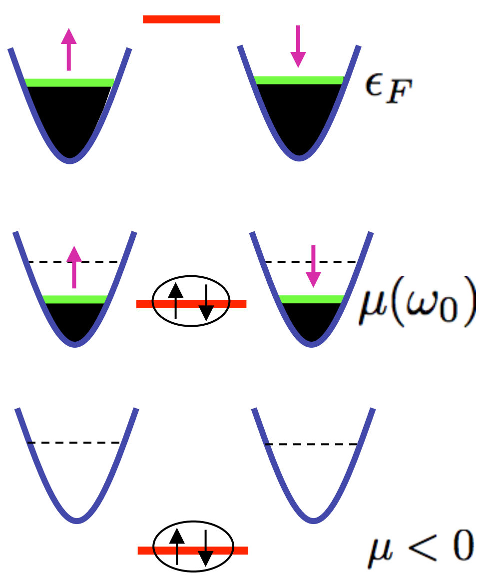
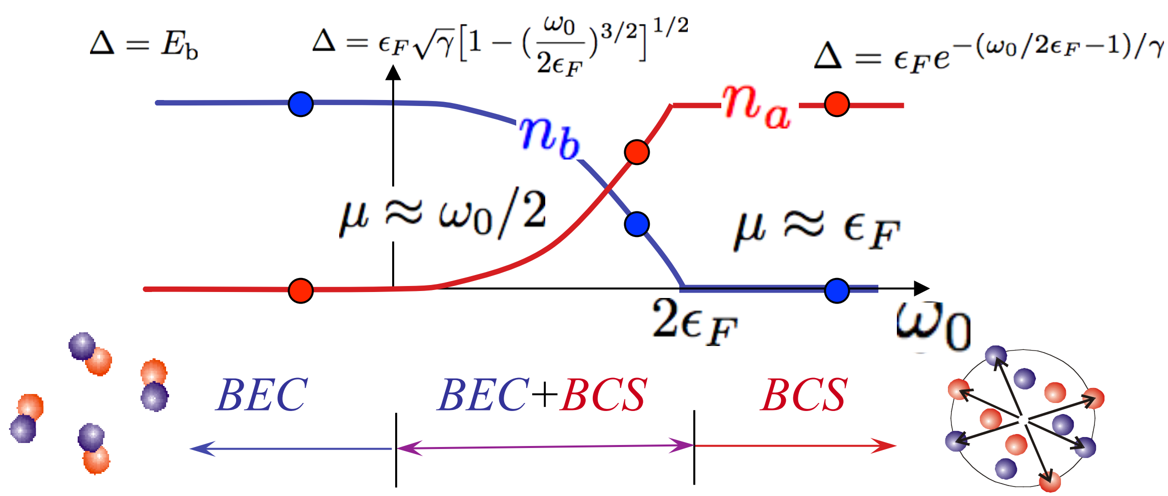




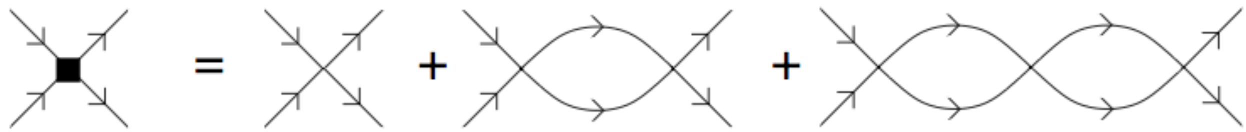
$\gamma >> 1$

Broad resonance scattering

$$\mathcal{H}_{2ch} \longrightarrow \mathcal{H}_{1ch} = \psi_\sigma^\dagger\left(\frac{p^2}{2m} - \mu_\sigma\right)\psi_\sigma + \lambda\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$$

- scattering T-matrix relates λ to a:

$T_{k\,k'}$ =

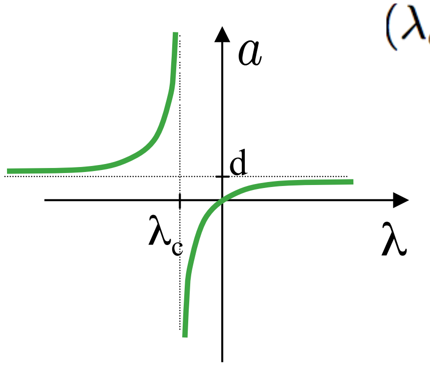


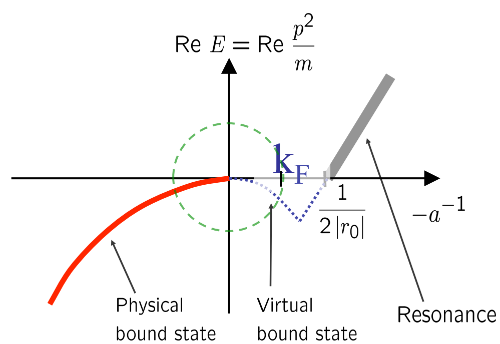
$+\ldots = \dfrac{\lambda}{1-\lambda\Pi}$

