Resonant atomic gases

Leo Radzihovsky

for details see: *Gurarie, L.R., Annals of Physics, 322, 2-119 (2007)*
Sheehy, L.R., Annals of Physics, 322, 1790 (2007)

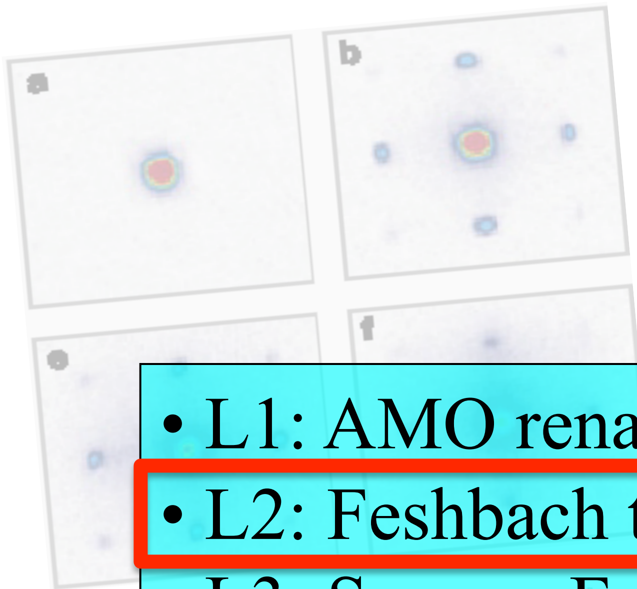
Giorgini, et al., RMP, 80, 885 (2008)
Ketterle and Zwierlein, Varenna lectures (2006)

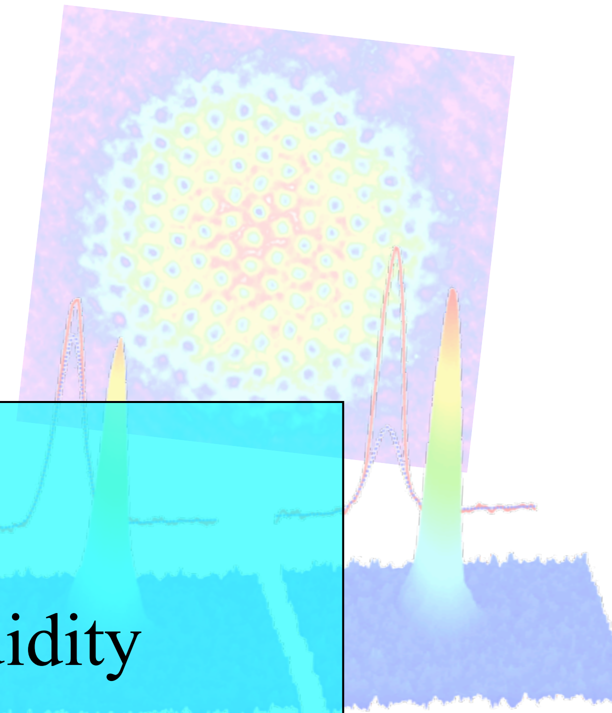
$: NSF

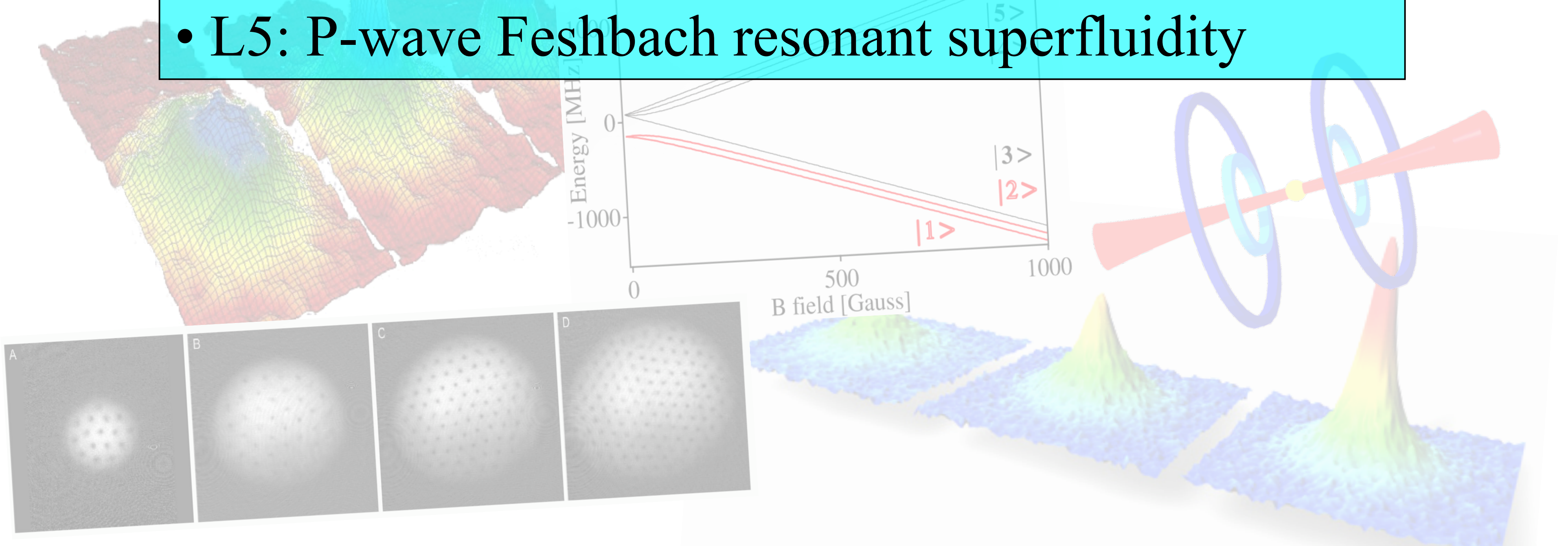
Mysore, India, Dec 2010

Course outline

- L1: AMO renaissance overview
- L2: Feshbach two-atom scattering
- L3: S-wave Feshbach resonant superfluidity
- L4: Imbalanced s-wave resonant Fermi gases
- L5: P-wave Feshbach resonant superfluidity

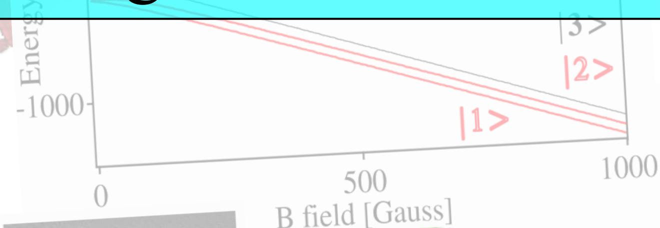


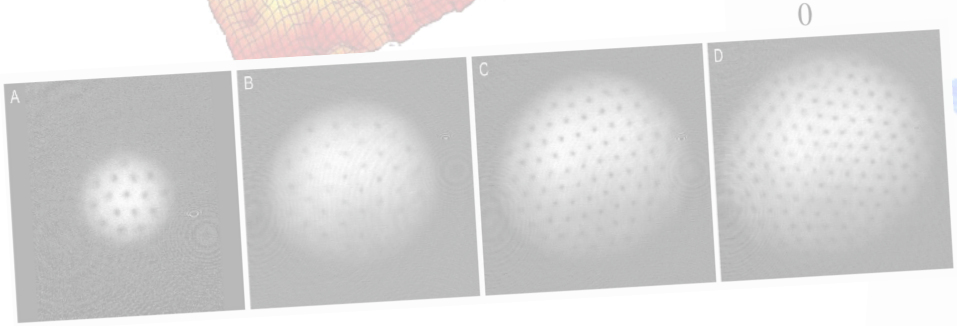




Lecture 2: Feshbach resonances

- review of scattering theory phenomenology:
  - o scattering f-amplitude, T-, S-matrix, phase shift
  - o low energy scattering universality
- hyperfine states of alkali atoms
- two-channel scattering model
- one-channel scattering model



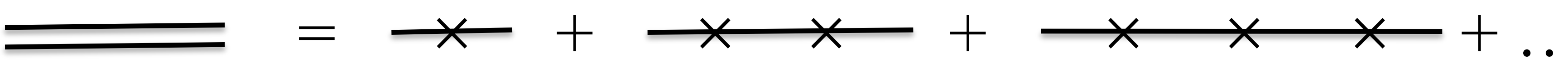


Review of scattering theory

- Schrodinger eqn via Greens function: $(H_0 + V)\psi = E\psi$

$$\psi = \psi_0 + \frac{1}{E - H_0} V\psi = \psi_0 + \frac{1}{E - H_0} T\psi_0$$

- T-matrix: $T = V + V\frac{1}{E - H_0}V + V\frac{1}{E - H_0}V\frac{1}{E - H_0}V \ldots = V + V\frac{1}{E - H_0}T$



$$\psi = e^{i\mathbf{k}\cdot\mathbf{r}} + \frac{2m_r}{\hbar^2}\int_{k'} \frac{T(k', k)e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2 - k'^2 + i\varepsilon} = e^{i\mathbf{k}\cdot\mathbf{r}} + \frac{e^{ikr}}{r}\underbrace{\frac{-2m_r}{4\pi\hbar^2}T(\mathbf{k}', \mathbf{k})}$$

- Scattering amplitude: $f(\mathbf{k}', \mathbf{k}) = \sum_{\ell}(2\ell + 1)f_\ell(k)P_\ell(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}})$

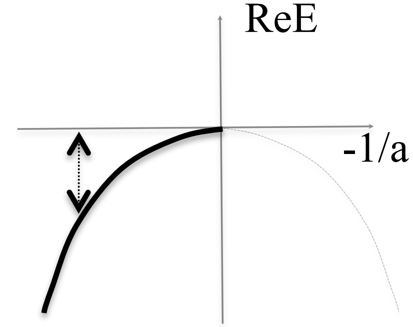
- Scattering matrix S and phase shift δ: $S_\ell = e^{i2\delta_\ell} = 2ikf_\ell + 1 = \frac{f_\ell}{f_\ell^*}$

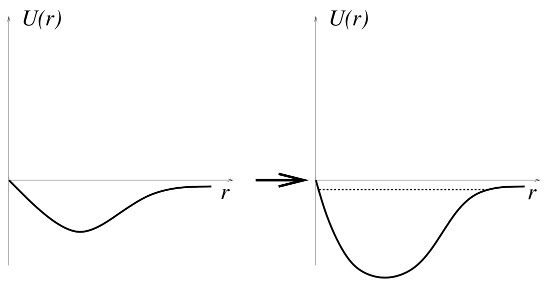
Low-E resonant scattering phenomenology

- unitarity: $$f_s = \frac{1}{F_s(k^2) - ik} \approx \frac{1}{-a_s^{-1} + \frac{r_0}{2}k^2 - ik} \qquad \left(f_\ell = \frac{1}{k^{-2\ell} F_\ell(k^2) - ik} \right)$$

$$-ka = \tan\delta_0 \qquad (F_\ell(k^2) = k^{2\ell+1}\cot\delta_\ell)$$

- low energy: $$k_{pole} = ia^{-1} \rightarrow E_{bound} = -\frac{\hbar^2}{2m_r a^2}$$

