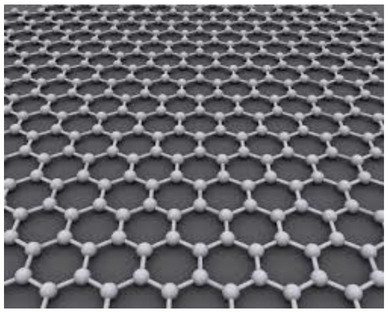
Disorder-driven quantum transition in semiconductors and Dirac semimetals



with: *S. Syzranov, V. Gurarie*

see: *PRL, PRB 2015*

$: NSF, Simons Foundation

University of Utah, May 7, 2015

Outline

- Motivation
- Results
- Qualitative physics
- RG for ‘dirty’ semiconductors and Dirac semimetals
- Critical density of states and transport
- Lifshitz tails
- Conclusions

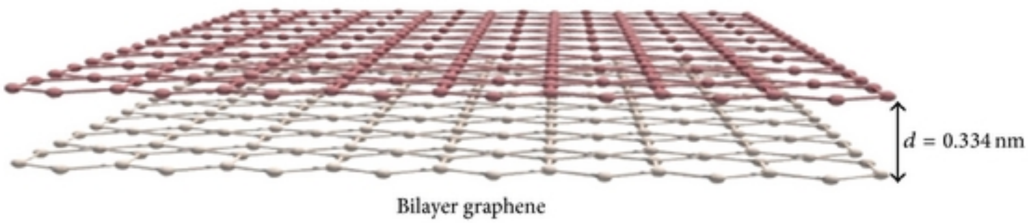
S. V. Syzranov, V. Gurarie, L. Radzihovsky, PRB 91, 035133 (2015)

S. V. Syzranov, L. Radzihovsky, V. Gurarie, PRL 114, 166601 (2015)

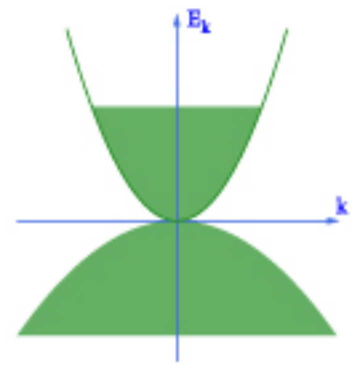
Motivation

Transport in lightly-doped (E_F << W) conductors

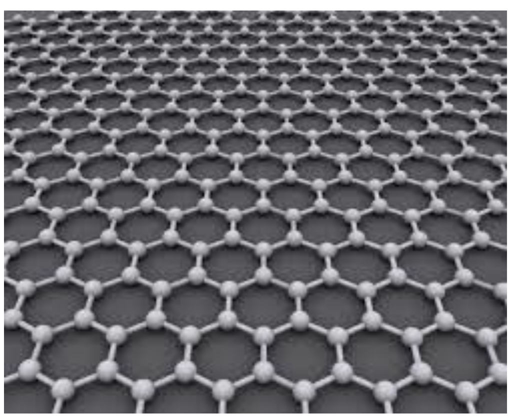
- *semiconductors, bilayer graphene:*

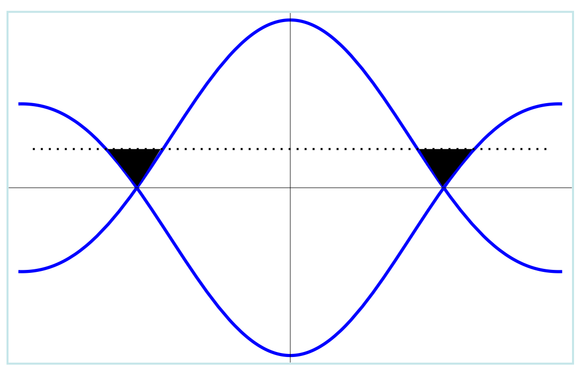


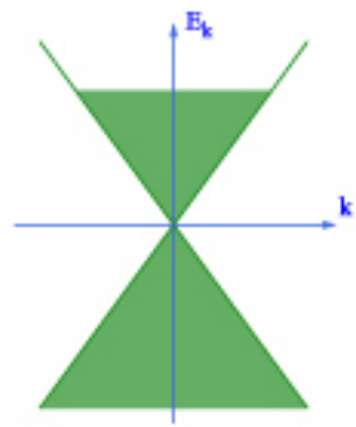
$$\varepsilon_k \sim k^2$$



- *graphene, Weyl semimetals, topological insulators:*







$$\varepsilon_k \sim \sigma \cdot k$$

Cd_3As_2

nature materials
LETTERS
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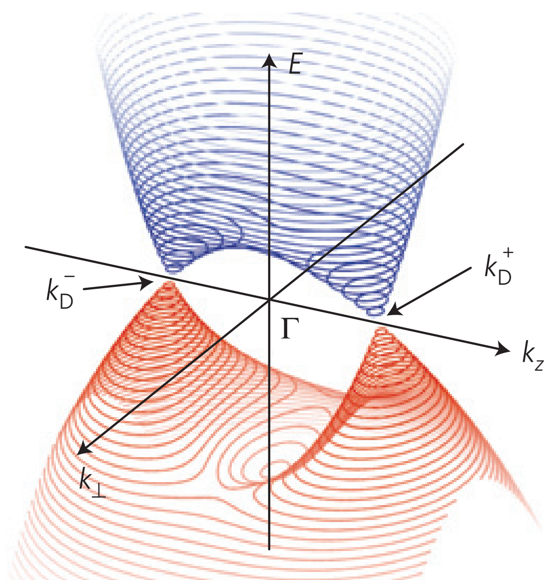
Landau quantization and quasiparticle interference in the three-dimensional Dirac semimetal Cd_3As_2

Sangjun Jeon[1†], Brian B. Zhou[1†], Andras Gyenis[1], Benjamin E. Feldman[1], Itamar Kimchi[2], Andrew C. Potter[2], Quinn D. Gibson[3], Robert J. Cava[3], Ashvin Vishwanath[2] and Ali Yazdani[1★]

d



Results

study:

$$\varepsilon(\hat{\mathbf{p}})\psi + V(\mathbf{r})\psi = E\psi$$

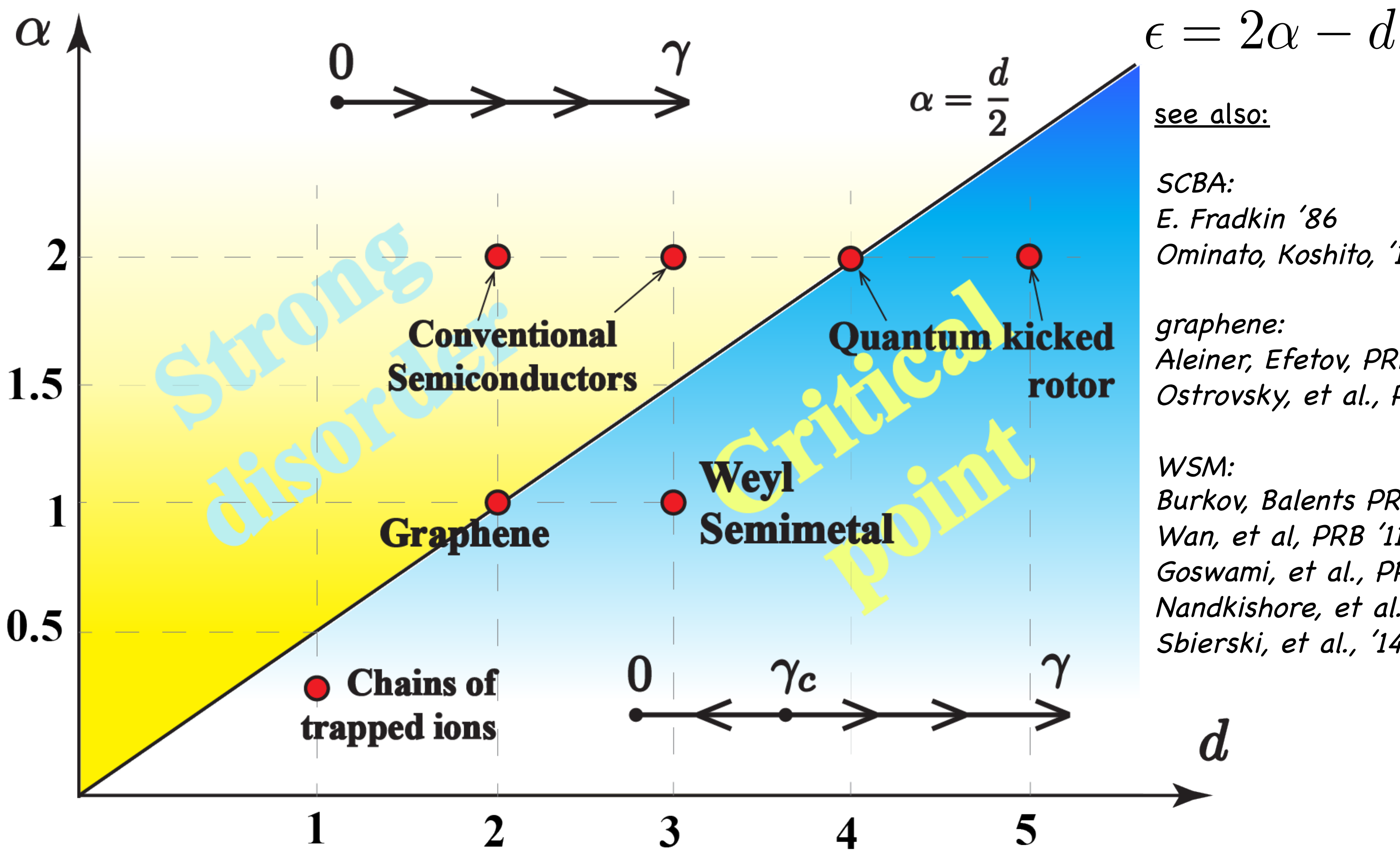
at bottom of the band, E -> 0

S. V. Syzranov, V. Gurarie, L. Radzihovsky, PRB 91, 035133 (2015)

S. V. Syzranov, L. Radzihovsky, V. Gurarie, PRL 114, 166601 (2015)

results: "phase diagram"