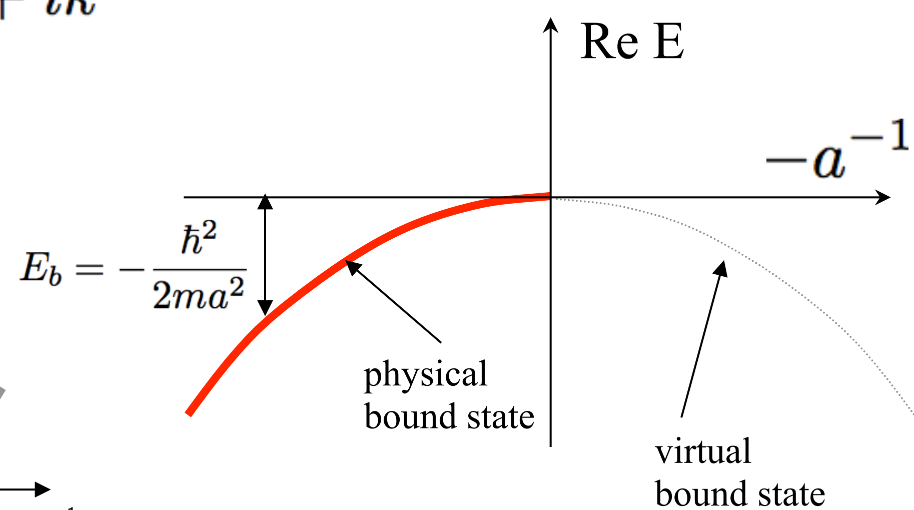
$$= -\frac{4\pi\hbar^2}{m} f_{kk'} \approx \frac{4\pi\hbar^2}{m}\frac{1}{a^{-1}+ik}$$

$$\longrightarrow \boxed{a = \frac{m}{4\pi\hbar^2}\frac{\lambda}{1+\lambda/\lambda_c}}$$

$(\lambda_c = \pi d/m)$

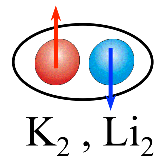




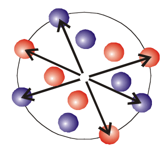


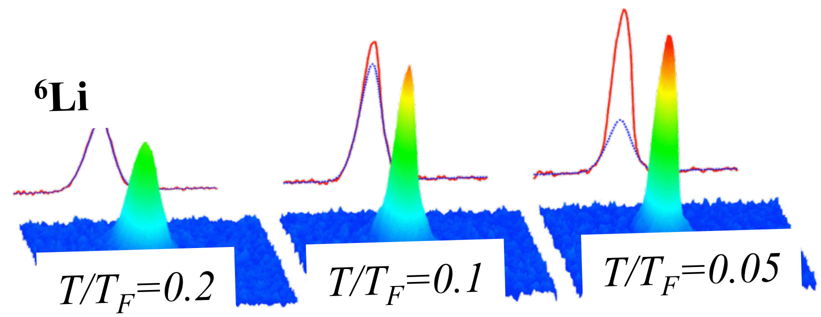
S-wave resonant fermionic superfluidity

- molecular BEC *(Regal, Jin '03)*



- BCS superfluid *(Regal, Jin 04 Zwierlein, Ketterle '04)*





- BCS-BEC crossover:

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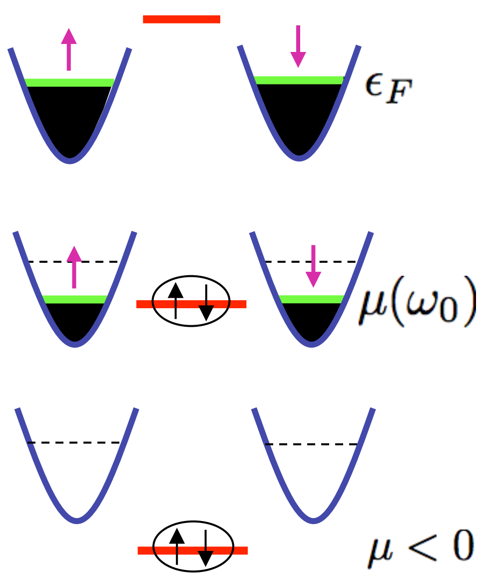