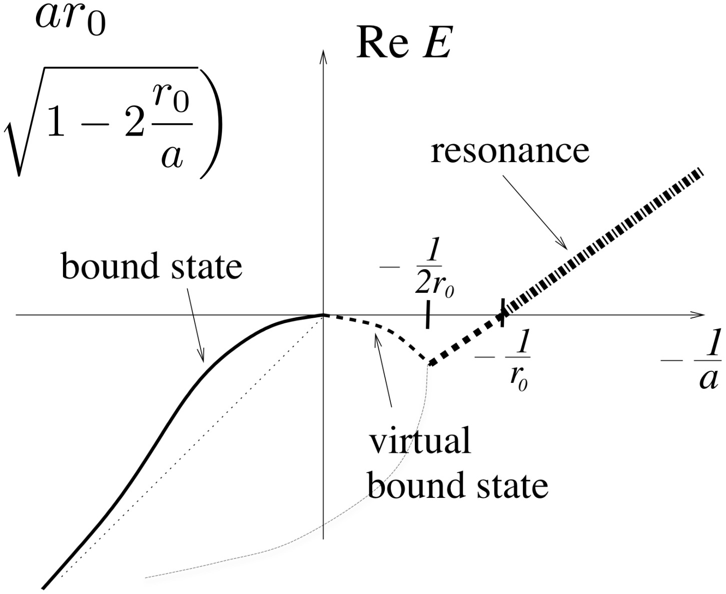


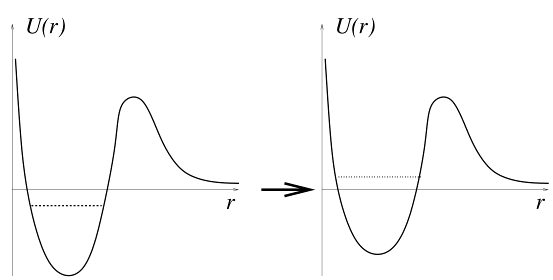


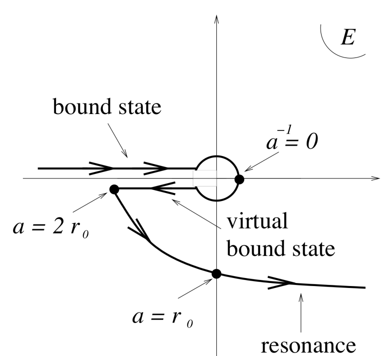
$$\psi \sim e^{ik_{\rm pole}r - iE_{\rm pole}t}$$

- intermediate energy ($r_0 < 0$): $$k_{\rm pole}^{\pm} = \frac{i}{r_0} \pm \frac{\sqrt{2ar_0 - a^2}}{ar_0}$$

$$\rightarrow E_{\rm pole} = \frac{1}{m_r r_0^2}\left(\frac{r_0}{a} - 1 + \sqrt{1 - 2\frac{r_0}{a}}\right)$$







Feshbach resonances

from C. Greene

- O. K. Rice, JCP 1, 375 (1933) - basic treatment of how a bound state autoionizes into a degenerate continuum
- U. Fano, Nuovo Cimento 12, 156 (1935) – shows that quantum interference has opposite signs above and below the resonance, leading to asymmetric line profiles analogous to anomalous dispersion
- G. Breit and E. Wigner, Phys. Rev. 49, 519 (1936) – Basic formula developed for symmetric resonance profile when only the “bound part” of the reaction dominates
- H. Feshbach, Ann. Phys. 5, 357 (1958) and 19, 287 (1962) – developed general projection operator formalism that cleanly separates “bound” and “continuum” subspaces and systematically treats their interaction
- U. Fano, Phys. Rev. 124, 1866 (1961) – more elegant reformulation of his 1935 theory of asymmetric line profiles from discrete-continuum interactions
- P. Anderson, Phys. Rev. 124, 41 (1961) – model of localized impurity state in a continuous band

Feshbach resonances

Feshbach resonances in neutron-sulfur scattering, from Blatt&Weisskopf, 1950s

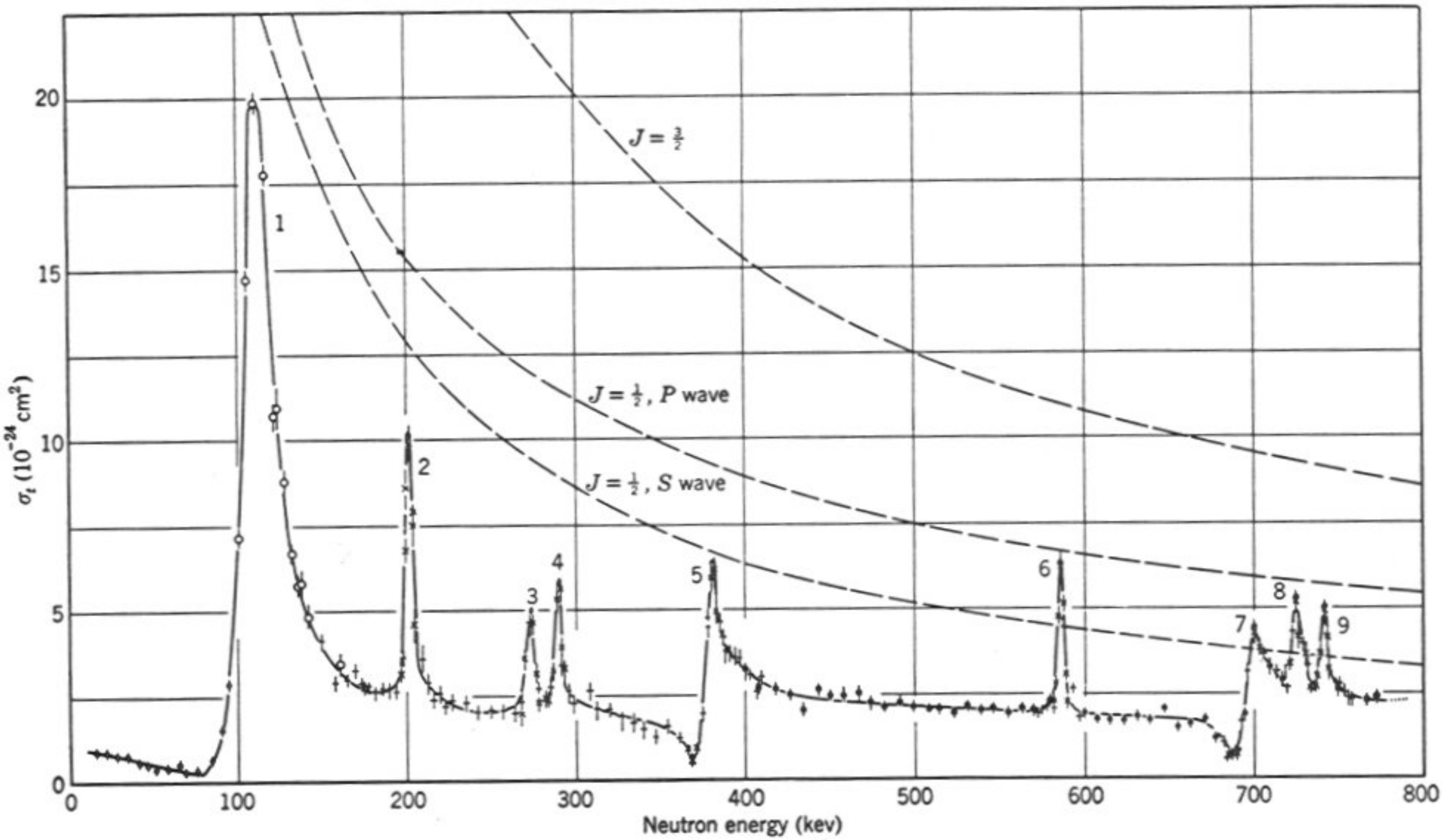
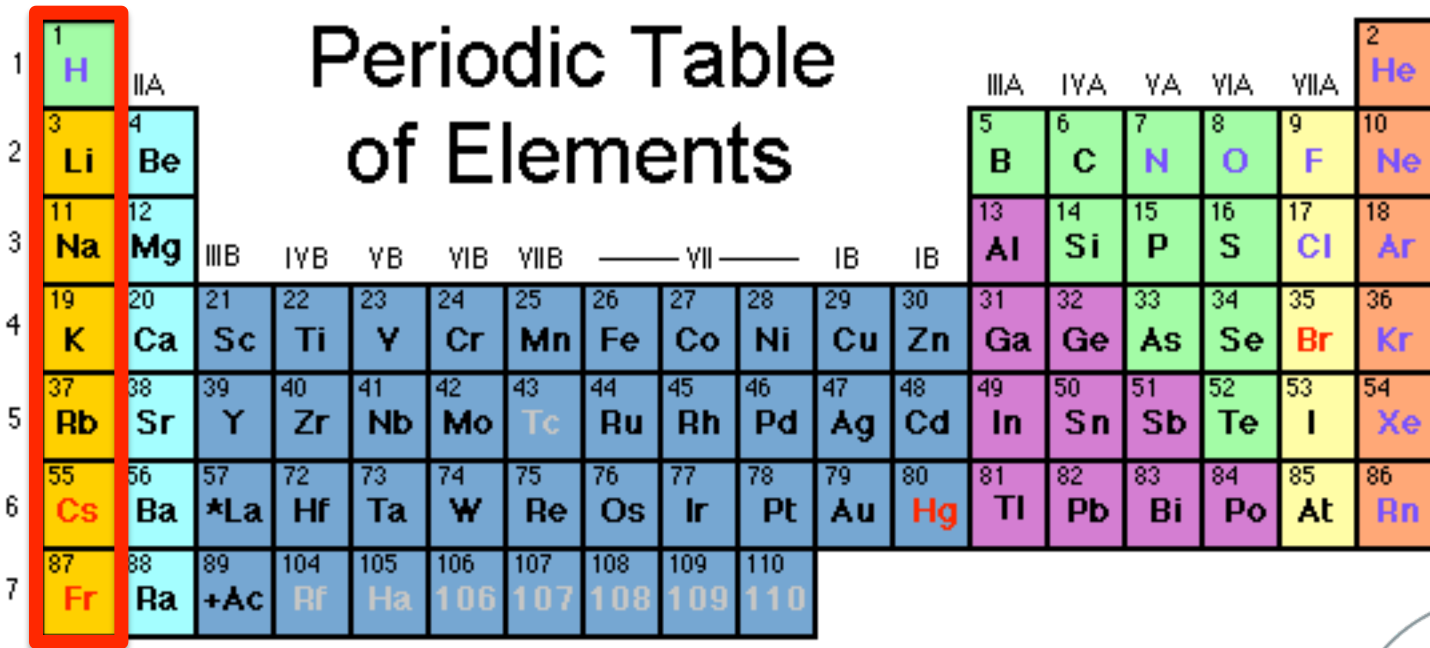
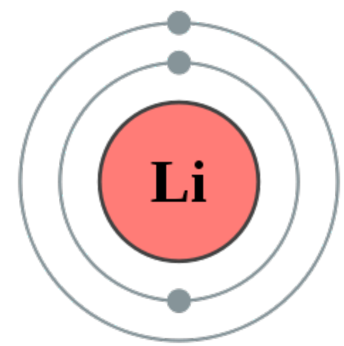


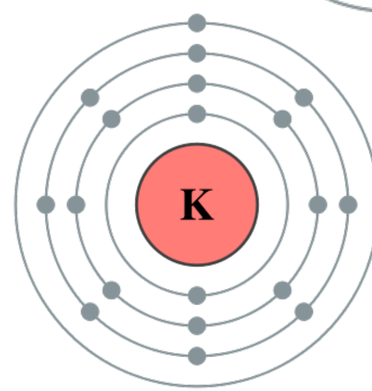
FIG. 2.2. Total neutron cross section for sulfur; experimental data taken from Adair (49) and Peterson (50).