non-interacting $\varepsilon_k \sim k^\alpha$ in random potential



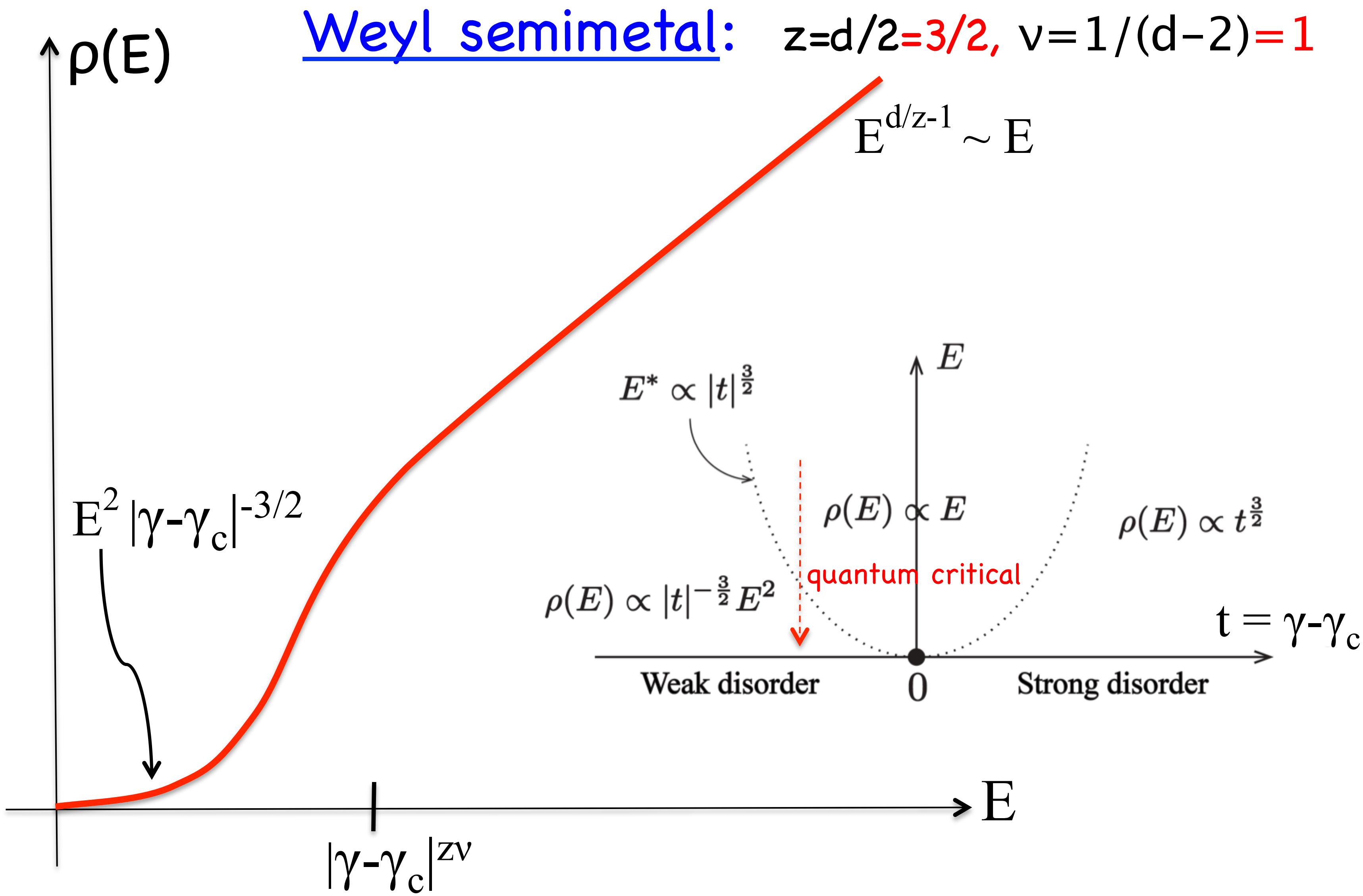
see also:

SCBA:
E. Fradkin '86
Ominato, Koshito, '14

graphene:
Aleiner, Efetov, PRL '06
Ostrovsky, et al., PRB '06

WSM:
Burkov, Balents PRL '11
Wan, et al, PRB '11
Goswami, et al., PRL '11
Nandkishore, et al., '14
Sbierski, et al., '14

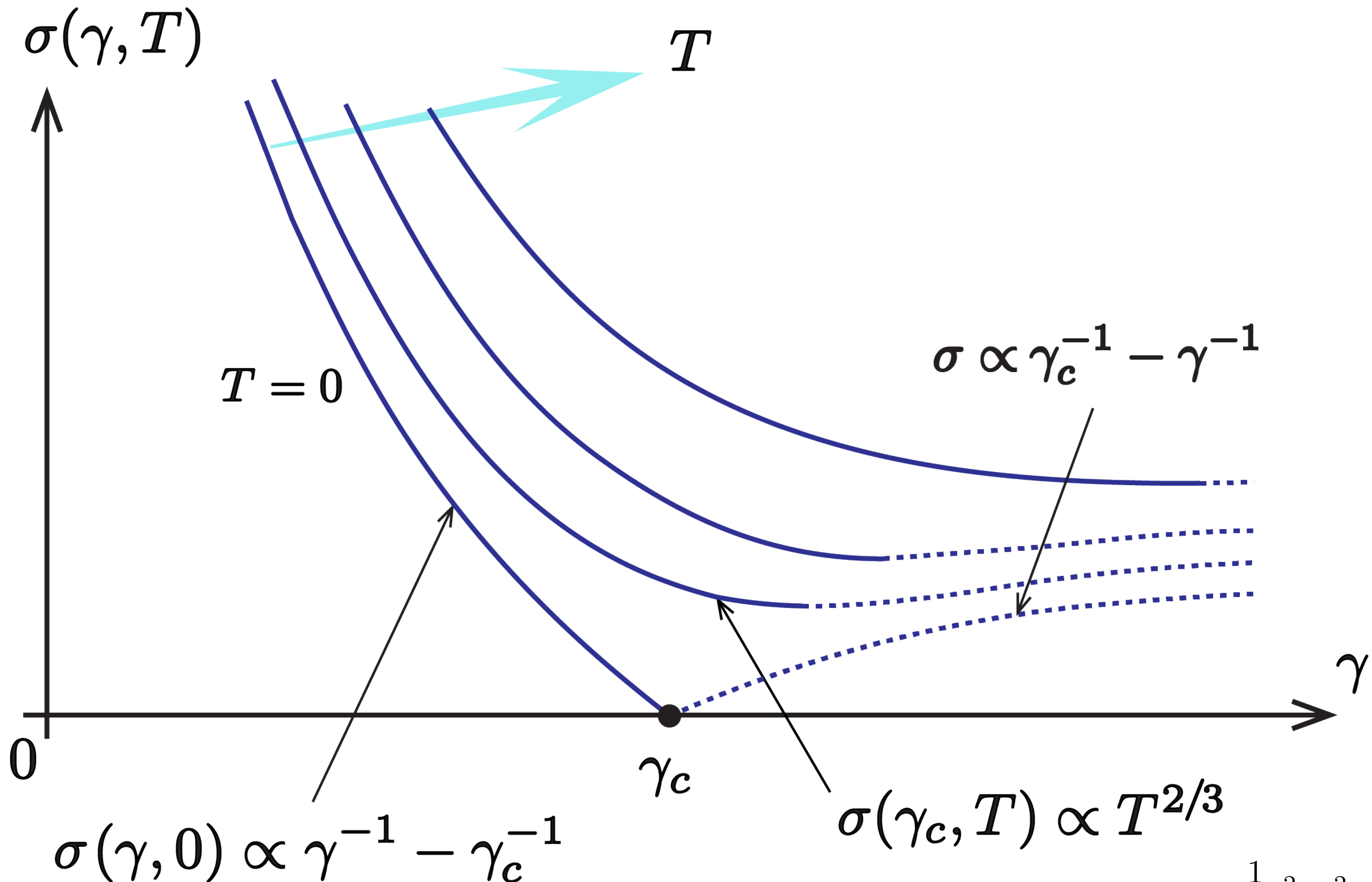
results: critical density of states



...ignoring rare regions of strong disorder Lifshitz tails

results: critical conductivity

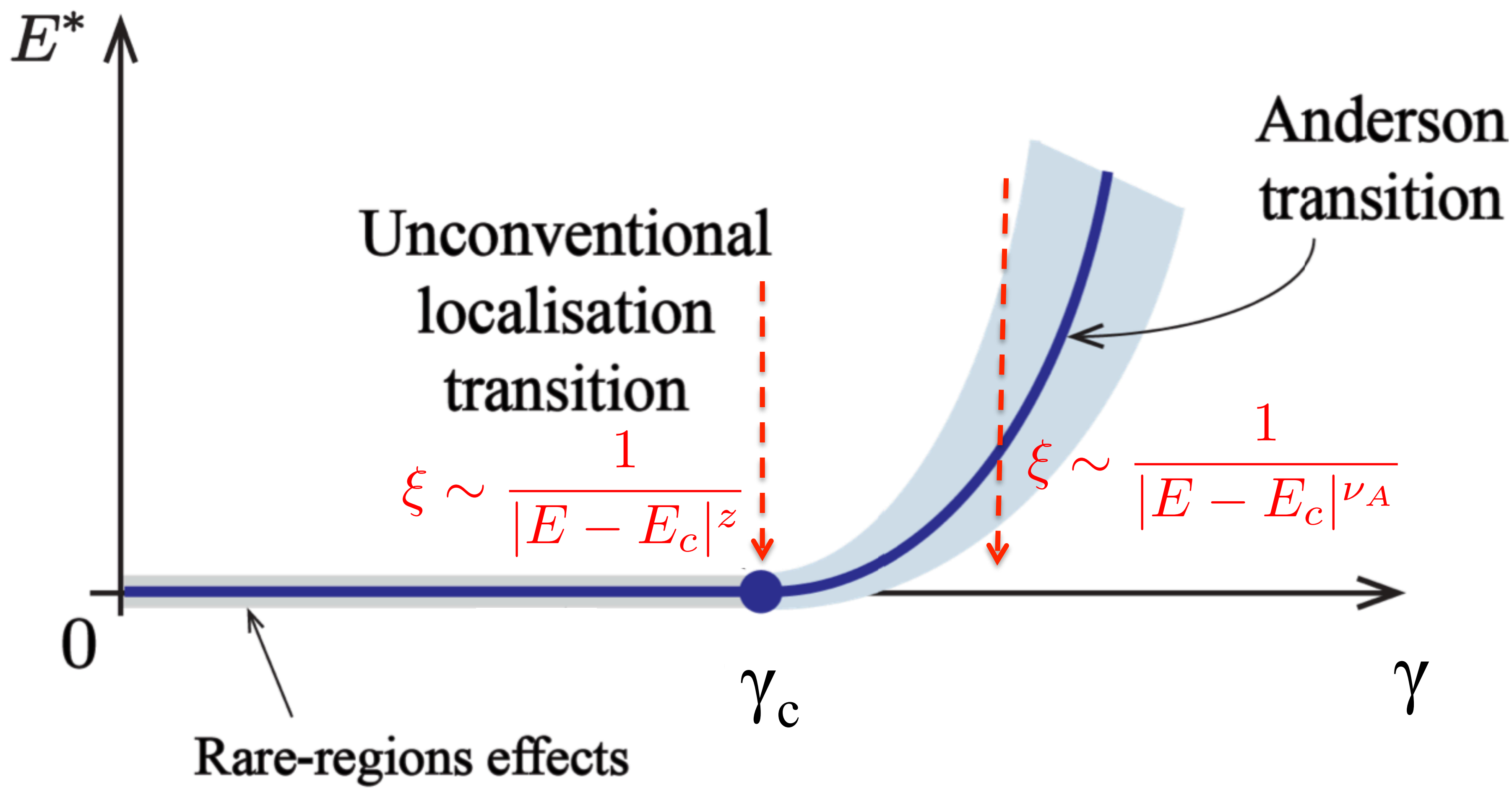
Weyl semimetal: $z=d/2=3/2$, $\nu=1/(d-2)=1$



$$\sigma_D = \frac{1}{2}e^2\nu v^2\tau = \frac{e^2v^2}{2\pi\Delta}$$

results: nonanalytic mobility edge

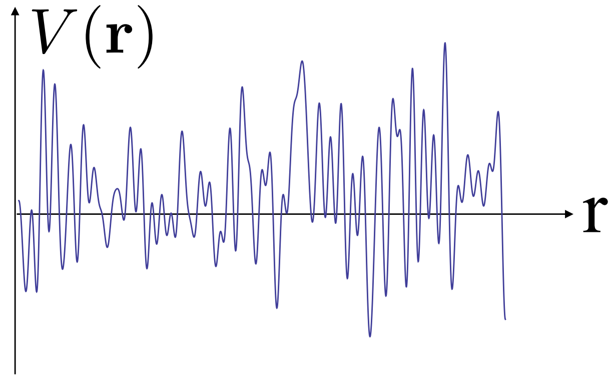
weakly-doped semiconductor



Anderson localization

- *noninteracting* quantum motion in a random potential

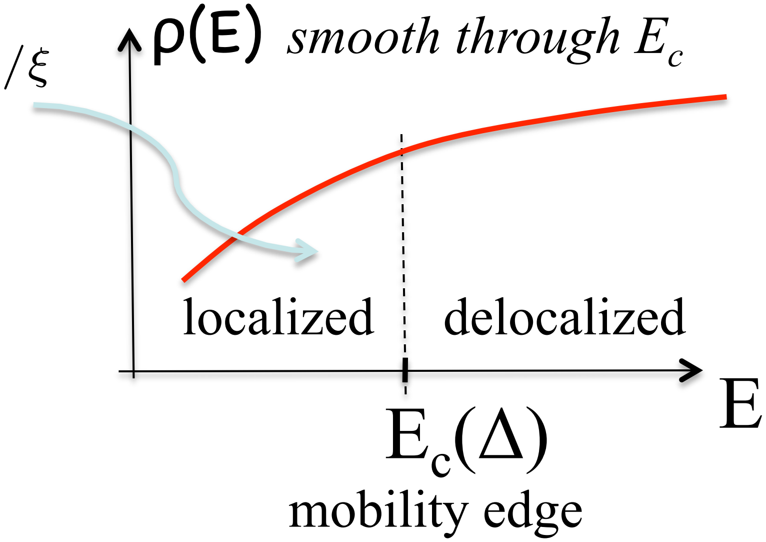
$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(\mathbf{r})\psi = E\psi$$