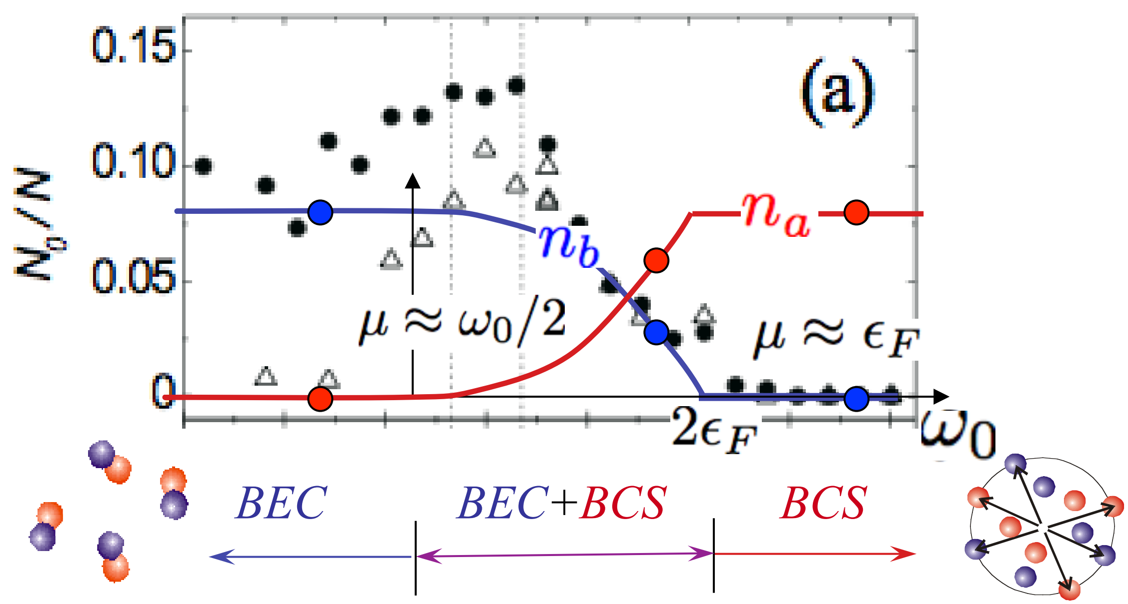

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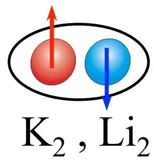

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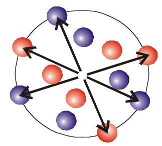

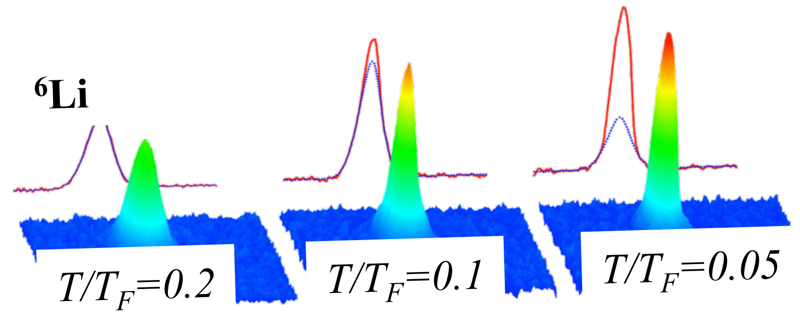

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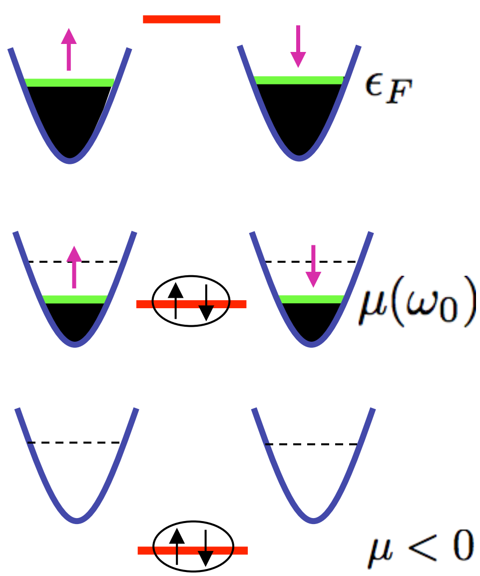