Alkali atoms





- Li 6: $2S_{1/2}$ $|n=2,l=0,s=1/2,s_z>|i=1,i_z>$



- K40: $4S_{1/2}$ $|n=4,l=0,s=1/2,s_z>|i=4,i_z>$

Hyperfine interaction in a B field

$$H_{HF} = \alpha_{HF} \vec{I} \cdot \vec{S} - (g_I \mu_N \vec{I} + g_S \mu_B \vec{S}) \cdot \vec{B}$$

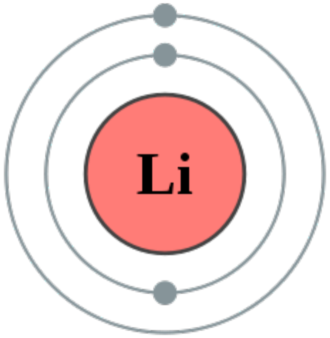
- Li 6: $2S_{1/2}$ $|n=2, l=0, s = 1/2, s_z>|i=1, i_z>$

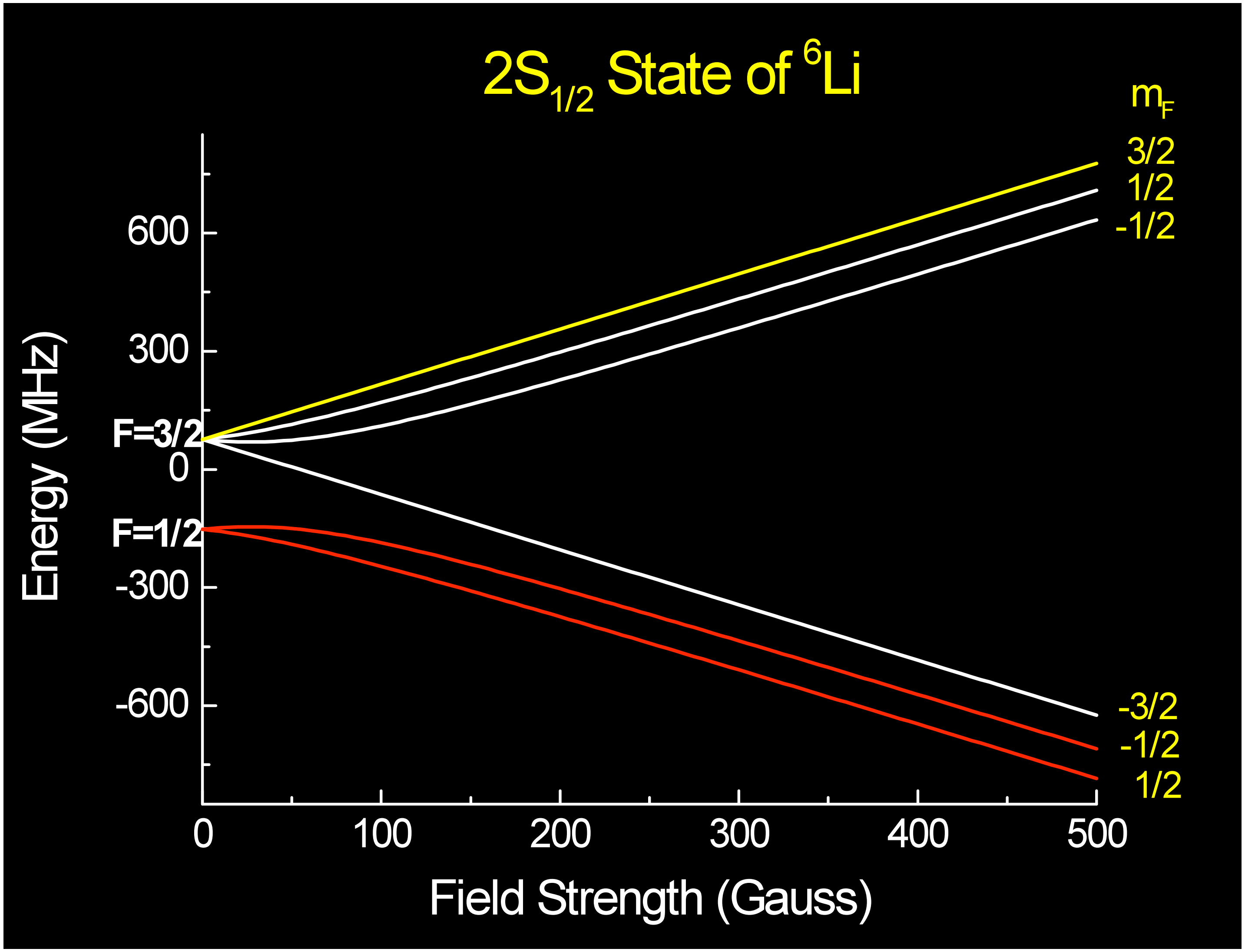
$\Longrightarrow \frac{1}{2} \otimes 1 = \frac{3}{2} \oplus \frac{1}{2}$ $|n=2, l=0, f, m_f>$ *(B=0)*

- K40: $4S_{1/2}$ $|n=4,l=0,s=1/2,s_z>|i=4,i_z>$

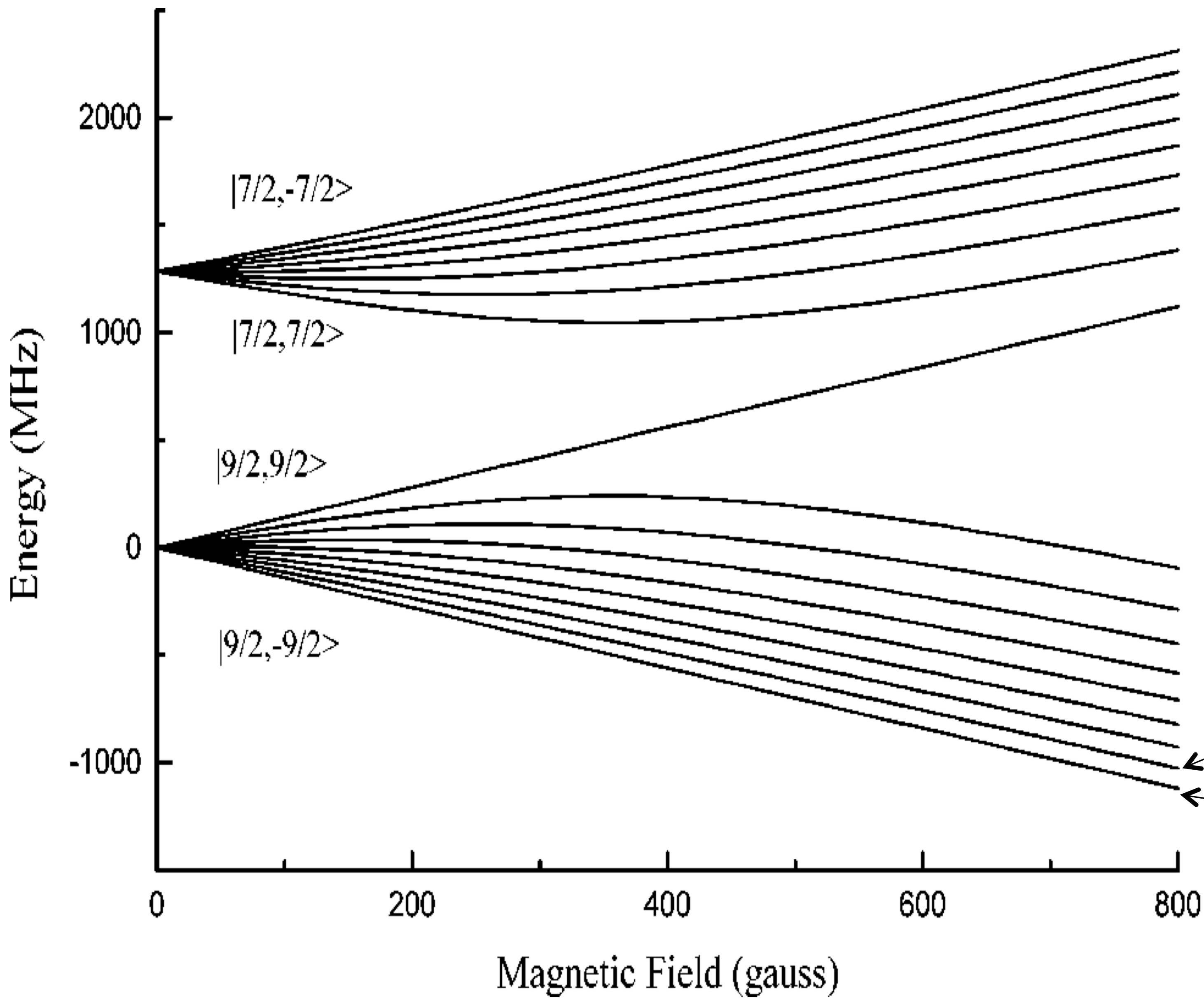
$\Longrightarrow \frac{1}{2} \otimes 4 = \frac{9}{2} \oplus \frac{7}{2}$ $|n=4, l=0, f, m_f>$ *(B=0)*