- conventional wisdom (orthogonal)
  - d < 2: all states are localized
  - d > 2: localization-delocalization Anderson transition

$$\psi \sim e^{-|\mathbf{r}-\mathbf{r}_0|/\xi}$$

$$\xi \sim \frac{1}{|E-E_c|^{\nu_A}}$$



Mobility edge E_c

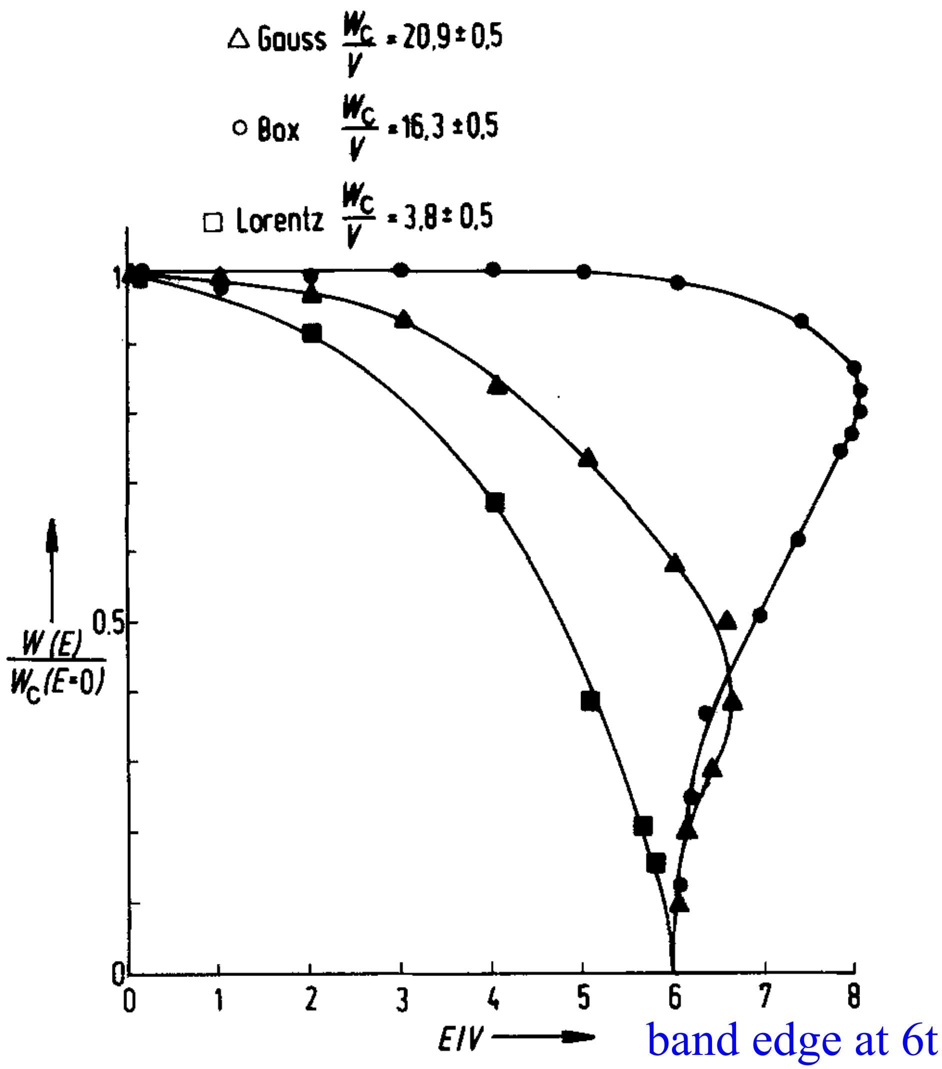


Fig. 1. Mobility edge trajectories $W_C(E)$ for the box (● $W_C(0)/V = 16.3 \pm 0.5$), Gaussian (▲ $W_C(0)/V = 20.9 \pm 0.5$), and the Lorentzian (■ $W_C(0)/V = 3.8 \pm 0.5$) distribution

Z. Phys. B – Condensed Matter 66, 21–30 (1987)

Localization, Quantum Interference, and the Metal-Insulator Transition

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Received September 10, 1986

mobility edge $E_c(\Delta)$ is a *smooth* function of disorder strength $\Delta \sim V_0^2$

Model

low-density electrons in a random potential, ***μ -> 0***

$$\varepsilon(\hat{\mathbf{p}})\psi + V(\mathbf{r})\psi = E\psi \quad \rightarrow \quad Z \sim \int [d\psi] e^{iS}$$

$$S = \int_{\mathcal{C}} dt d^d r \overline{\psi} \left[\lambda i \partial_t - \varepsilon(\hat{\mathbf{p}}) - V(\mathbf{r})\right] \psi$$

$$\varepsilon(p) = \kappa p^\alpha \qquad \overline{V(\mathbf{r})V(0)} = \Delta \delta^d(\mathbf{r})$$

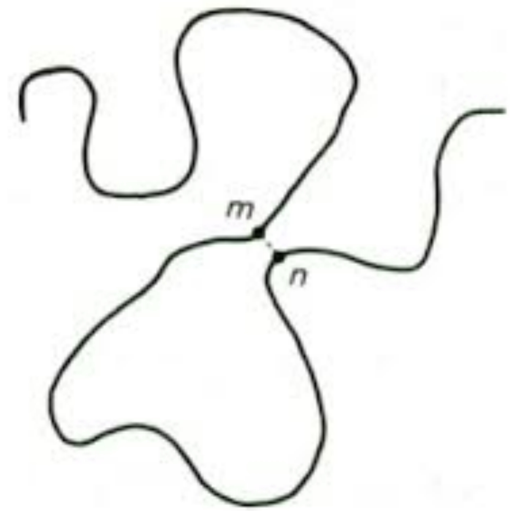
$$\begin{aligned} \varepsilon(\hat{\mathbf{p}}) &= \frac{\hat{p}^2}{2m}, \quad \text{semiconductor} \\ &= v\sigma \cdot \hat{\mathbf{p}}, \quad \text{Weyl semimetal} \end{aligned}$$

treat via *scaling, perturbation* theory, and *RG*
with ε=(2α-d)-expansion

Related problems

- *Feynman's polaron coupled to a slow boson:*
  - *world-line* ⟶ *self-avoiding polymer* (deGennes '72)

$$H[r(t)] = \frac{1}{2}\int dt \left(\frac{d\mathbf{r}}{dt}\right)^2 + g\int dt \int dt' \delta^d(\mathbf{r}(t) - \mathbf{r}(t'))$$



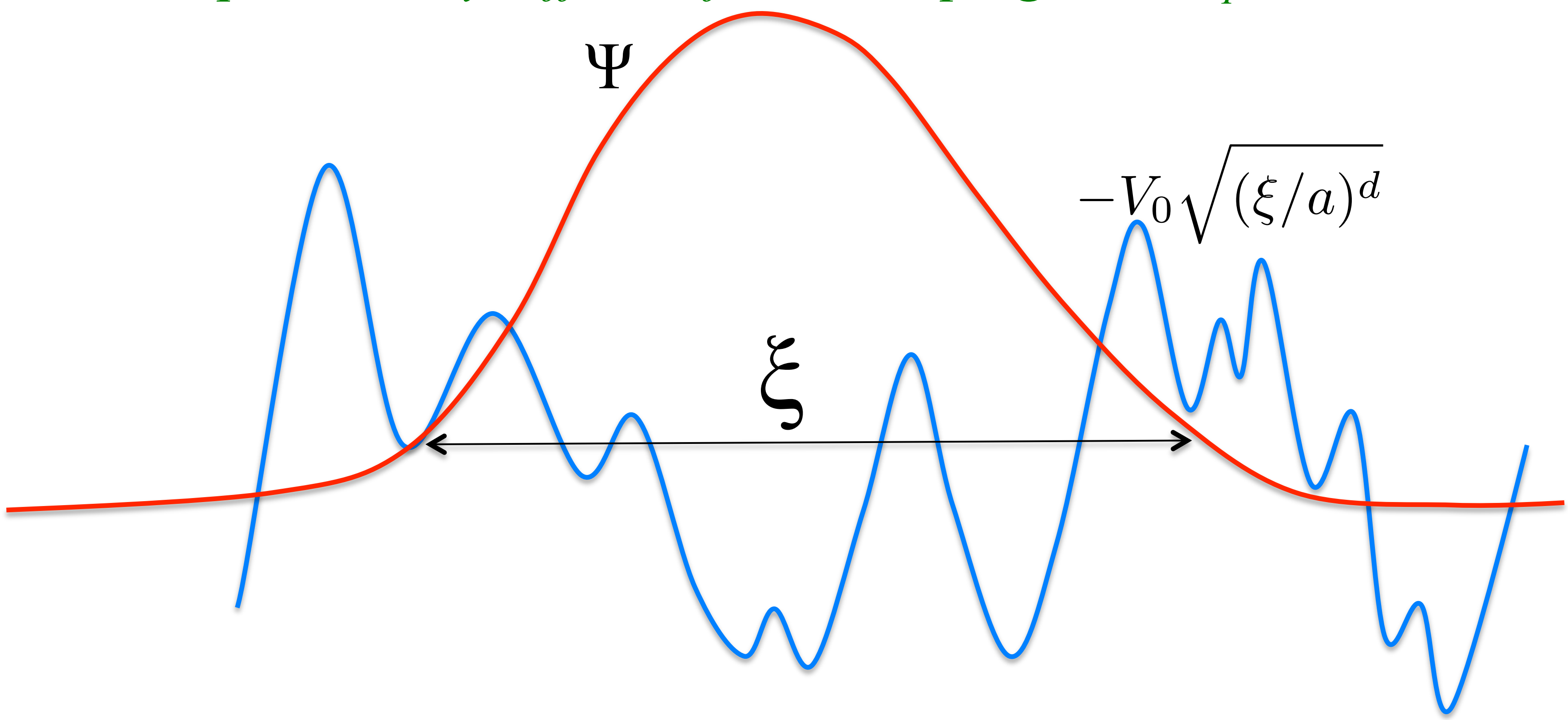
  - *singular corrections for d < 4 (=2α, α=2)*
  - $R_G = (\langle r^2 \rangle)^{1/2} \sim L^{\nu}$, $\nu = 1/2 + O(4-d)$