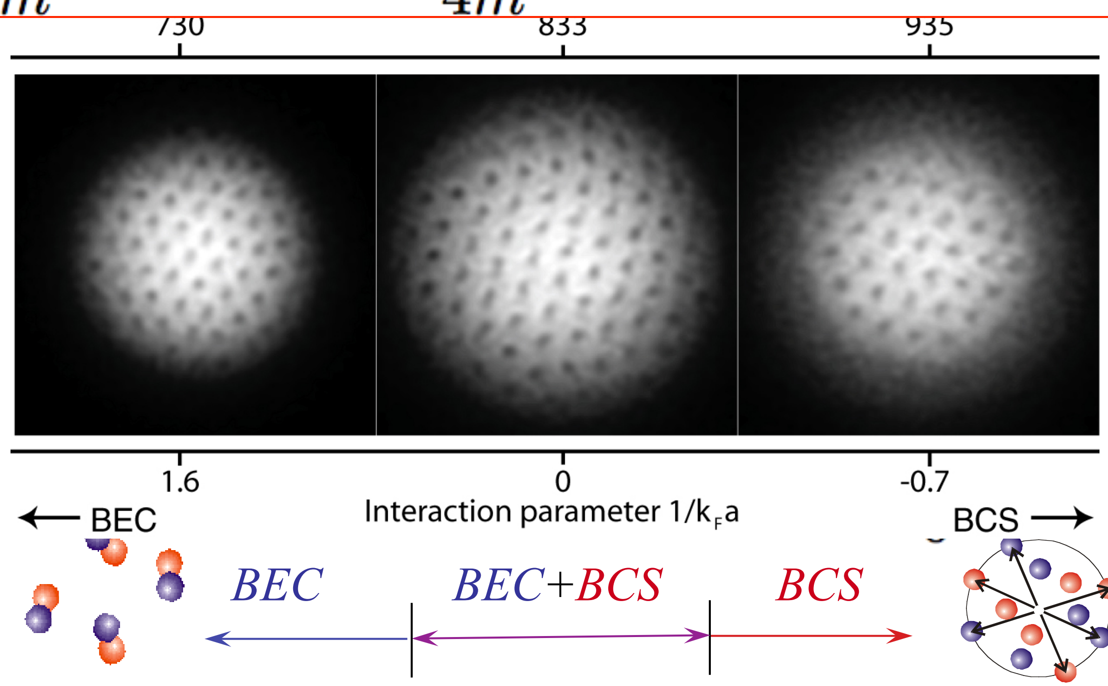

S-wave resonant fermionic superfluidity

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dimensionless coupling: $\boxed{\gamma \sim \left(\frac{g\sqrt{n}}{\epsilon_F}\right)^2 \sim \frac{g^2}{\epsilon_F^{1/2}} \sim \frac{1}{r_0 n^{1/3}}}$

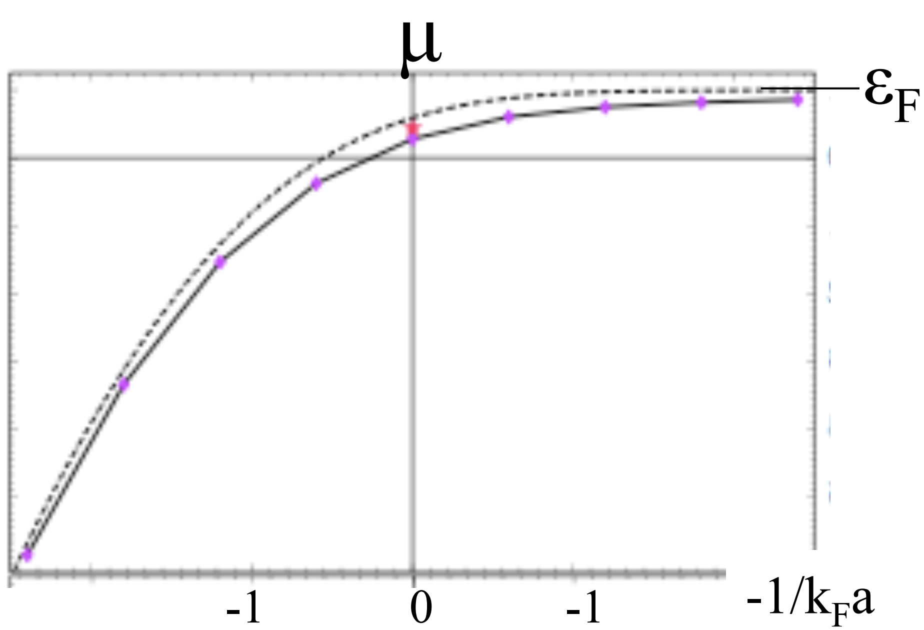
$\gamma^{^{40}K}_{202G} \approx 5,\ \Delta B \sim 1G \sim 100\mu K$