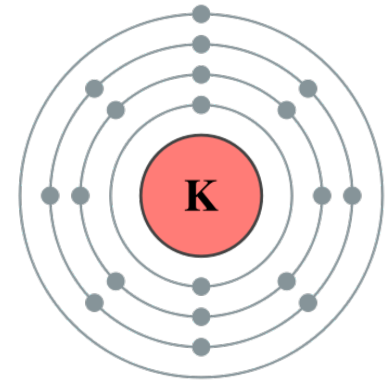
Hyperfine states of Li6

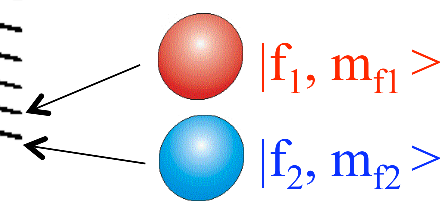




Hyperfine states of K40

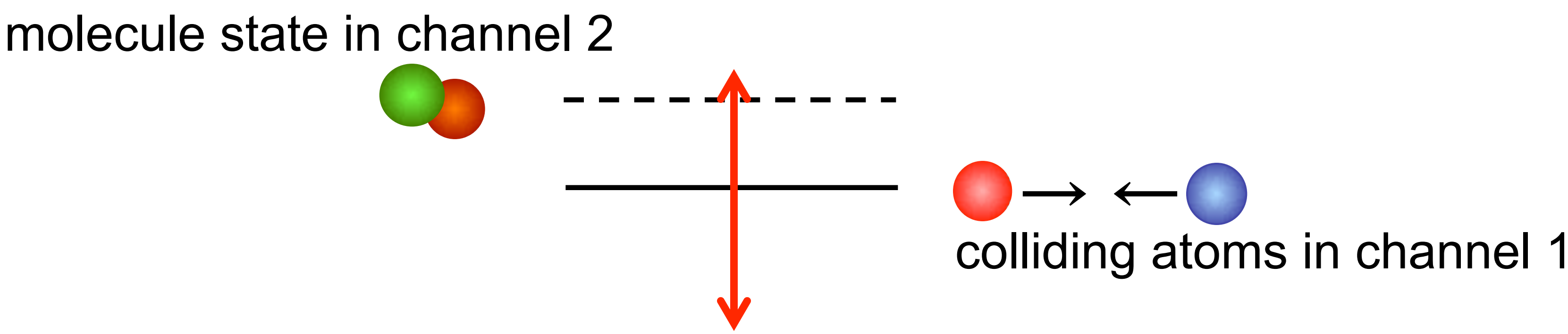






Atomic Feshbach resonances

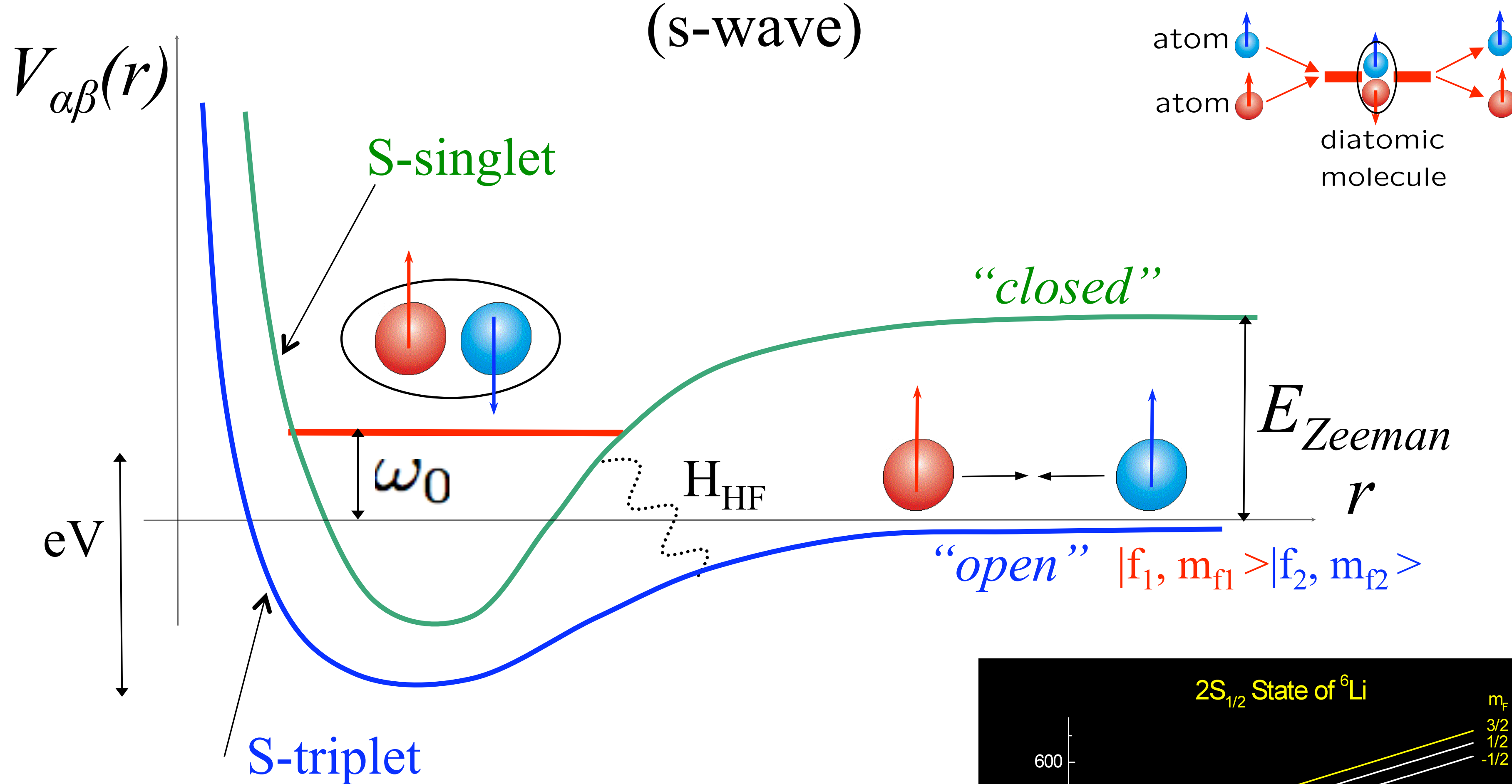
A magnetic-field tunable atomic scattering resonance



Channels are coupled by the hyperfine interaction

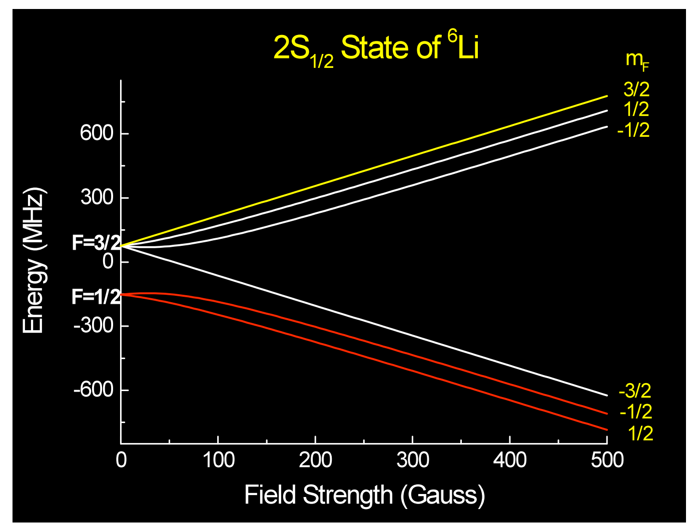
Atomic Feshbach resonances

(s-wave)



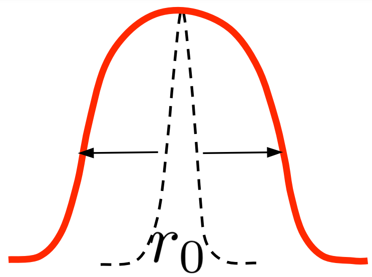
microscopics: project quantum chemistry short-scale calculation of $V^{s/t}(r)$ onto hyperfine states at long scales → *diagonalize 36 X 36 (e.g., for Li6)*

$$V_{\alpha\beta}(r) = \langle \alpha_1 \alpha_2 | \hat{V}_s + \hat{V}_t | \beta_1 \beta_2 \rangle$$



Detect Feshbach resonances in alkali atoms

- cloud size:

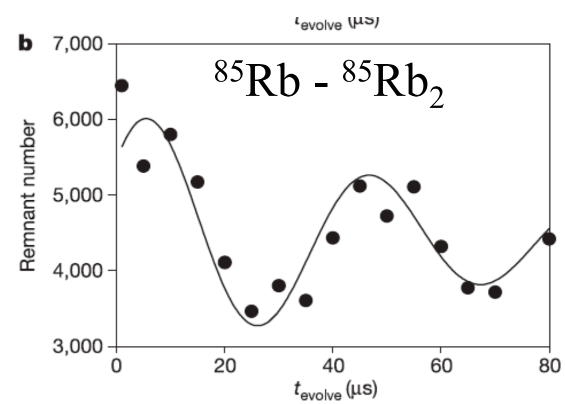


$$R(N) \sim (ar_0^4)^{1/5} N^{1/5}$$

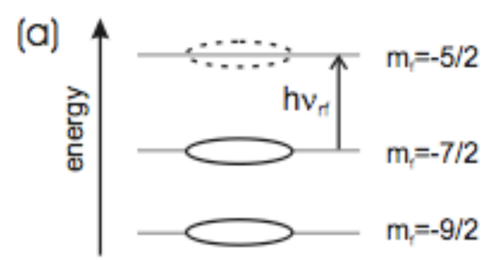
- atom loss via enhanced three-body decay rate: $\Gamma_3 \sim \frac{\hbar^2}{m} a^4 n^2$

- bound state Rabi oscillations (Ramsi fringes):

$$E_{\text{bound}} = -\frac{\hbar^2}{ma^2}$$



- RF spectroscopy resonance interaction shifts:



Rb85-Rb85 Feshbach resonance

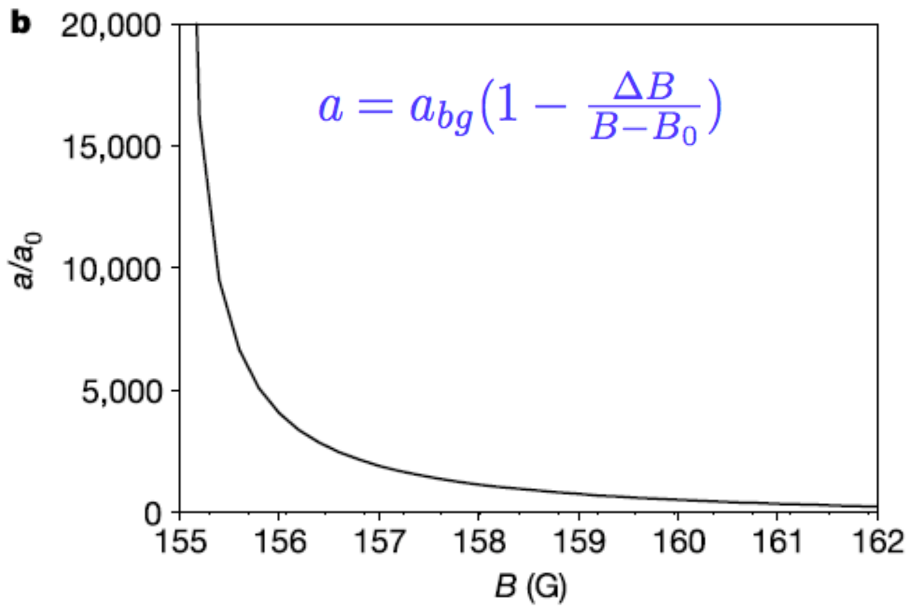
Atom–molecule coherence in a Bose–Einstein condensate

Elizabeth A. Donley, Neil R. Claussen, Sarah T. Thompson & Carl E. Wieman

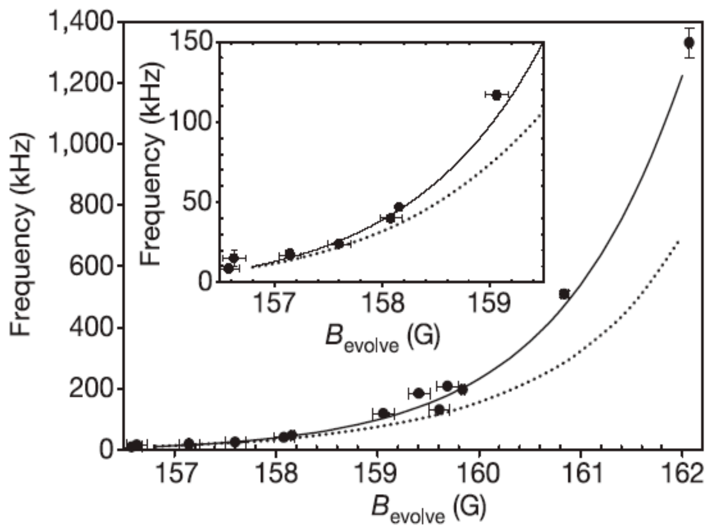
JILA, University of Colorado and National Institute of Standards and Technology, Boulder, Colorado 80309-0440, USA