- *Interacting particles at low density (BCS-BEC):*
  - *exact T-matrix* ⟶ $\dfrac{dg}{d\ell} = (2-d)g - g^2$

The "Physics" via scaling

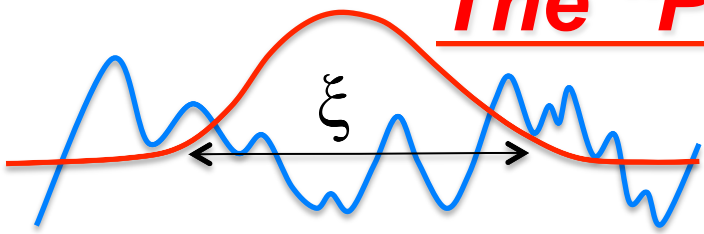
(distinct from interference effects near Fermi surface: e.g. weak localization)

Fermi liquids are "protected" by Fermi sea, with $E_F \sim W$, qualitatively different for low doping, with $E_F << W$



bound-state formation in random potential $-V_0\sqrt{(\xi/a)^d}$ *on scale* ξ

The "Physics" via scaling



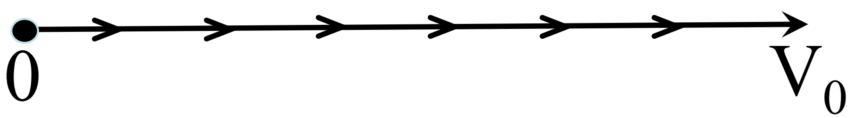
V(r) averaged over wavelength 1/k ~ ξ

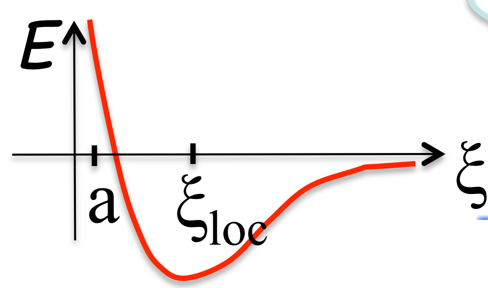
$$E \sim \underbrace{\kappa\xi^{-\alpha}}_{E_K} - \underbrace{V_0(\xi/a)^{-d/2}}_{V_{\mathrm{eff}}}$$

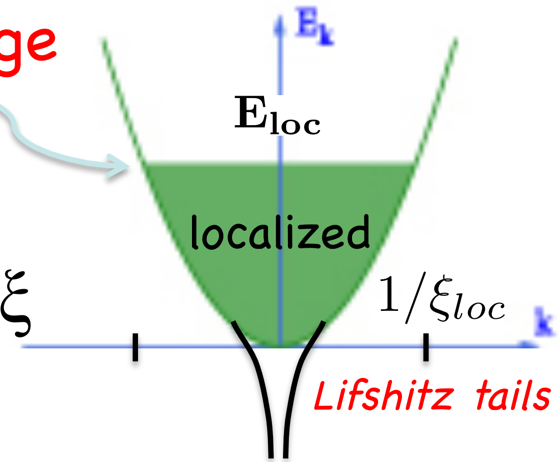
- *$d < 2\alpha$: disorder dominates -> bound state for arbitrarily weak V_0*

$$\xi_{loc} = a\left(\frac{\kappa}{V_0 a^\alpha}\right)^{\frac{2}{2\alpha-d}}$$

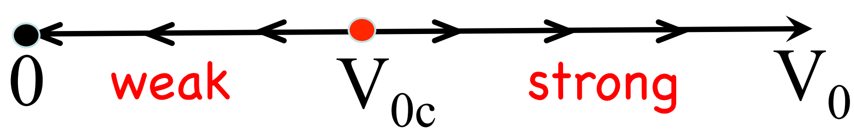
"conventional" mobility edge







- *$d > 2\alpha$: weak disorder irrelevant below critical* $\boldsymbol{V_{0c} = \kappa\, a^{-\alpha} = W}$

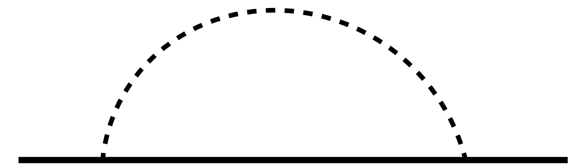


weak-to-strong disorder quantum phase transition

Perturbation theory

$$S = \int_{\mathcal{C}} dt d^d r \overline{\psi} \left[\lambda i \partial_t - \varepsilon(\hat{\mathbf{p}}) - V(\mathbf{r}) \right] \psi$$

- scattering rate 1/τ(E):



$$\Sigma(E) \sim \int d^d q \frac{1}{(E - q^\alpha) q^\alpha} \sim E^{d/\alpha - 1} \sim \frac{1}{\tau(E)}$$

$$\rightarrow \quad G(E) \sim \frac{1}{E - k^\alpha - E^{d/\alpha - 1}}$$

usually E-dependence ignored at E_F

- Ioffe-Regel criterion γ < 1 vs γ > 1 :

$$\gamma(E) \equiv \frac{\Sigma(E)}{E} \sim E^{d/\alpha - 2} \sim \frac{1}{k\ell(k)} \sim \frac{1}{kv(k)\tau(k)}$$

d < 2α: (typical) disorder *dominates* at low E

d > 2α: (typical) disorder is *irrelevant* at low E

$$\boxed{\frac{\partial \gamma}{\partial \ell} = (2\alpha - d)\gamma + \ldots}$$

Perturbation theory

$$S = \int_{\mathcal{C}} dt d^d r \overline{\psi} \left[\lambda i \partial_t - \varepsilon(\hat{\mathbf{p}}) - V(\mathbf{r})\right] \psi$$

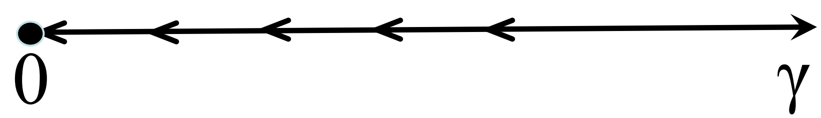
- Ioffe-Regel criterion, $\gamma < 1$ vs $\gamma > 1$:

$$\gamma(E) \equiv \frac{\Sigma(E)}{E} \sim E^{d/\alpha - 2} \sim \frac{1}{k\ell(k)} \sim \frac{1}{kv(k)\tau(k)}$$

$d < 2\alpha$: (typical) disorder *dominates* at low E

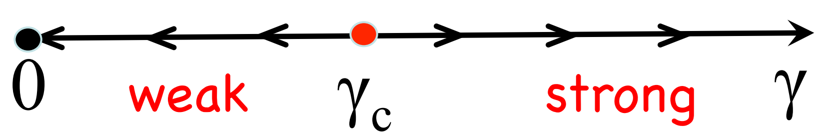
$\boxed{d > 2\alpha\text{: (typical) disorder is } \textit{irrelevant} \text{ at low E (e.g., d > 4 } \textit{new physics}\text{)}}$

$$\frac{\partial \gamma}{\partial \ell} = (2\alpha - d)\gamma + \ldots$$



What happens for stronger disorder for $d > 2\alpha$?

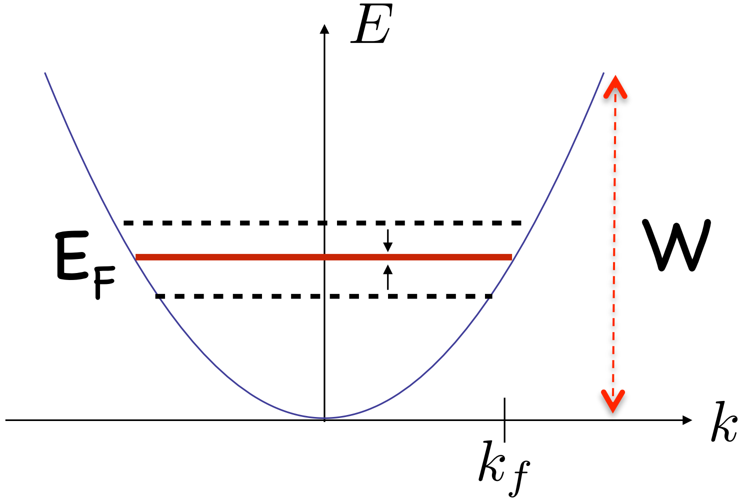
$$\frac{\partial \gamma}{\partial \ell} = (2\alpha - d)\gamma + c\gamma^2$$



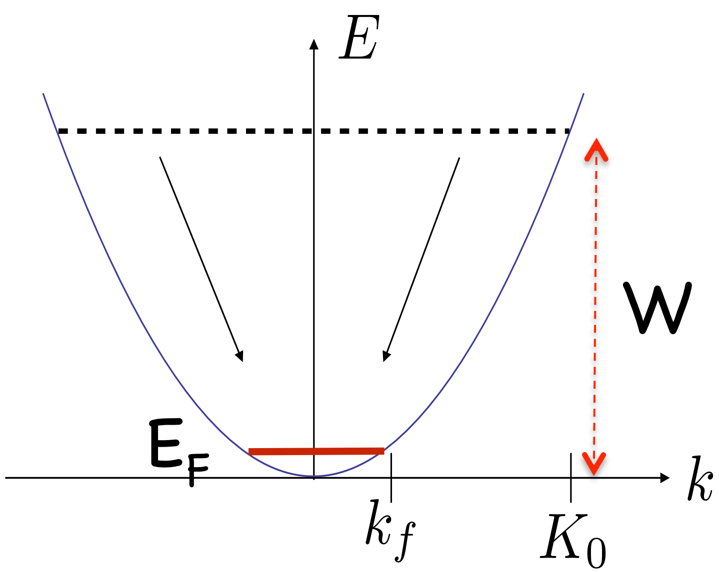
RG

$$S = \int_{\mathcal{C}} dt d^d r \overline{\psi} \left[\lambda i \partial_t - \varepsilon(\hat{\mathbf{p}}) - V(\mathbf{r}) \right] \psi$$

- "conventional" (Shankar) RG at Fermi surface



- "vacuum" RG down to the bottom of the band



RG

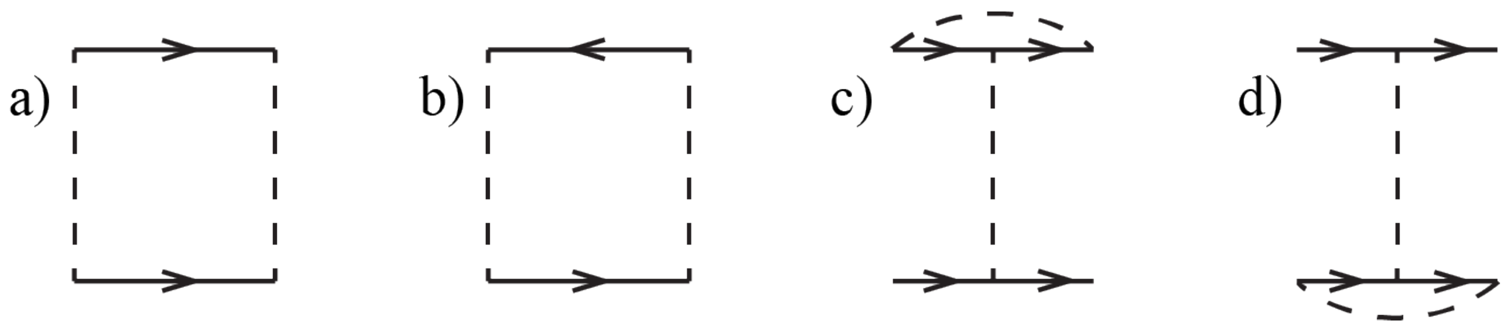
$$S = \int_{\mathcal{C}} dt d^d r \overline{\psi} \left[\lambda i \partial_t - \varepsilon(\hat{\mathbf{p}}) - V(\mathbf{r}) \right] \psi$$