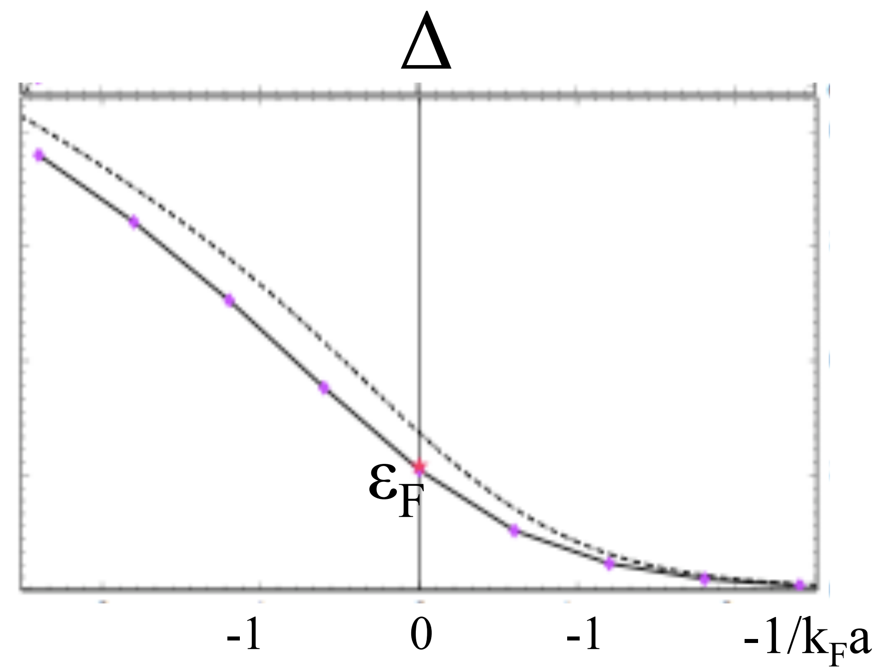
$\gamma^{^{6}Li}_{544G} \approx 0.1,\ \Delta B \sim 0.1G \sim 10\mu K$

$\epsilon_F \sim 1\mu K$

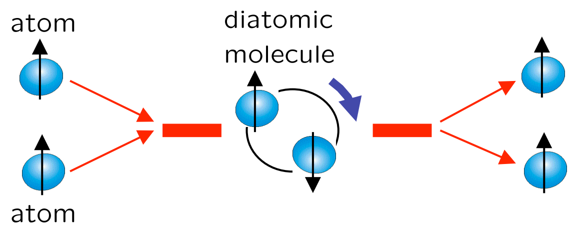
- **narrow** resonance $\gamma \ll 1 \;\rightarrow\; \mathrm{MFT}: \phi(x) = B$
- **broad** resonance $\gamma \gg 1 \Longrightarrow \mathcal{H}_{1ch} = \psi_\sigma^\dagger(\frac{p^2}{2m} - \mu)\psi_\sigma + \lambda\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$

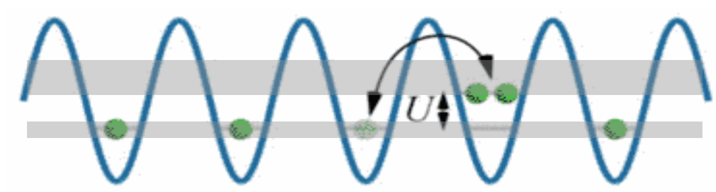
Strongly coupled ϕ and ψ $\Longrightarrow$ *MFT uncontrolled*





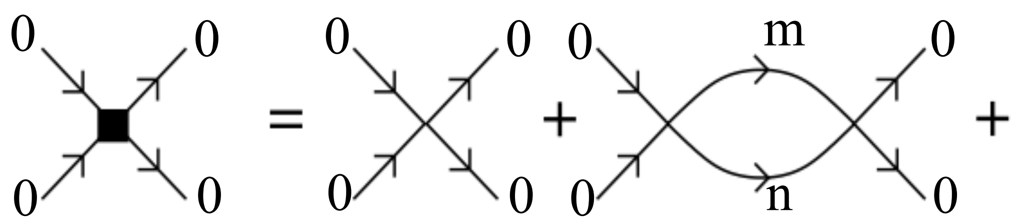
Resonant fermions in a periodic potential





Fedichev, et al., ’04
Zhai, Ho ’07
Buchler, ’10
Cui, et al., ’10
von Stecher, et al., ‘11