NATURE | VOL 417 | 30 MAY 2002

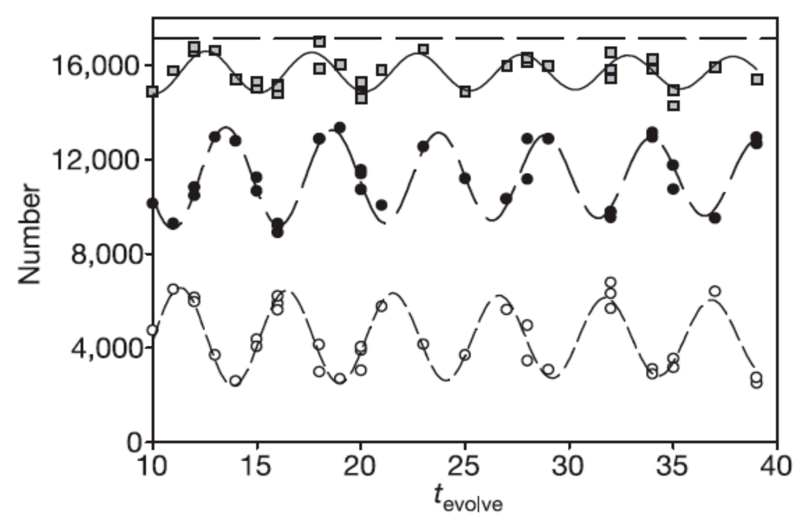
$|F = 2, m_F = -2\rangle$



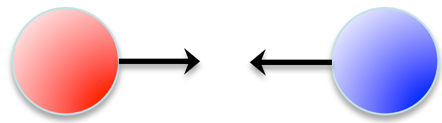
molecular binding energy

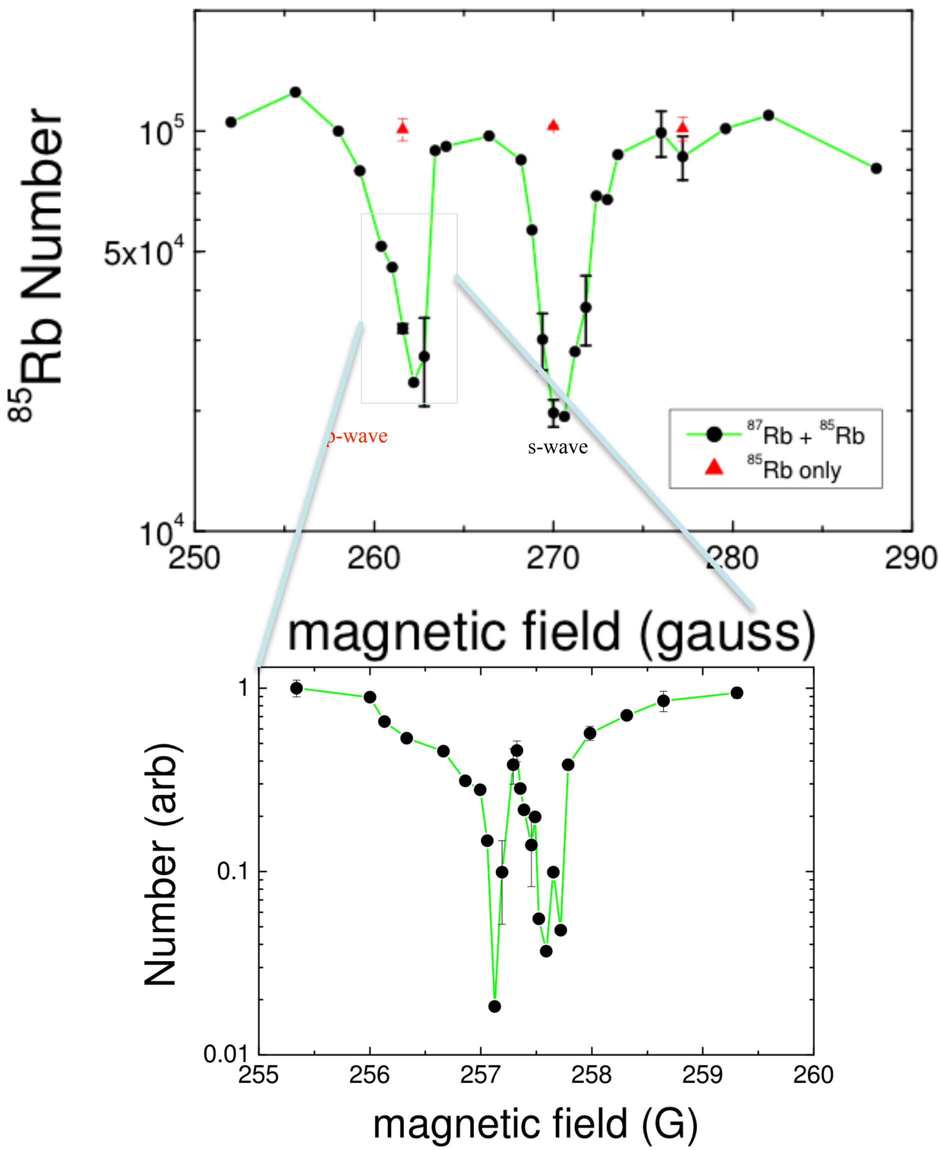


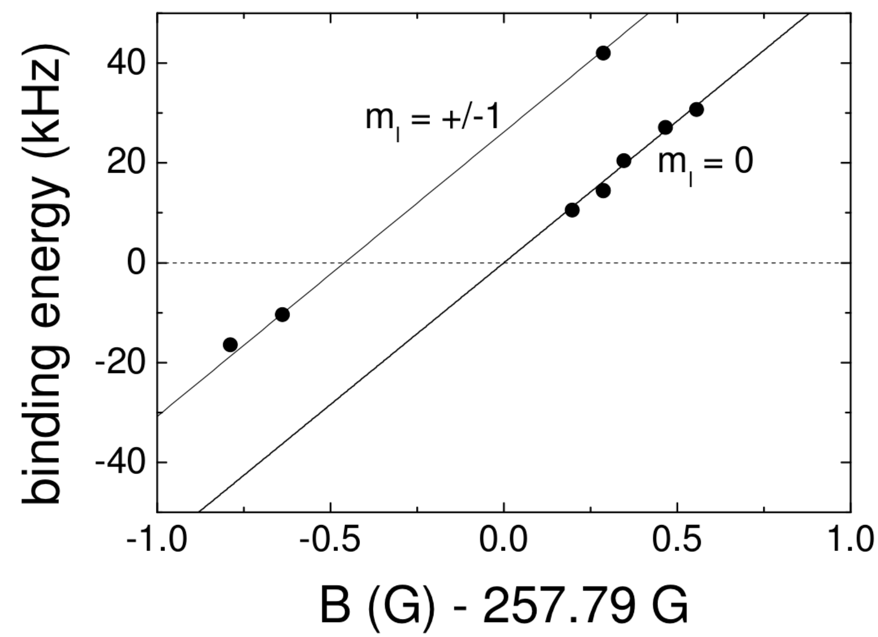
Rabi oscillations



Rb85-Rb87 s- and p-wave Feshbach resonance

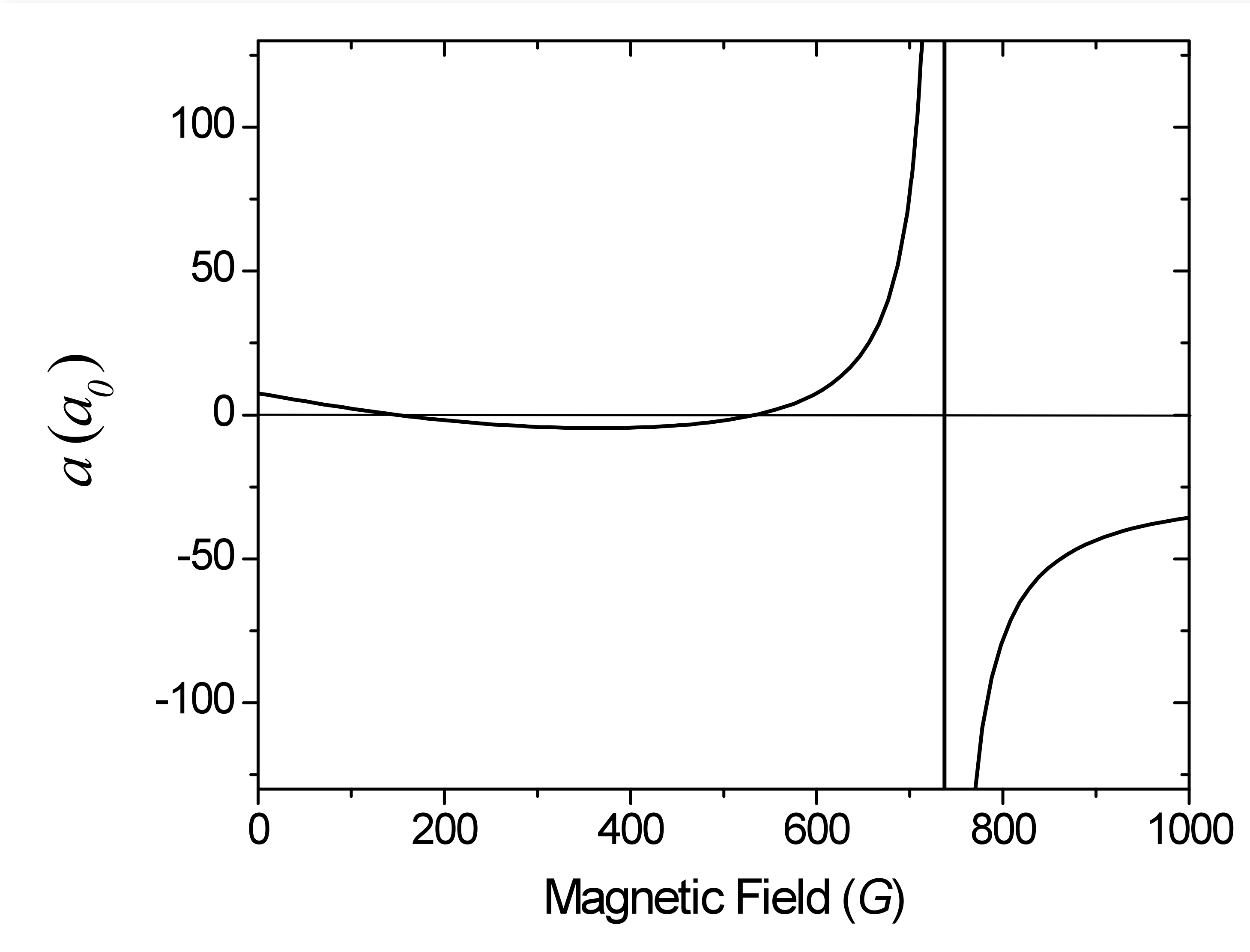
^{85}Rb : F=2, m_F= - 2  ^{87}Rb : F=1, m_F= - 1