- rescaling and tree-level RG flow: $r = r' \, b$, $t = t' \, b^z$

$\lambda(b) = \lambda$, $\kappa(b) = \kappa \, b^{z-\alpha}$, $\Delta(b) = \Delta \, b^{2z-d}$

$$\longrightarrow \quad \gamma(b) = \frac{\Delta(b)}{\kappa(b)^2} = \gamma b^{2\alpha - d} \equiv \gamma e^{\epsilon \ell}$$

- momentum-shell RG flow: *integrate out* $K_0/b < k < K_0$



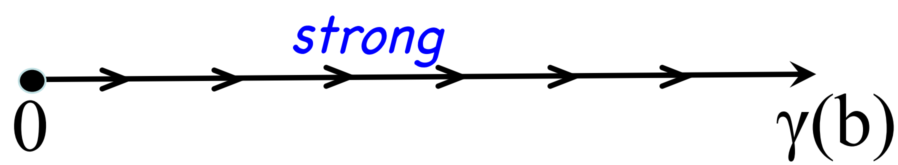
$$\frac{d\lambda}{d\ell} = \gamma \lambda, \quad \frac{d\gamma}{d\ell} = \epsilon \gamma + 2\alpha \gamma^2$$

<u>***Mobility threshold & transition***</u>

$$\gamma(b) = \frac{\gamma b^{\epsilon}}{1-\gamma/\gamma_c + (\gamma/\gamma_c) b^{\epsilon}}, \quad \lambda(b) = \left[\frac{\gamma(b)}{\gamma} b^{-\epsilon}\right]^{1/2\alpha}, \quad \gamma_c = -\frac{\epsilon}{2\alpha}$$

renormalized Drude conductivity (γ < 1): $\sigma_D(b) = \frac{v(b)^2}{2\pi\gamma(b) b^{-\epsilon}}$

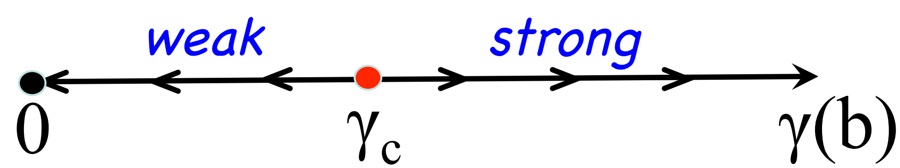
- $\epsilon > 0$ (d < 2α):



--> *disorder dominates for k <* $K_{loc} = K_0 (1 + \frac{|\gamma_c|}{\gamma})^{-1/\epsilon}$

--> <u>*mobility threshold of Anderson transition*</u>

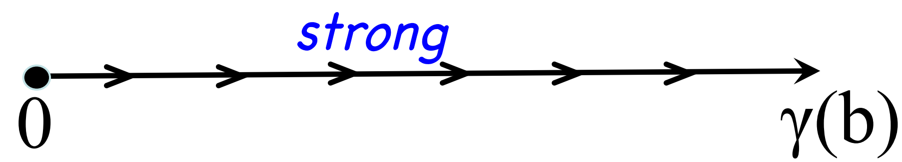
- $\epsilon < 0$ (d > 2α) :

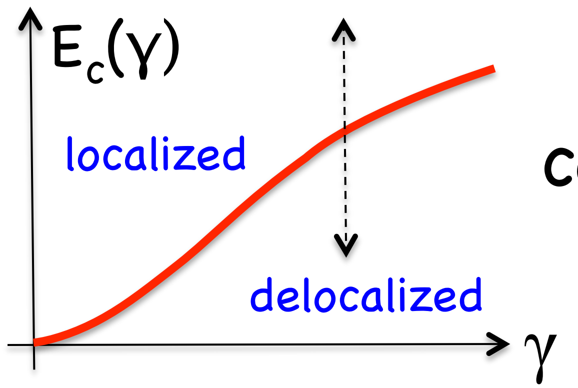


--> <u>*disorder-driven quantum transition*</u> *T=0, μ->0*

Mobility Edge

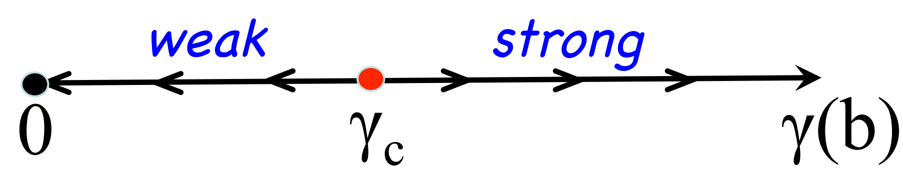
- $\epsilon > 0$ $(d < 2\alpha)$:

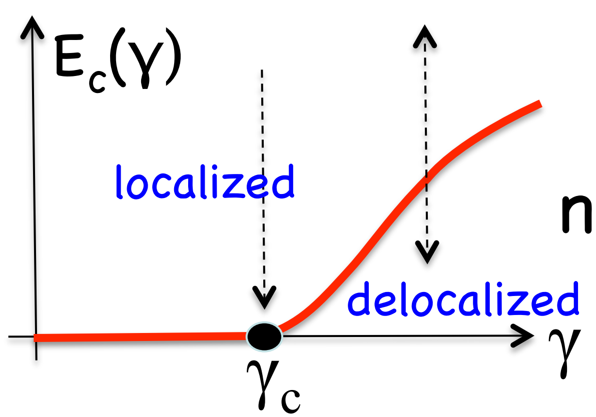




conventional Anderson transition

- $\epsilon < 0$ $(d > 2\alpha)$:





nonanalytic mobility edge

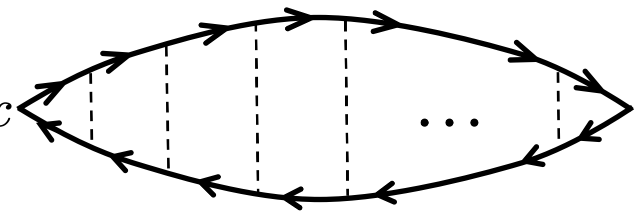
$$E_c(\gamma) \sim (\gamma - \gamma_c)^{z\nu}\ , \ \gamma > \gamma_c$$

Conductivity

$$\sigma_{ij}(\omega) = \frac{e^2}{2\pi\omega}\int dE\left[n_F(E) - n_F(E+\omega)\right]\int d^d r' \mathrm{Tr}\overline{\hat{v}_{i\mathbf{r}} G^A(E+\omega,\mathbf{r},\mathbf{r}')\hat{v}_{j\mathbf{r}'}G^R(E,\mathbf{r}',\mathbf{r})}$$

Weyl semimetal:

$$\longrightarrow \quad \sigma_{xx} = \frac{e^2v^2}{2\pi}\sum Tr\Big[\hat{\sigma}_x$$



$$\hat{\sigma}_x\Big]$$

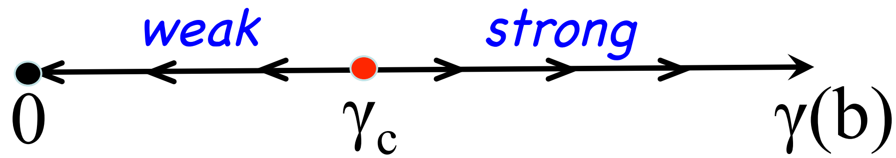
$$= \frac{1}{2}e^2v^2\nu\tau = \frac{e^2v^2}{2\pi\Delta}$$

$$\longrightarrow \quad \tau^{-1} = -2Im\Sigma_R = -2\Delta\int\frac{1}{\varepsilon - v\vec{\sigma}\cdot\vec{k} + \frac{i}{2\tau}}$$

$$= \pi\nu\Delta$$

Scaling theory of critical transport

- $\epsilon < 0$ (d > 2α):



$$\xi \sim |\gamma - \gamma_c|^{-\nu} \equiv |\delta\gamma|^{-\nu} \qquad \sigma \sim \xi^{2-d}$$

$$\longrightarrow \quad \sigma(\gamma, T, \mu) = |\delta\gamma|^{\nu(d-2)} g(T|\delta\gamma|^{-z\nu}, \mu/T)$$

$$\sigma(\gamma, 0, 0) \sim |\gamma - \gamma_c|^{\nu(d-2)} \qquad \sigma(\gamma_c, T, 0) \sim T^{(d-2)/z}$$

- comparing with RG calculation, find *(for 3d WSM)*:

$\longrightarrow$ $z = 3/2, \quad \nu = -1/(2\alpha - d) = 1$

see also: Goswami, Chakravarty, PRL 107, 2011

$(E \sim k^{\alpha}/\lambda(k) \sim k^{d/2})$

Density of states

$$\rho(E) = -\frac{1}{\pi}\mathrm{Im}\int \frac{d^d r}{V}\overline{G_R(\mathbf{r},\mathbf{r},E)}$$

$$= \lambda(K_E)\cdot\rho_{\mathrm{clean}}[\lambda(K_E)E]$$

where:

$$\gamma(b) = \frac{\gamma b^{\epsilon}}{1-\gamma/\gamma_c+(\gamma/\gamma_c)b^{\epsilon}}, \quad \lambda(b) = \left[\frac{\gamma(b)}{\gamma}b^{-\epsilon}\right]^{1/2\alpha}$$

$$\lambda(K_E)E \sim \kappa K_E^{\alpha}$$

Scaling theory of density of states

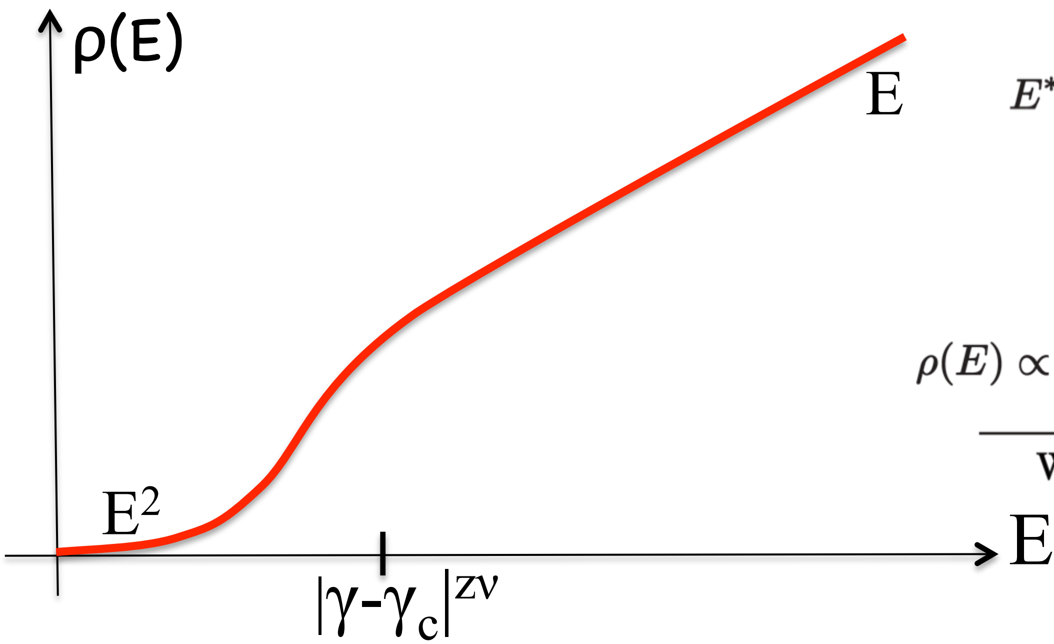
$$\xi \sim |\gamma - \gamma_c|^{-\nu}, \quad E \sim \xi^{-z}$$