- 2-body problem:



$$U = \frac{\lambda}{\ell_W^3} + \Pi_{\mathrm{higher}}$$

- band hybridization

- lattice induced resonances

- deep lattice $\Longrightarrow$ single band tight binding model

- repulsively bound pairs

- bound states at BZ boundary for arbitrarily weak attraction

S-wave resonant fermionic superfluidity

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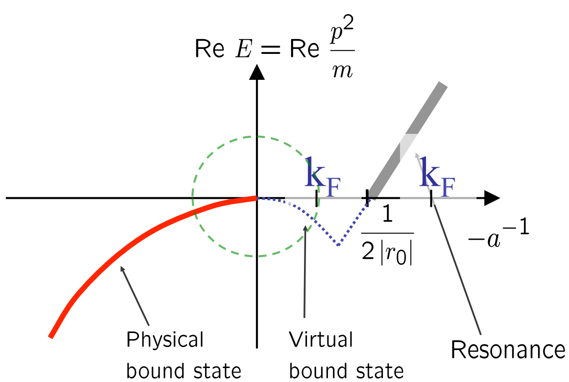
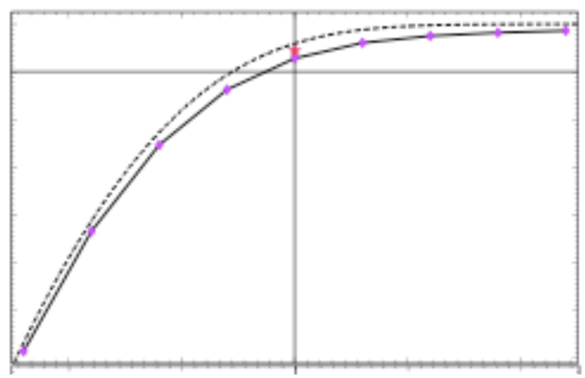
$\epsilon_F \sim 1\mu K$

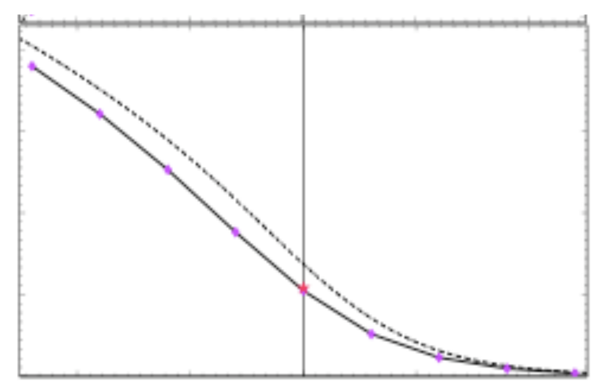
- **narrow** resonance $\gamma \ll 1 \ \rightarrow\ \mathrm{MFT}: \phi(x) = B$
- **broad** resonance $\gamma \gg 1 \Longrightarrow \mathcal{H}_{1ch} = \psi_\sigma^\dagger\left(\frac{p^2}{2m} - \mu\right)\psi_\sigma + \lambda\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$

Strongly coupled ϕ and ψ

$\Longrightarrow$ *MFT uncontrolled*







$$\gamma \approx |T_{k_F}|n/\epsilon_F \approx \frac{1}{(k_F a)^{-1} - k_F r_0 + 1}$$

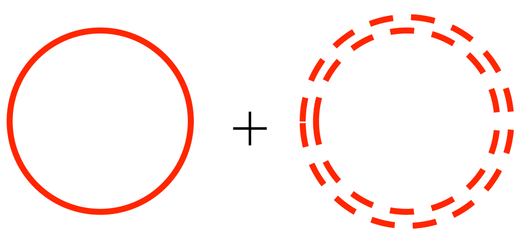
$\gamma >> 1$ ## Broad resonance superfluidity: Large N

- no small parameter for $k_F a \sim n^{1/3} a >> 1 \rightarrow$ *introduce 1/N*

$$\mathcal{H}_{1ch} \xrightarrow{Sp(2N)} \mathcal{H}_N = \psi^\dagger_{\sigma\alpha}\left(\frac{p^2}{2m} - \mu_\sigma\right)\psi_{\sigma\alpha} + \frac{\lambda}{N}\psi^\dagger_{\uparrow\alpha}\psi^\dagger_{\downarrow\alpha}\psi_{\downarrow\beta}\psi_{\uparrow\beta}$$

$$S[\phi] = -\frac{N}{\lambda}\int_0^\beta d\tau d^3r|\phi|^2 - N\mathrm{Tr}\log\left[-G_\phi^{-1}\right] \qquad G_\phi^{-1} = \begin{pmatrix} -\partial_\tau + \frac{\nabla^2}{2m} + \mu_\uparrow & \phi_x \\ \phi_x^* & -\partial_\tau - \frac{\nabla^2}{2m} - \mu_\downarrow \end{pmatrix}$$

$$\begin{aligned} f &= -\frac{1}{\beta V}\log\int D\phi e^{-S[\phi]}, \\ &= N f^{(0)} + f^{(1/N)} + \ldots \end{aligned}$$