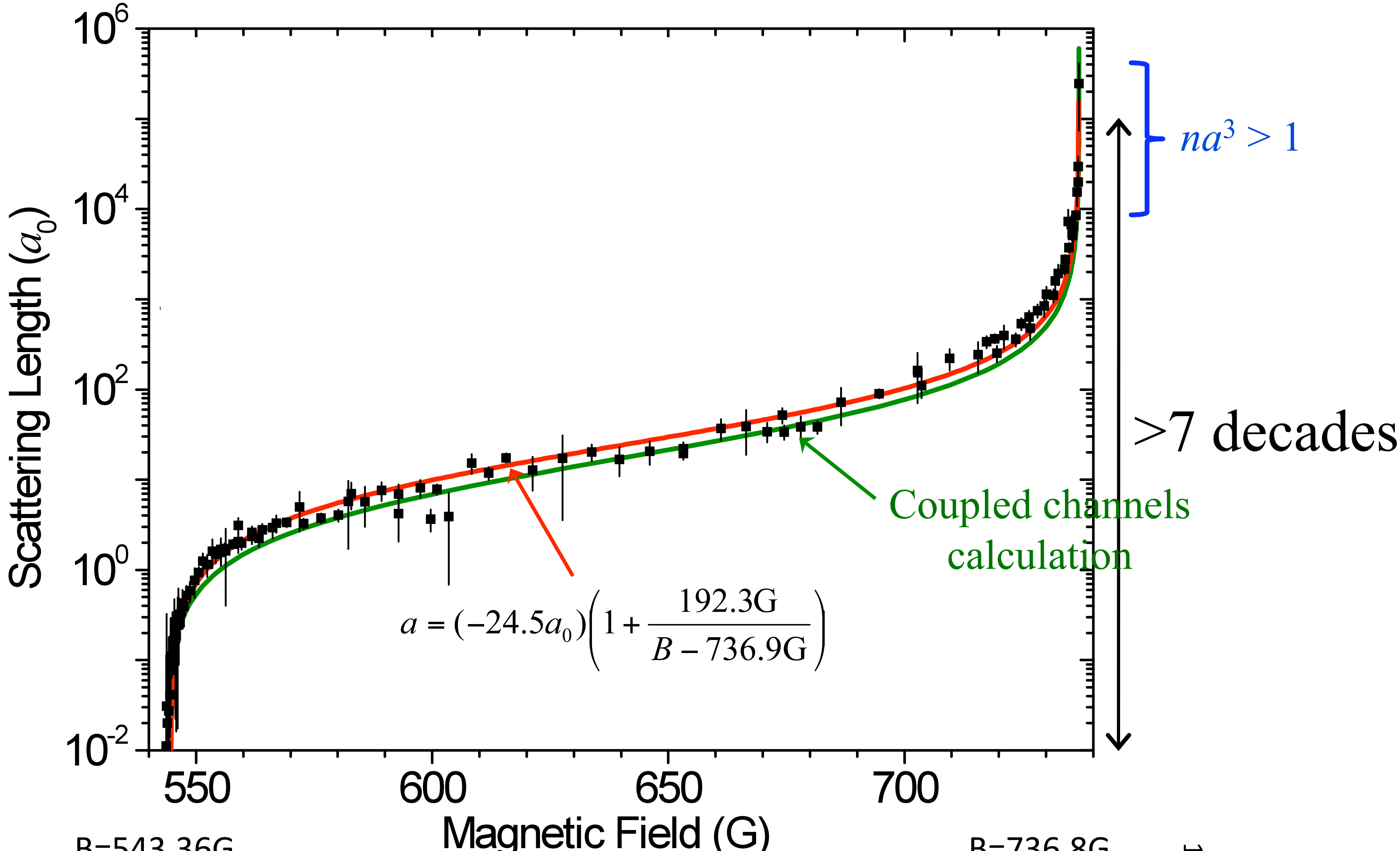


Papp's Thesis
Papp, Pino, Wieman

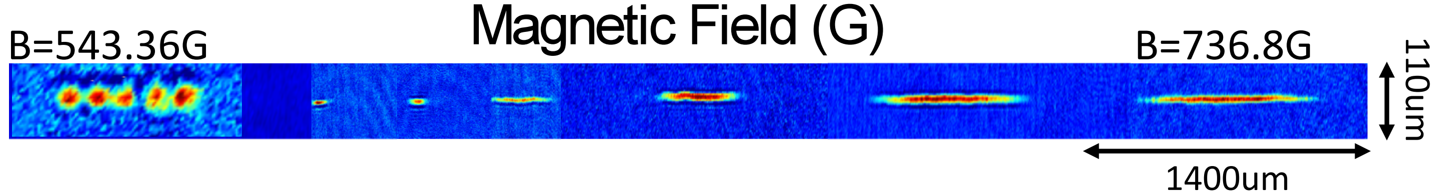
Li7-Li7 s-wave Feshbach resonance



Li7-Li7 s-wave Feshbach resonance

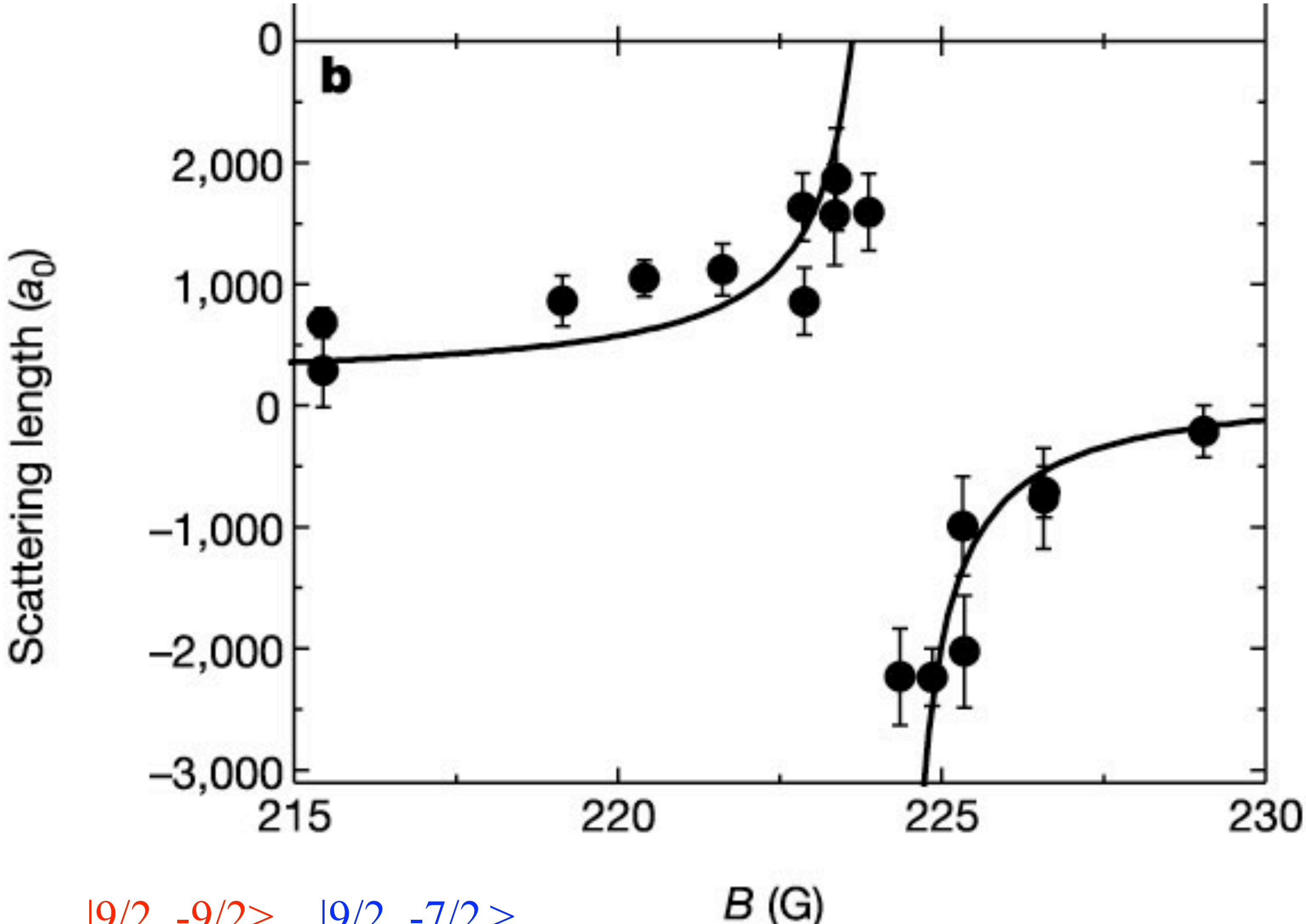


Solitons

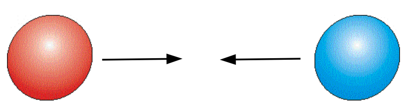


Pollack, Dries, Junker, Chen, Corcovilos, Hulet, PRL **102**, 090402 (2009)

K40-K40 s-wave Feshbach resonance



|9/2, -9/2> |9/2, -7/2 >



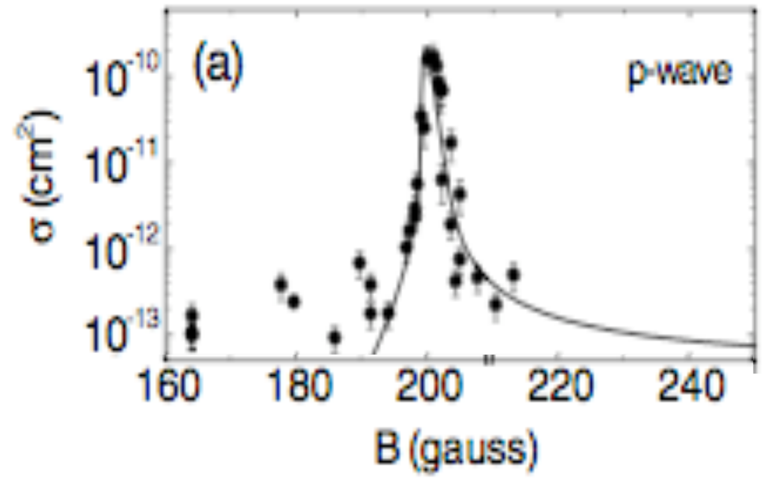
Regal, Jin 2003

K40-K40 p-wave Feshbach resonance

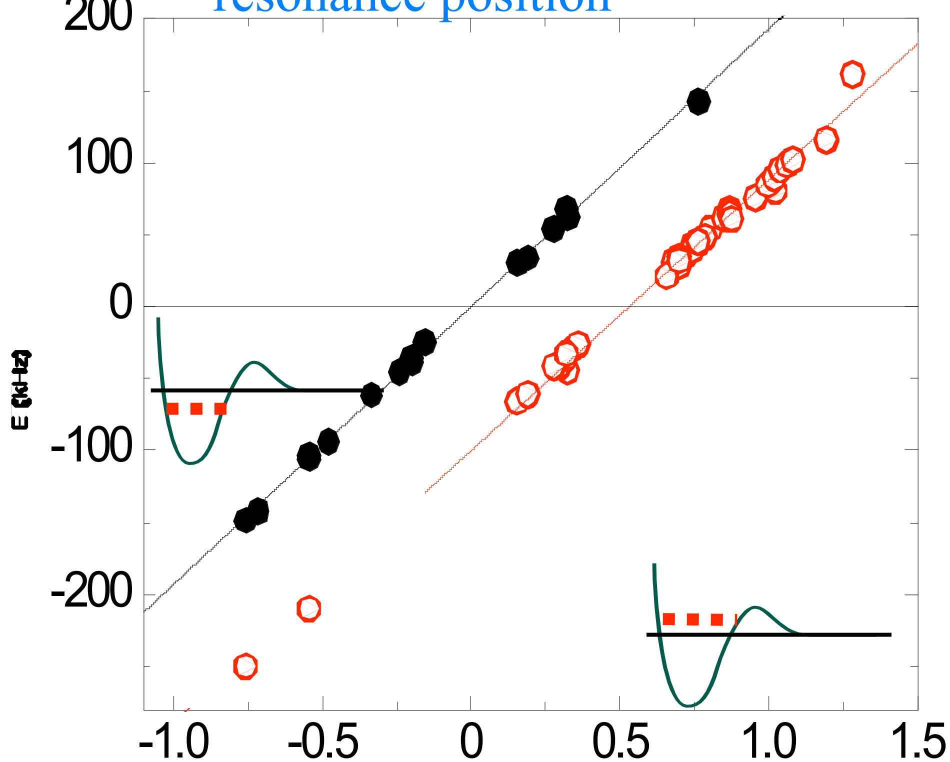
Regal, et al.'03

- p-wave Feshbach resonance in ^{40}K,
- making p-wave molecules:

  resonant disappearance of atoms
  with oscillating B(t)

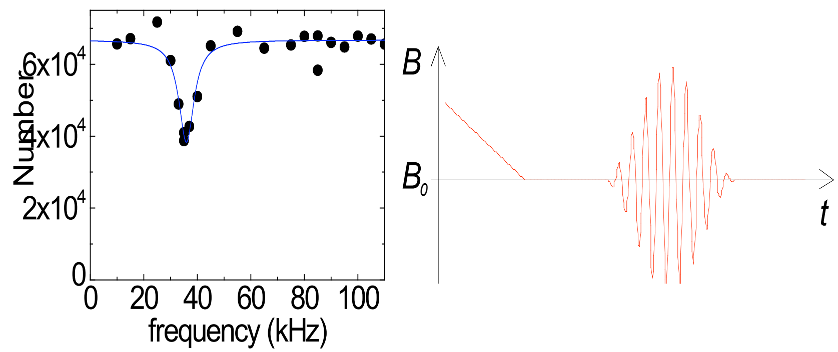


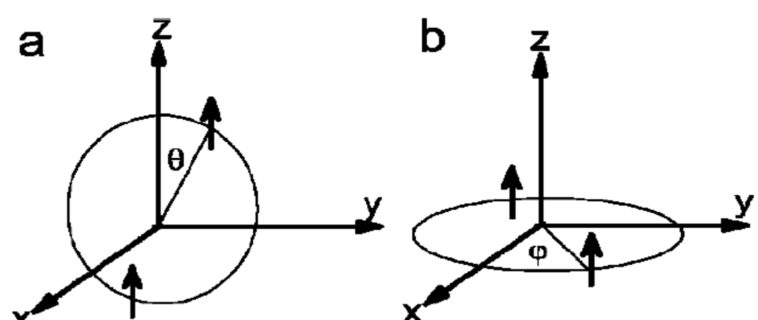
resonance position



B (Gauss)