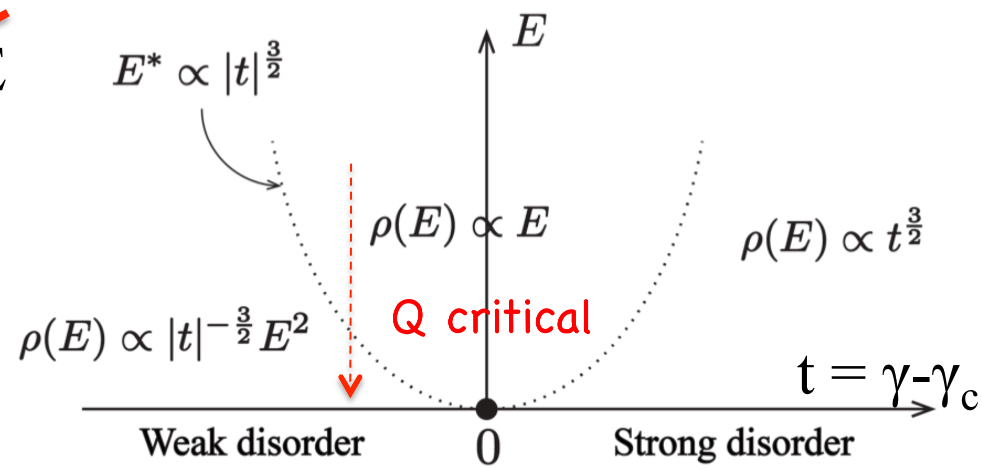
WSM: z=d/2=3/2
ν=1/(d−2)=1

$$\rho(E,\gamma) \sim \frac{k^d \lambda_k}{\epsilon_k} \sim E^{d/z-1} f(E|\gamma - \gamma_c|^{-z\nu})$$

$$\sim E^2 |\gamma - \gamma_c|^{-3/2}, \text{ for } \; E \ll |\gamma - \gamma_c|^{z\nu}$$

$$\sim E, \text{ for } \; E \gg |\gamma - \gamma_c|^{z\nu}$$





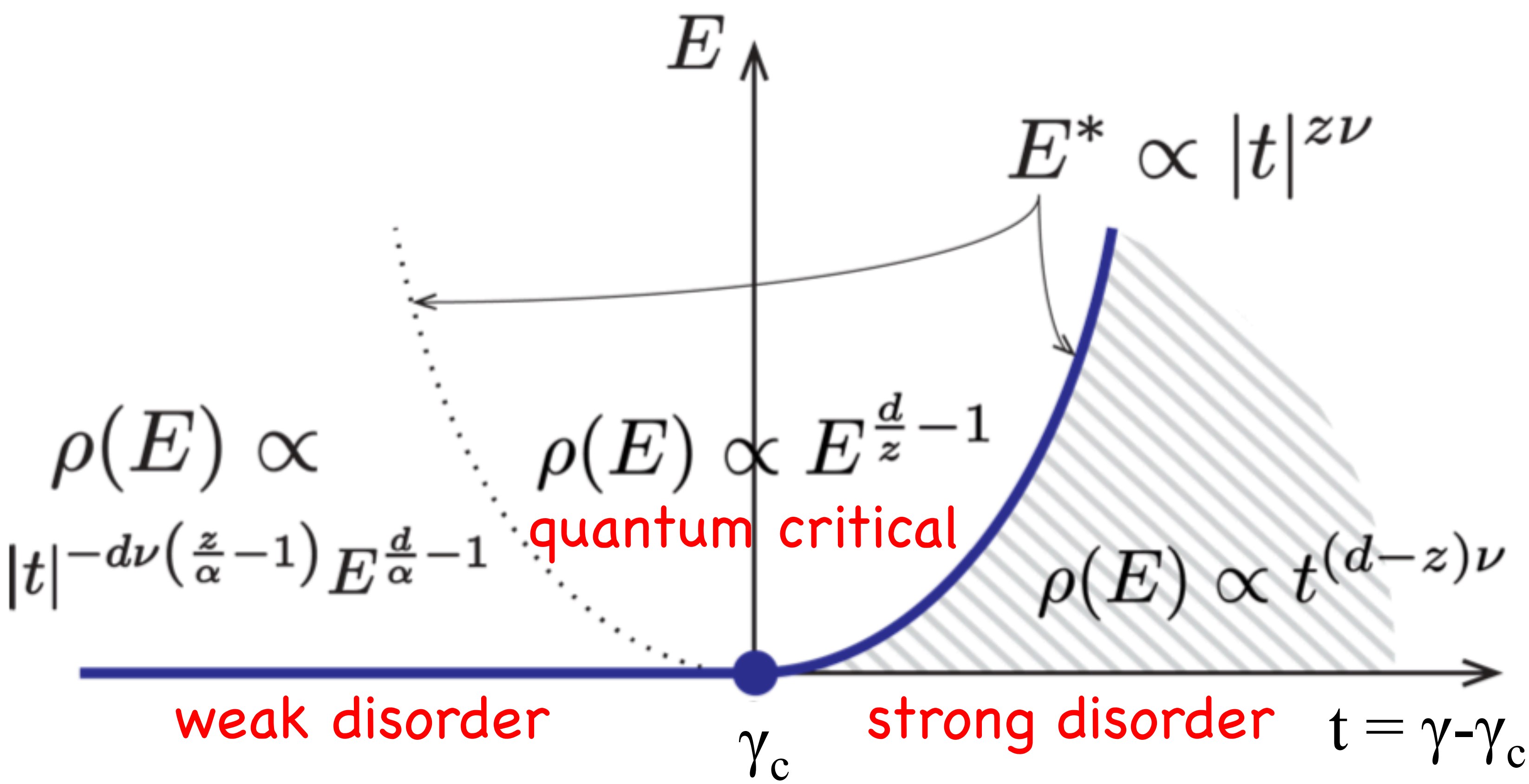
…ignoring rare regions of strong disorder Lifshitz tails (Nandkishore, et al.)

Scaling theory of density of states

$$\xi \sim |\gamma - \gamma_c|^{-\nu}, \quad E \sim \xi^{-z}$$

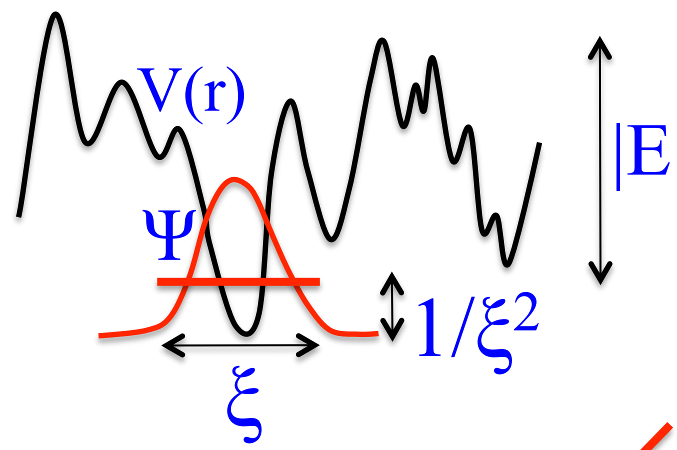
semiconductor:
$z = \alpha - \varepsilon/4$, $\nu = -1/\varepsilon$

$$\rho(E,\gamma) \sim \frac{k^d \lambda_k}{\epsilon_k} \sim E^{d/z-1} f(E|\gamma-\gamma_c|^{-z\nu})$$



Lifshitz tails

- deeply lying states E < 0: $\varepsilon(\hat{\mathbf{p}})\psi + V(\mathbf{r})\psi = E\psi$



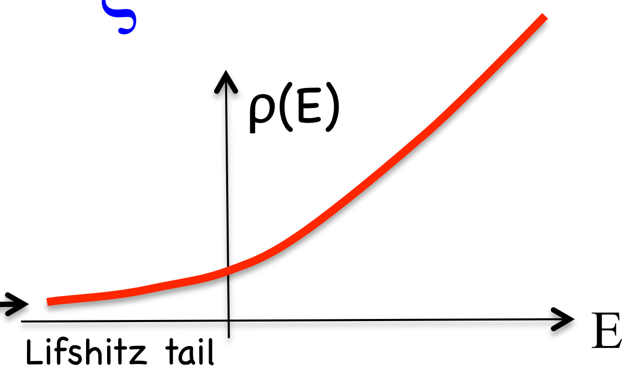
- probability of V, size ξ:

$$P(V) \sim e^{-\frac{1}{\Delta}\int d^d r V^2}$$

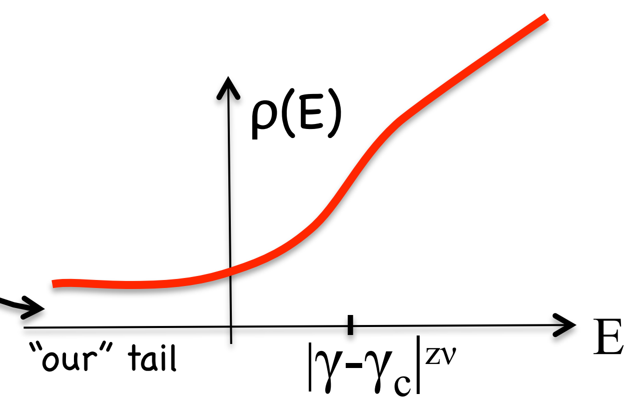
$$\sim e^{-\frac{\xi^d}{\Delta}(\kappa/\xi^\alpha + |E|)^2}$$

- maximize over ξ:

d < 2α: $\xi \sim E^{-1/\alpha}$ --> $\rho(E) \sim e^{-|E|^{(2\alpha-d)/\alpha}}$

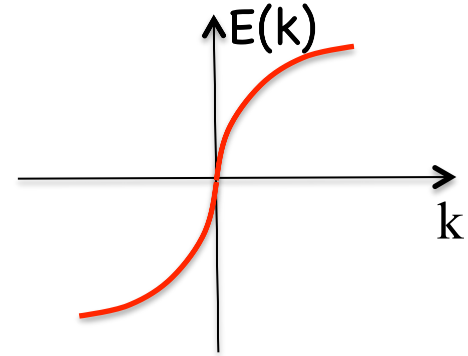


d > 2α: $\xi \sim r_0$ --> $\rho(E) \sim e^{-\frac{r_0^{d-2\alpha}}{\gamma}}$



1d chiral model

M. Gartner, et al. 2015



- model: $\hat{h} = |\hat{p}|^{\alpha}\mathrm{sgn}(\hat{p}) + V(x)$

  -> long-range hopping $J_{xx'}$

$$\hat{H} = \sum_{x,x'} J_{xx'}\hat{a}_x^{\dagger}\hat{a}_{x'} + \sum_x V_x \hat{a}_x^{\dagger}\hat{a}_x$$

- high-d transition in 1d: $\alpha < \alpha_c = d/2 = 1/2$
  - $z = 1/2$
  - $\nu = 1/(1-2\alpha)$
  - $\rho(E) \sim E^{\theta}$
  - $\theta = d/z - 1 = 1$, for $\gamma = \gamma_c$
  - $\theta = d/\alpha - 1$, for $\gamma < \gamma_c$