MFT

$$f^{(0)} = -\frac{|\Delta|^2}{\lambda} - \int_k (E_k - \xi_k) - \sum_{\sigma=\pm}\int_k \log\left[1 + e^{-\beta(E_k + \sigma h)}\right]$$

Veillette, Sheehy, LR
Nikolic, Sachdev
also Nishida, Son
ε-expansion

$\gamma >> 1,\ k_F a \to \infty$

Universality at unitary point

T.L. Ho '04

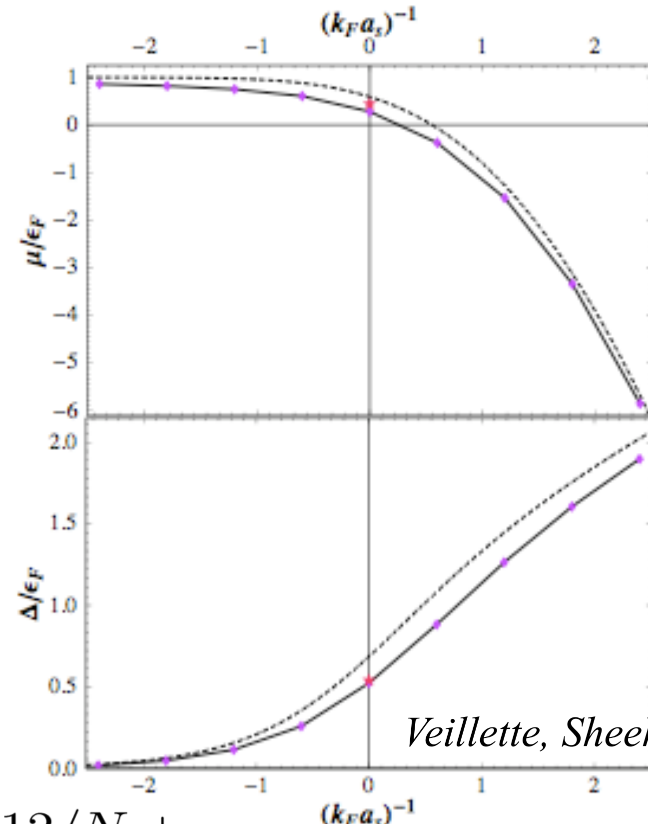
- $f_k = -1/(a^{-1} + i\,k) \to i/k$, k_F is the only scale

$$\boxed{f(T,n) = n\epsilon_F \hat{f}(k_B T/\epsilon_F)}$$

check in $N \to \infty$ (BCS) limit:

$$\frac{m}{2\pi\hbar^2 a} \to 0 = \int_k \left(\frac{1}{E_k} - \frac{1}{\epsilon_k}\right)$$

$$\begin{aligned} \epsilon &= \xi \frac{3}{5} \epsilon_F \\ \mu &= \xi \epsilon_F \\ \Delta &= \alpha \epsilon_F \\ \Delta_{exc} &= \alpha_{exc} \epsilon_F \\ k_B T_c &= \gamma \epsilon_F \\ B &= \xi \frac{2}{3} n \epsilon_F \end{aligned}$$