Gaebler, et al.'07

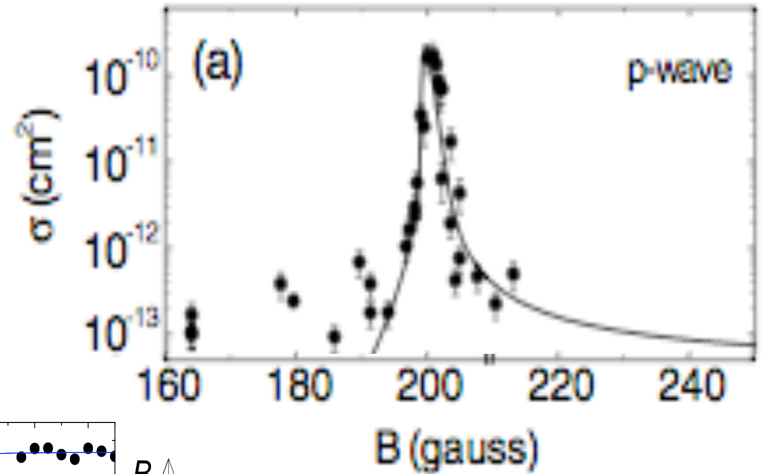




$m_\ell = 0$ $\quad$ $m_\ell = \pm 1$

K40-K40 p-wave Feshbach resonance

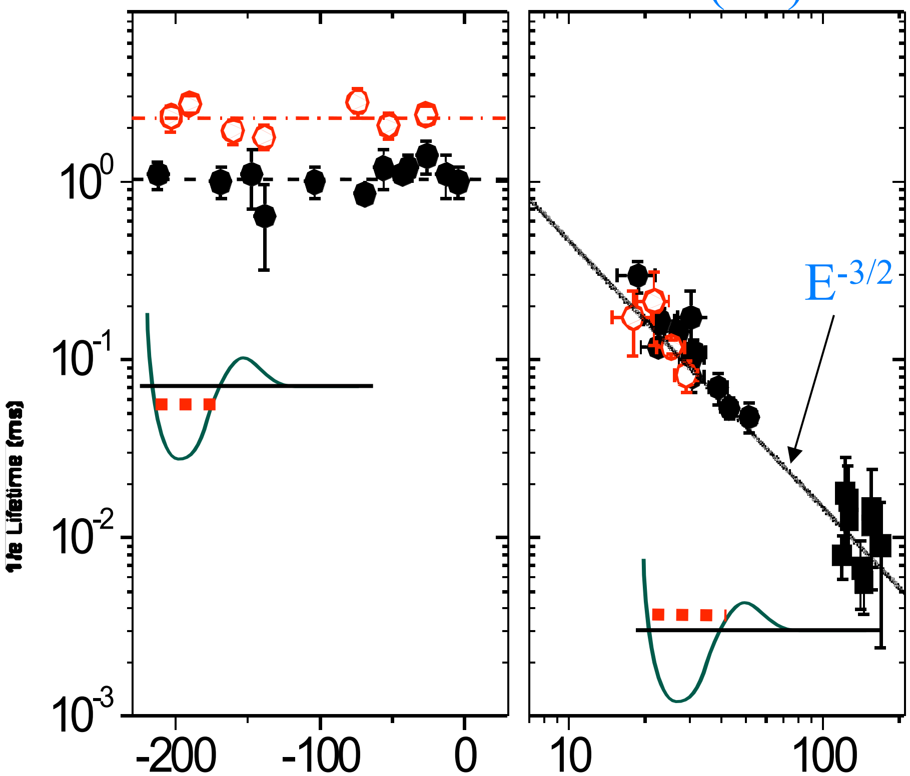
Regal, et al.'03



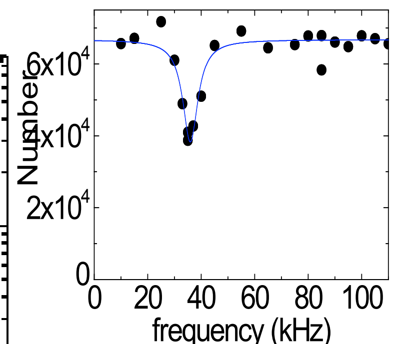
- p-wave Feshbach resonance in ^{40}K
- making p-wave molecules:

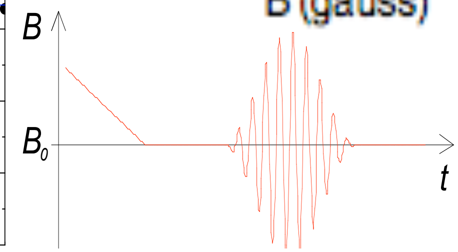
  resonant disappearance of atoms
  with oscillating B(t)

resonance lifetime (ms)

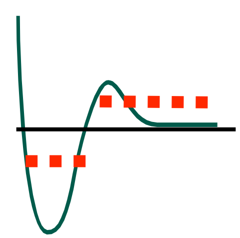


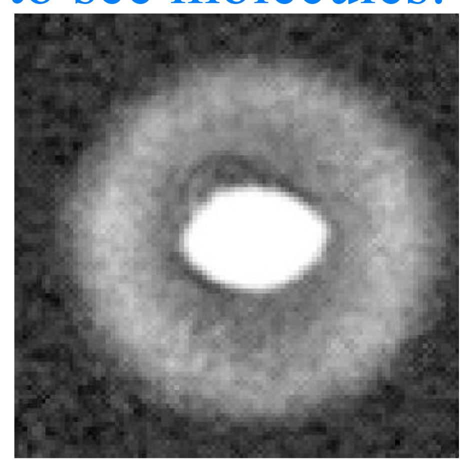
Gaebler, et al.'07





to see molecules:

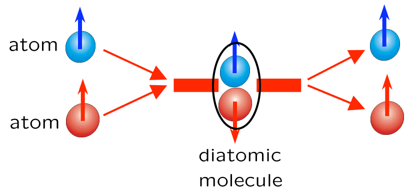


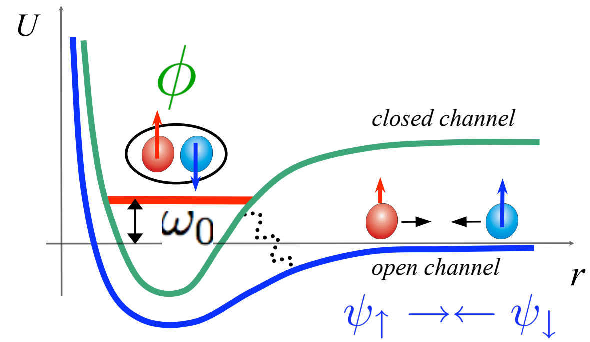


look for energetic atoms

S-wave Feshbach resonant scattering

- tunability (strength and sign) of interactions (sudden and adiabatic)



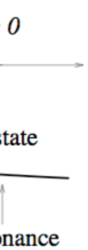


two-channel model:

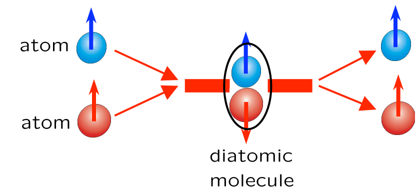
("bare") detuning

$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger \frac{\hat{p}^2}{2m}\psi_\sigma + \phi^\dagger\Big(\frac{\hat{p}^2}{4m} + \epsilon_0\Big)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \text{h.c.}$$

atoms (open channel) | molecules (closed channel) | atom – molecules interconversion

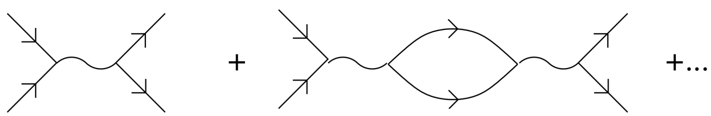


S-wave FR scattering:details



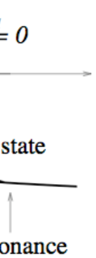
$$\mathcal{S}_{2ch} = \overline{\psi}_\sigma (i\partial_t - \frac{p^2}{2m})\psi_\sigma + \overline{\phi}(i\partial_t - \frac{p^2}{4m})\phi + g\overline{\phi}\psi_\uparrow\psi_\downarrow + c.c.$$

$$T(\mathbf{k},\mathbf{k}') = gD_0g + gD_0g\Pi g D_0 g + \cdots = \frac{g^2}{\omega - \frac{p^2}{4m} - \epsilon_0 - g^2\Pi}$$



$$\begin{aligned}\Pi(k) &= \int_{\nu,\mathbf{q}} \frac{i}{(\omega - \nu - \frac{k_1^2}{2m} + i0)(\nu - \frac{k_2^2}{2m} + i0)} \\ &= \int \frac{d^3q}{(2\pi)^3}\frac{1}{(\omega - \frac{p^2}{4m}) - \frac{q^2}{m} + i0} = -\frac{m\Lambda}{2\pi^2} - i\frac{m}{4\pi}k\end{aligned}$$

$$f_s(k) = \frac{1}{F(k^2) - ik}$$

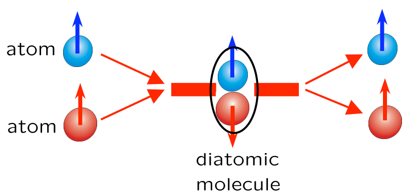


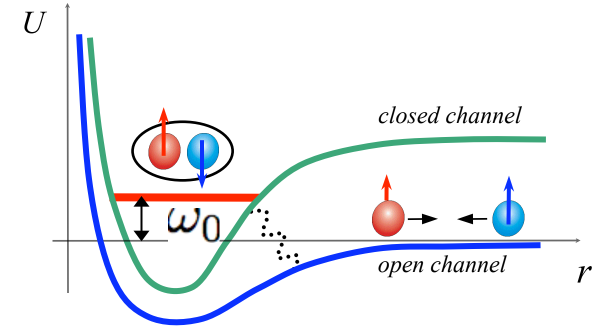
$$\longrightarrow f_s(k) = \frac{1}{-a^{-1} + \frac{r_0}{2}k^2 - ik}, \text{ with } a \sim -\frac{g^2}{\omega_0} \sim a_{bg}\frac{\Delta B}{B_0 - B}, \quad r_0 \sim -\frac{1}{g^2}$$

$$= \frac{-\sqrt{\Gamma_0/m}}{E - \omega_0 + i\sqrt{\Gamma_0 E}} \qquad (\omega_0 = \epsilon_0 - g^2\Lambda m, \quad \Gamma_0 = g^4m^3)$$

S-wave Feshbach resonant scattering

- tunability (strength and sign) of interactions (sudden and adiabatic)