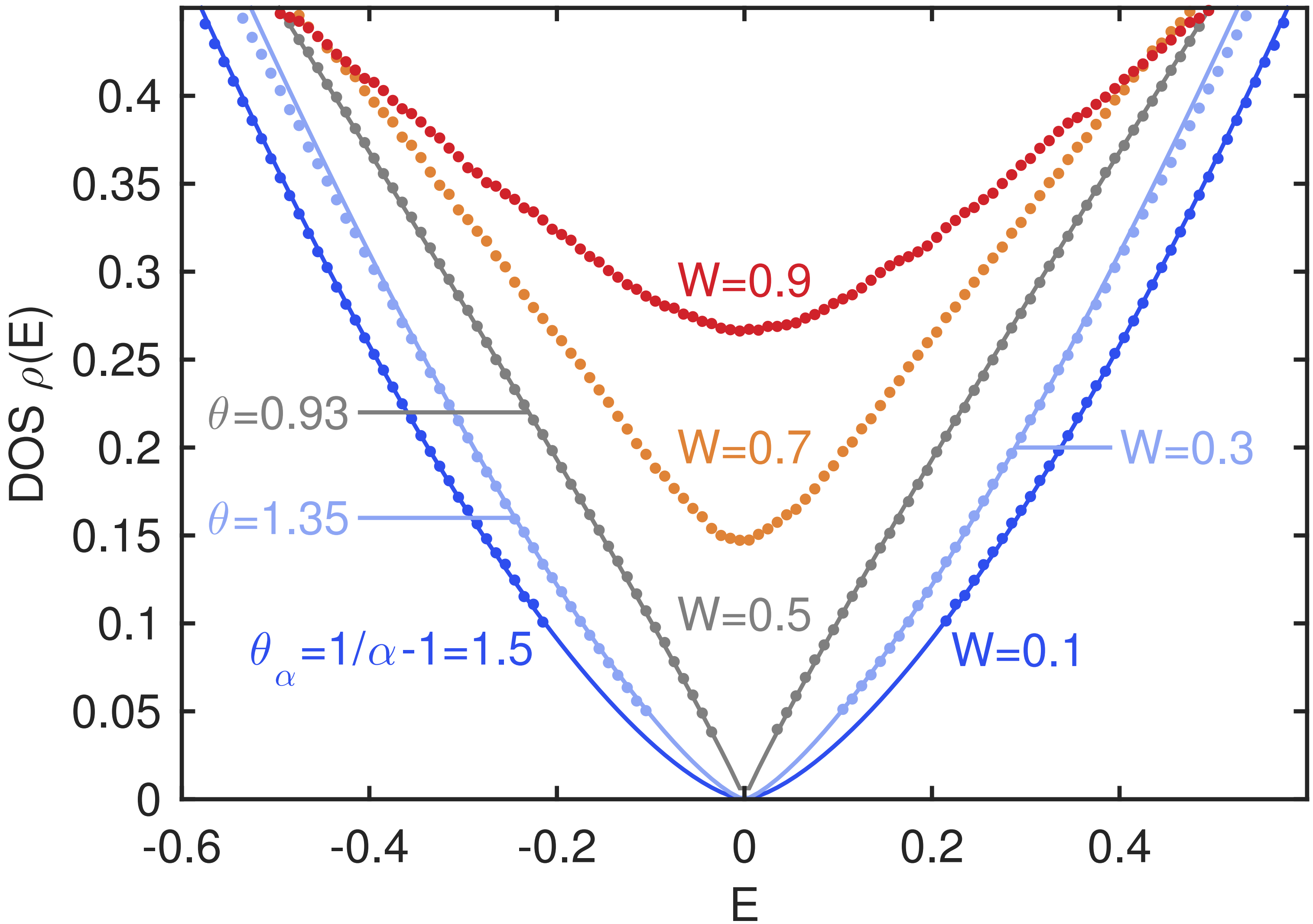
1d chiral model

M. Gartner, et al. 2015

Density of states $\rho(E) \sim E^{\theta}$

$\theta = d/z - 1 = 1$, for $\gamma = \gamma_c$

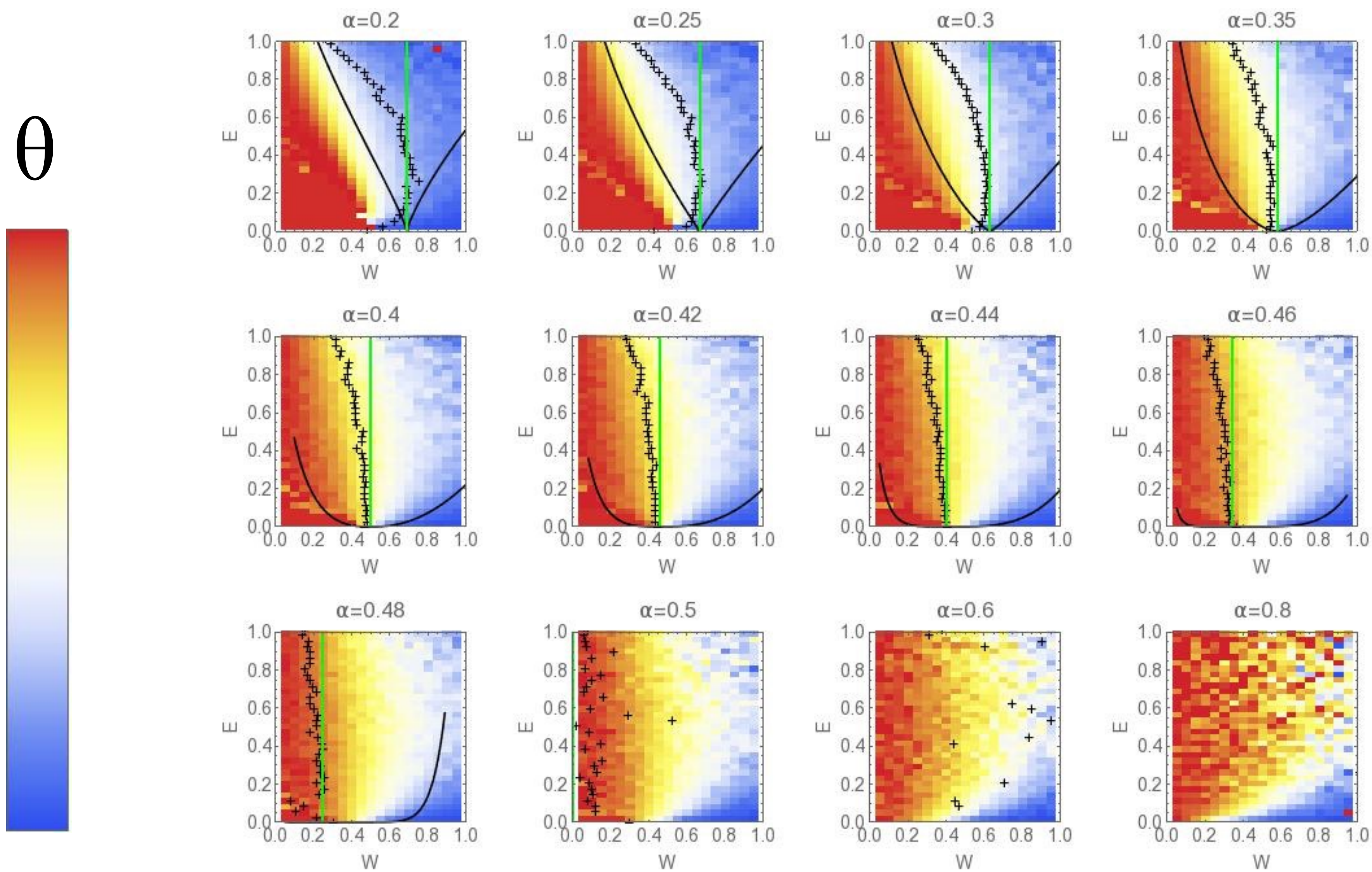
$\theta = d/\alpha - 1$, for $\gamma < \gamma_c$



1d chiral model

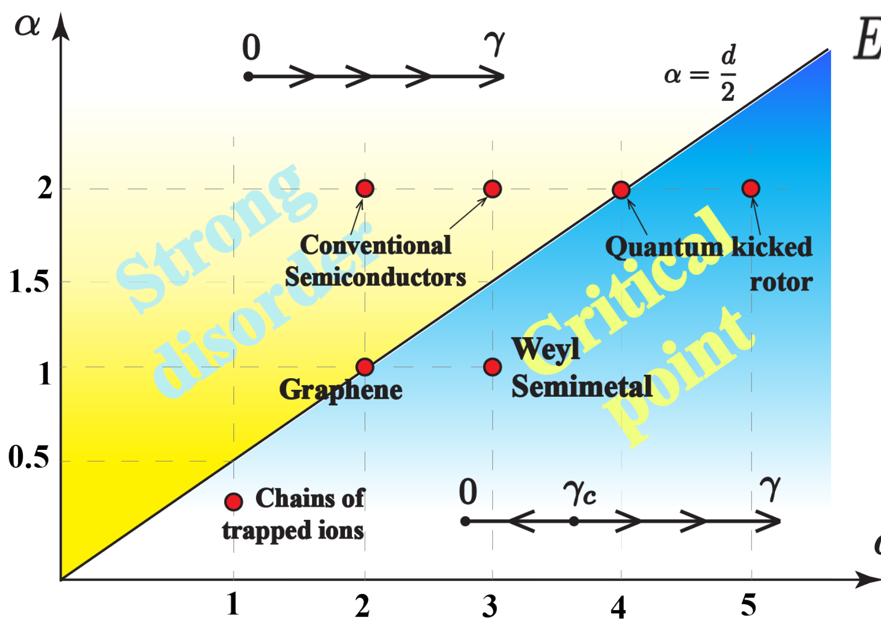
M. Gartner, et al. 2015

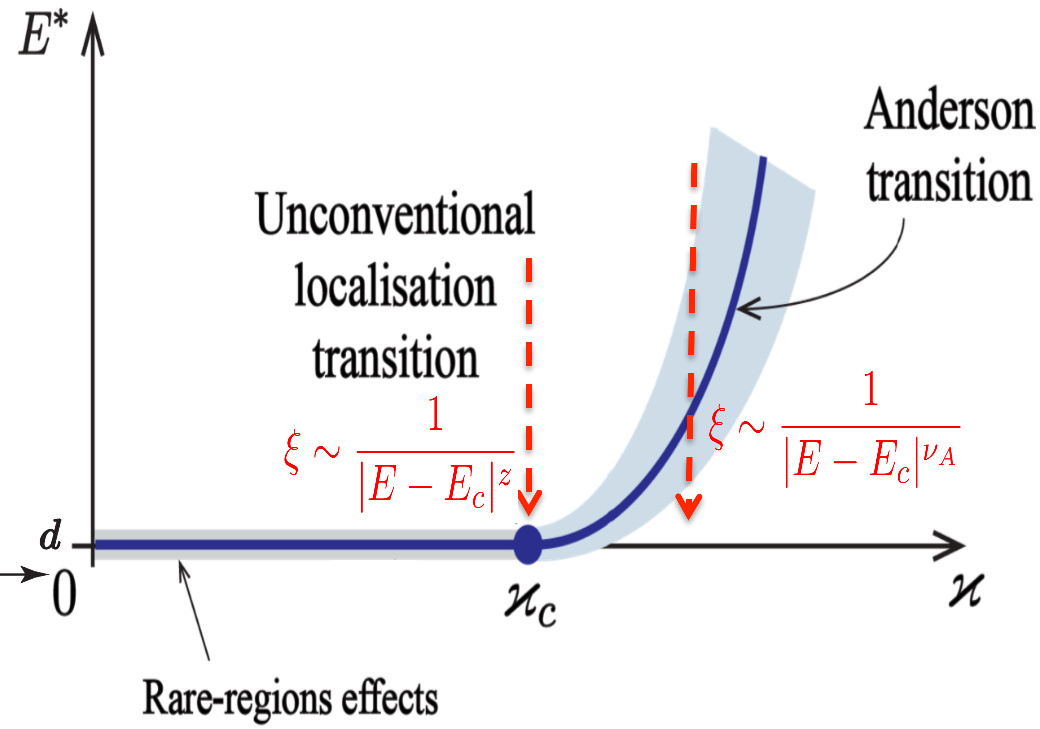
Density of states: $\rho(E) \sim E^{\theta}$