Veillette, Sheehy, LR

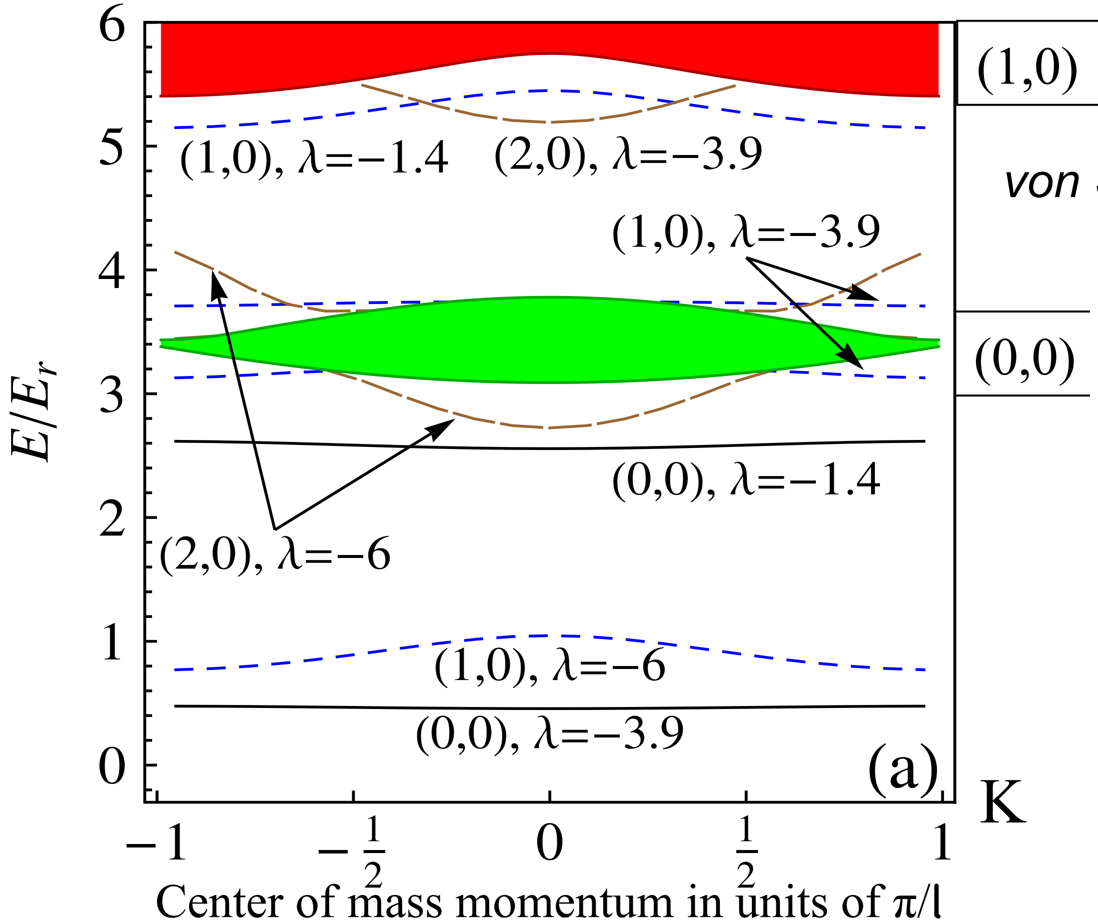
Result from 1/N $\xi = 0.5906 - 0.312/N + \cdots$

Exp with ^{40}K $\xi = 0.46^{+0.05}_{-0.12}$

Band hybridization: periodic potential

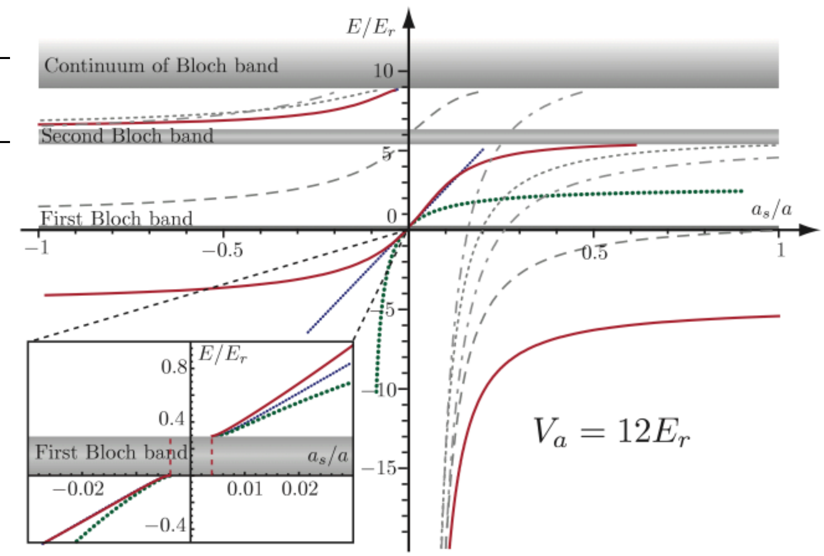
two atoms in a periodic potential with attractive interactions λ in 1D:

two particle bands (n,m) and bound states



- *band hybridization*
- *lattice induced resonances*

von Stecher, Gurarie, L.R., Rey, PRL '11



Buchler, Arxiv '09

Possible to reduce to single band resonant lattice model?

yes: $U = \lambda/l_w^{\,3} + \Pi_{n>0}$ ⟵ *need to convert* λ *to* a_s