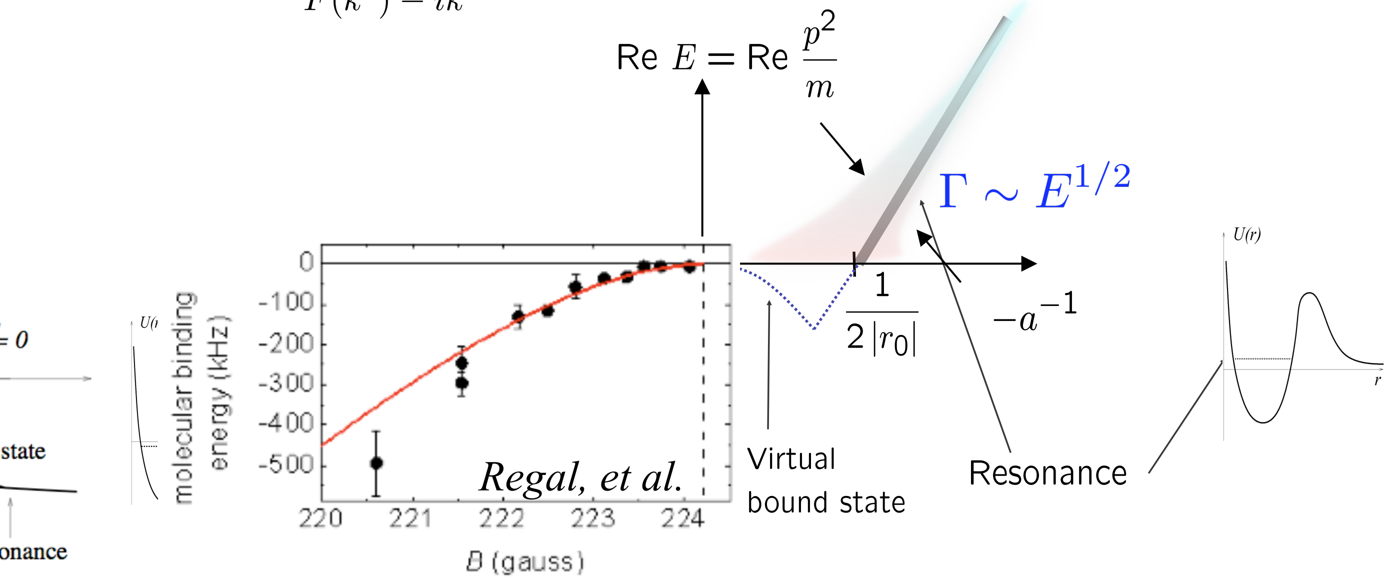


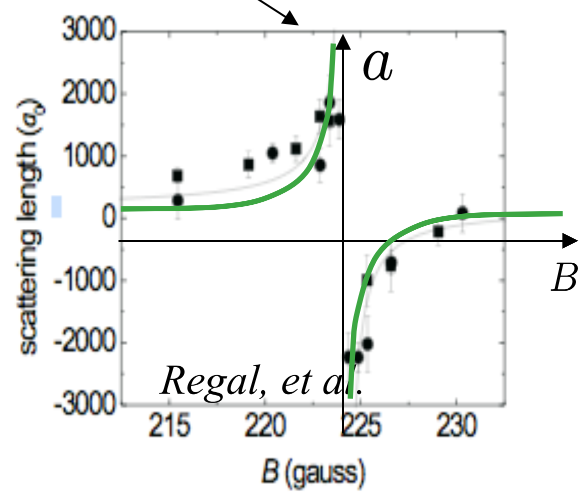


$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger \frac{\hat{p}^2}{2m}\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} + \epsilon_0\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \text{h.c.}$$

$$\longrightarrow f_s(k) = \frac{1}{-a^{-1} + \frac{r_0}{2}k^2 - ik}, \text{ with } a \sim -\frac{g^2}{\omega_0} \sim a_{bg}\frac{\Delta B}{B_0 - B}, \quad r_0 \sim -\frac{1}{g^2}$$

$$f_s(k) = \frac{1}{F(k^2) - ik}$$





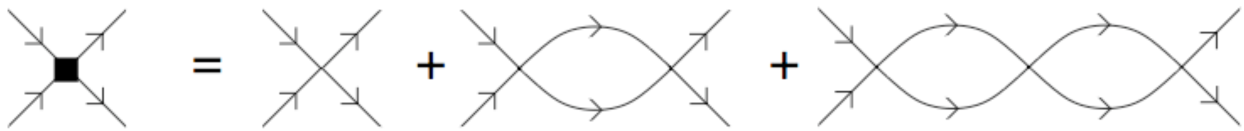
$\gamma >> 1$

Broad resonance scattering

$$\mathcal{H}_{2ch} \longrightarrow \mathcal{H}_{1ch} = \psi_\sigma^\dagger\left(\frac{p^2}{2m} - \mu_\sigma\right)\psi_\sigma + \lambda\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$$

- scattering T-matrix relates λ to a:

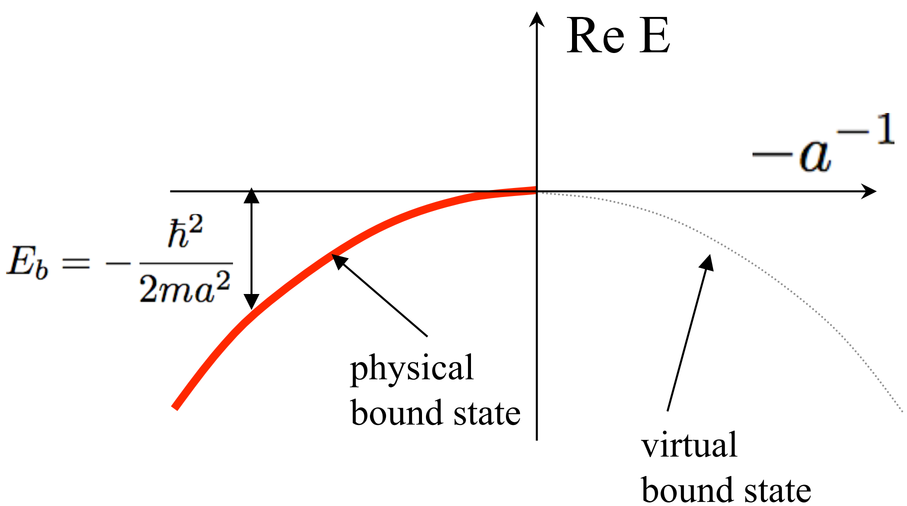
$T_{k\,k'} =$

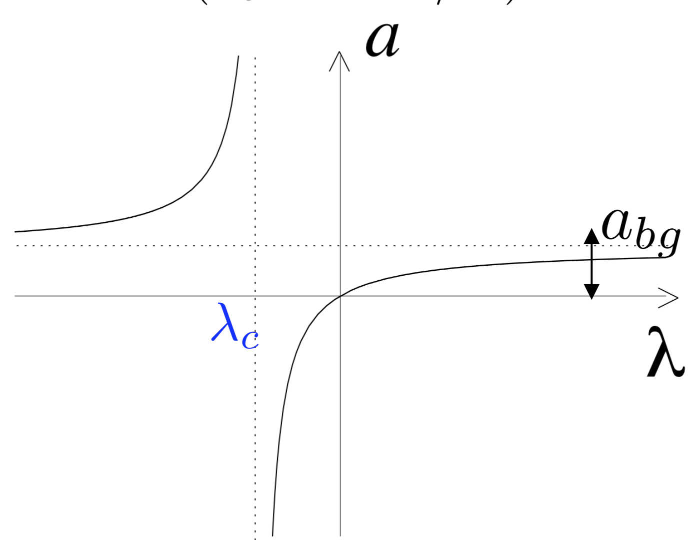


$+\ldots = \dfrac{\lambda}{1-\lambda\Pi}$

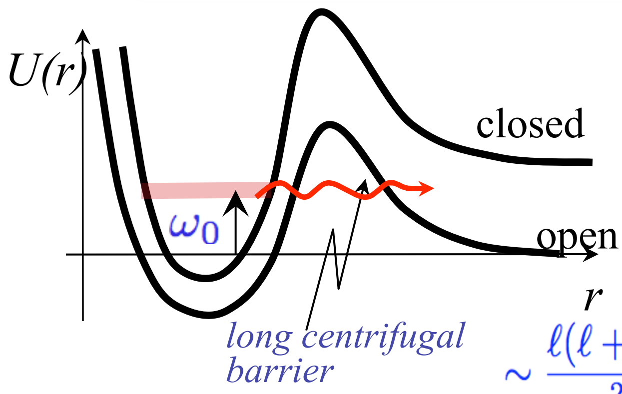
$$= -\frac{4\pi\hbar^2}{m} f_{kk'} \approx \frac{4\pi\hbar^2}{m}\frac{1}{a^{-1}+ik} \longrightarrow \boxed{a = \frac{m}{4\pi\hbar^2}\frac{\lambda}{1+\lambda/\lambda_c}}$$

$(\lambda_c = \pi\hbar^2 d/m)$



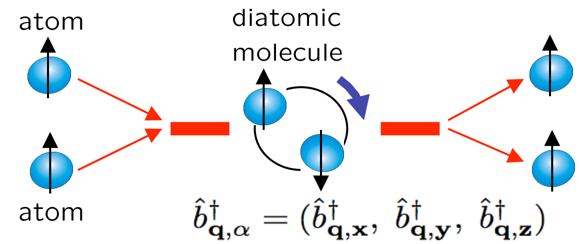


P-wave Feshbach resonant scattering



$\sim \frac{\ell(\ell+1)}{r^2}$

naturally ***narrow!***



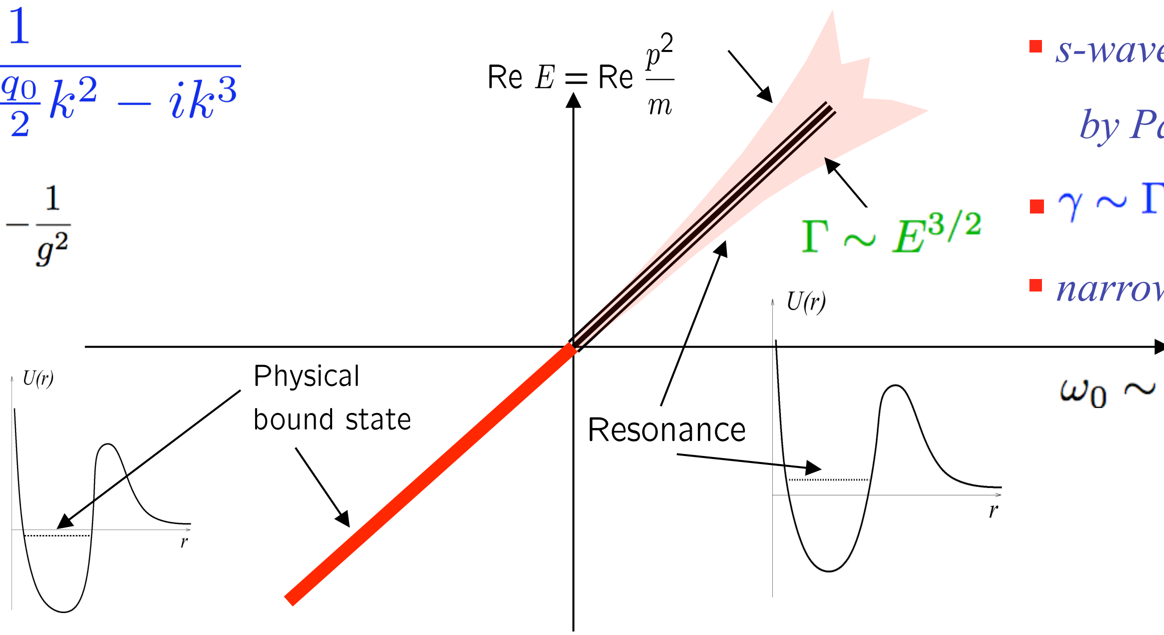
$\Rightarrow$ escape (molecular life) time $\tau \sim \Gamma^{-1} \sim E^{-\frac{3}{2}} \gg E^{-1}$, for $E \to 0$

$$\mathcal{H}_{2ch} = \psi^\dagger \frac{\hat{p}^2}{2m}\psi + \vec{\phi}^\dagger\left(\frac{\hat{p}^2}{4m} + \epsilon_0\right)\vec{\phi} - ig\vec{\phi}\cdot\psi^\dagger\nabla\psi^\dagger + \text{h.c.}$$

$$f_p(k) = \frac{1}{-v^{-1} + \frac{q_0}{2}k^2 - ik^3}$$

with $v^{-1} \sim -\frac{g^2}{\omega_0}$, $q_0 \sim -\frac{1}{g^2}$

$$f_p(q) = \frac{q^2}{F(q^2) - iq^3}$$



$\omega_0 \sim B - B_0$

- *s-wave suppressed by Pauli principle*
- $\gamma \sim \Gamma/E \sim E^{\frac{1}{2}} \ll 1$
- *narrows with* ϵ_F, n

Next time

- L1: AMO renaissance overview
- L2: Feshbach two-atom scattering
- L3: S-wave Feshbach resonant superfluidity
- L4: Imbalanced s-wave resonant Fermi gases
- L5: P-wave Feshbach resonant superfluidity