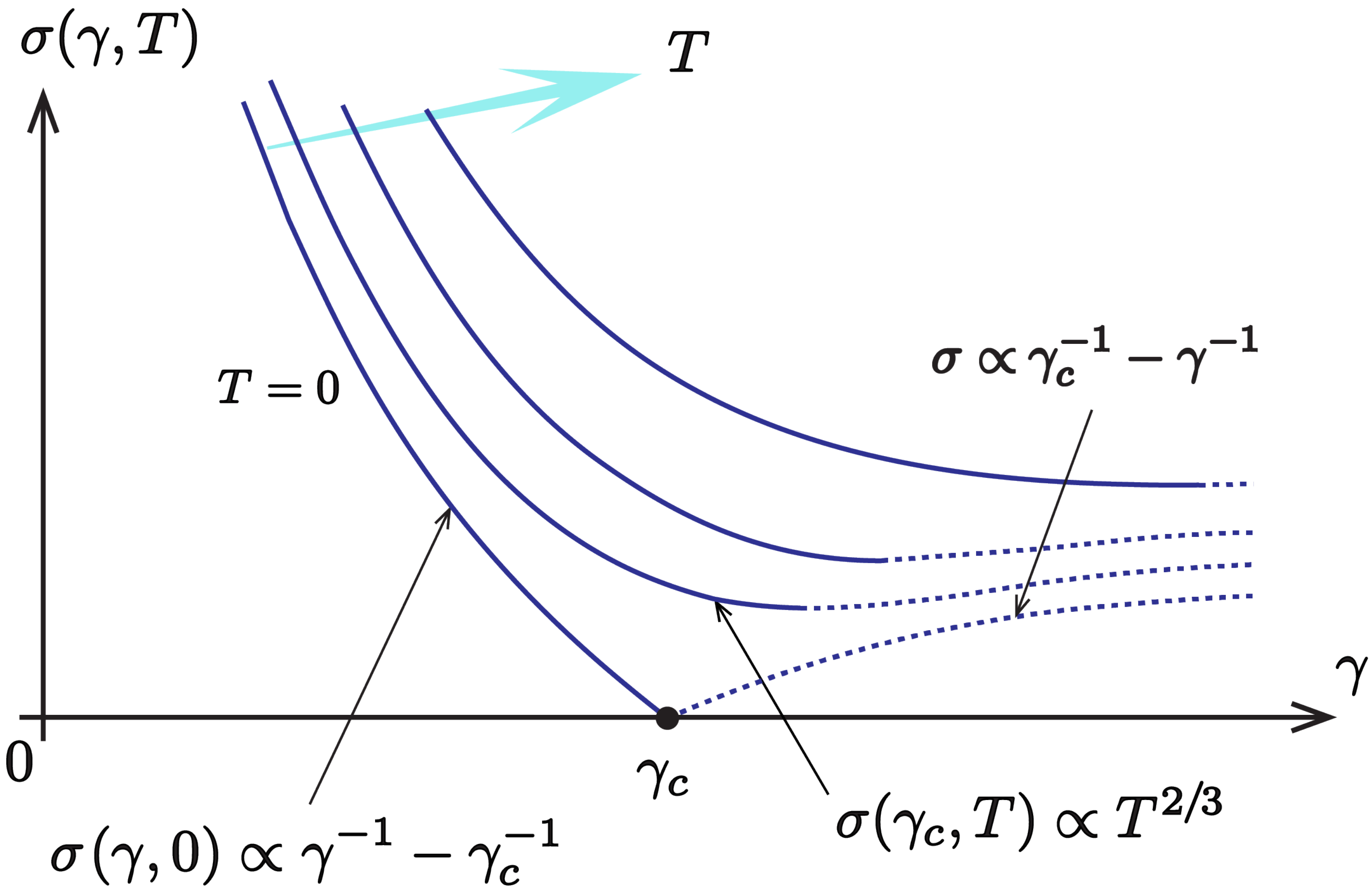
Summary and open questions

- studied transport in lightly-doped semiconductors, semimetals
- new weak-to-strong disorder-driven quantum phase transition
- RG and scaling theory --> conductivity, density of states





- numerics, experiments ?
- role of rare regions near critical point ?
- interplay with conventional Anderson localization ?
- Coulomb impurities ?



$\sigma(\gamma, T)$
T
$T = 0$
$\sigma \propto \gamma_c^{-1} - \gamma^{-1}$
γ
0
γ_c
$\sigma(\gamma_c, T) \propto T^{2/3}$
$\sigma(\gamma, 0) \propto \gamma^{-1} - \gamma_c^{-1}$

Results: critical density of states

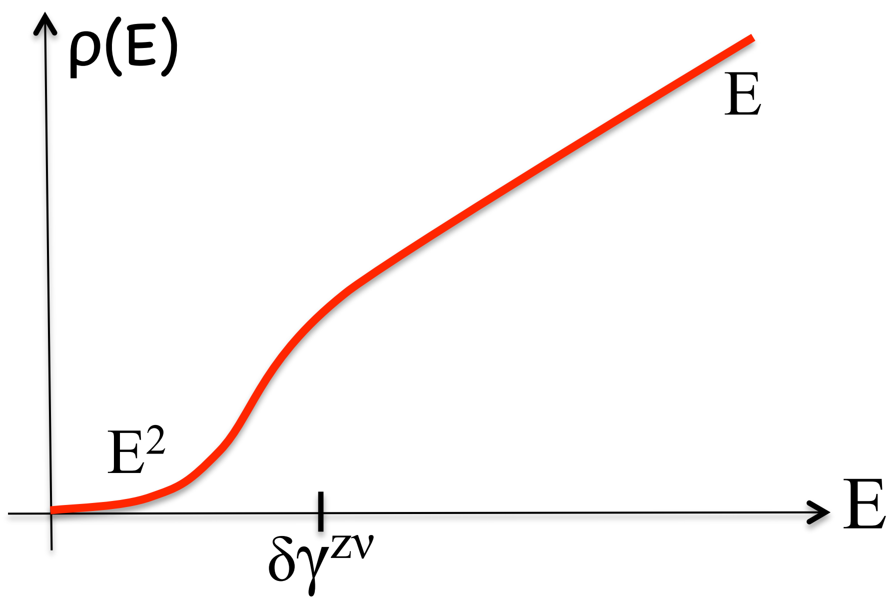
$$\xi \sim |\gamma - \gamma_c|^{-\nu}, \quad \omega \sim \xi^{-z}$$

WSM: z=d/2=3/2
ν=1/(d−2)=1

$$\rho(\omega,\gamma) = \omega^{d/z-1} f(\omega|\delta\gamma|^{-z\nu})$$

$$\sim \omega^2 |\gamma - \gamma_c|^{-3/2}, \text{ for } \omega \ll |\delta\gamma|^{z\nu}$$

$$\sim \omega, \quad \text{for } \omega \gg |\delta\gamma|^{z\nu}$$



...ignoring rare regions of strong disorder Lifshitz tails (Nandkishore, et al.)

Results: critical conductivity of Weyl SM

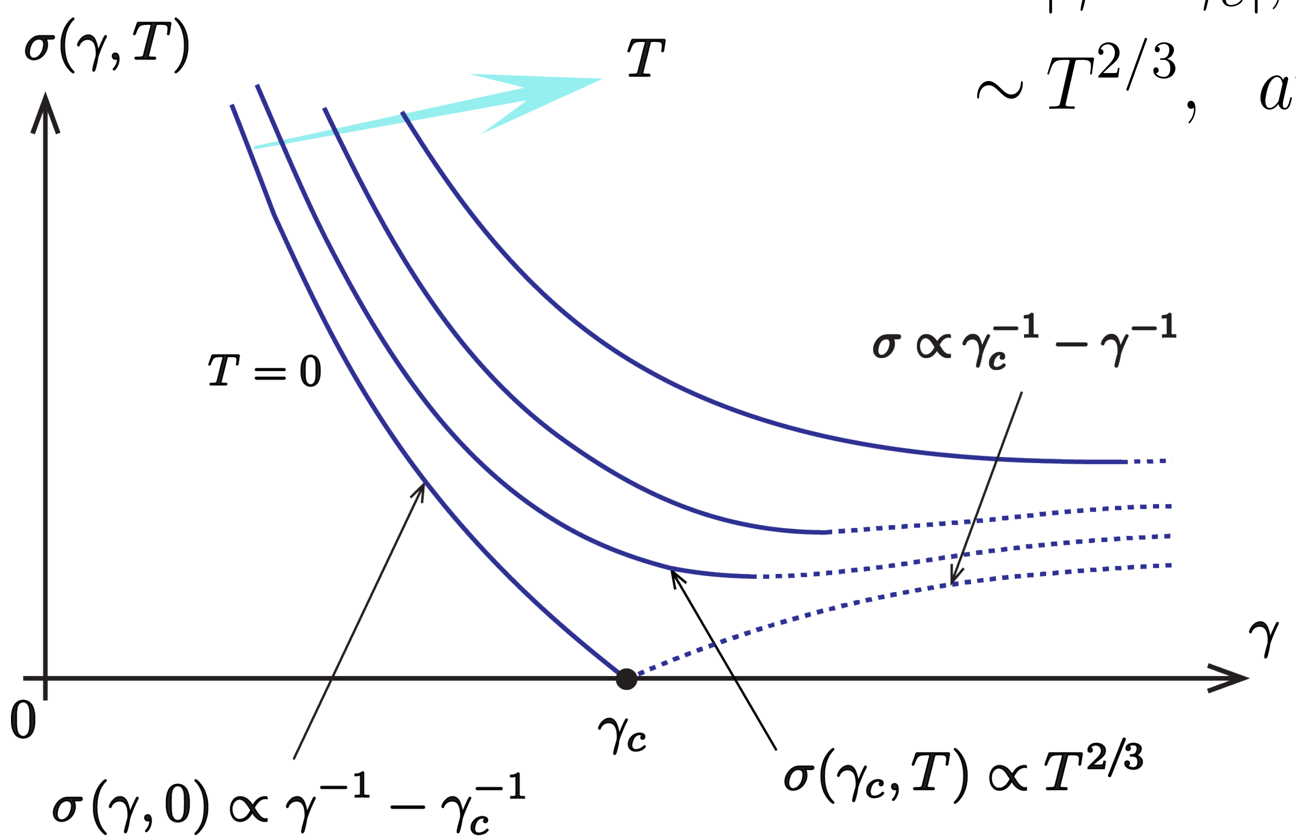
$$\xi \sim |\gamma - \gamma_c|^{-\nu}, \quad \omega \sim \xi^{-z}$$

WSM: z=d/2=3/2
ν=1/(d-2)=1

$$\sigma(\delta\gamma, T) \sim |\delta\gamma|^{\nu(d-2)} g\left[T|\delta\gamma|^{-z\nu}, \mu|\delta\gamma|^{-z\nu}\right]$$

$$\sim |\gamma - \gamma_c|, \quad at\ T = 0$$

$$\sim T^{2/3}, \quad at\ \gamma = \gamma_c$$



$$\sigma_D = \frac{1}{2} e^2 \nu v^2 \tau = \frac{e^2 v^2}{2\pi\Delta}$$