Resonant atomic gases

Leo Radzihovsky

for details see: *Gurarie, L.R., Annals of Physics, 322, 2-119 (2007)*
Sheehy, L.R., Annals of Physics, 322, 1790 (2007)

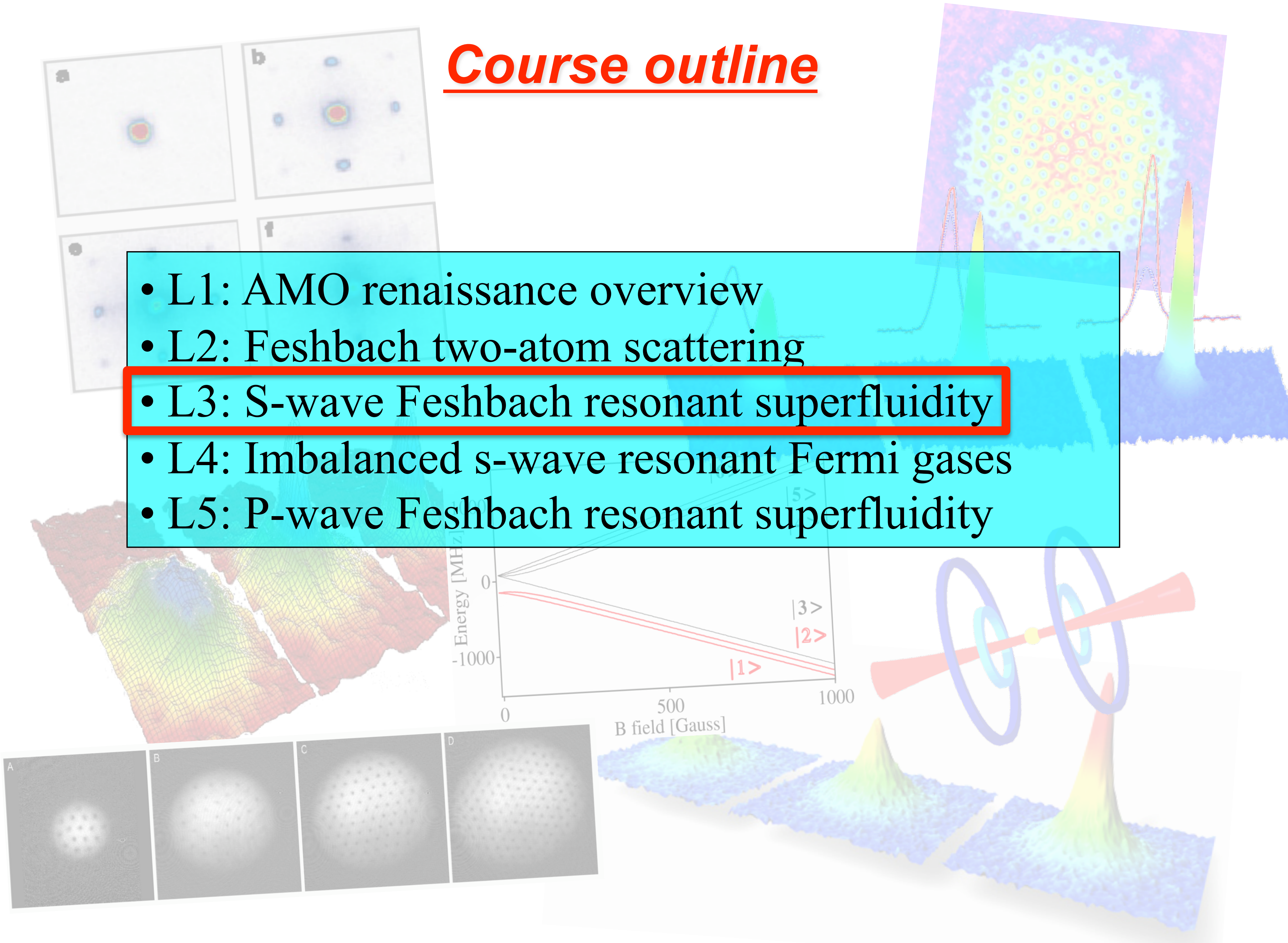
Giorgini, et al., RMP, 80, 885 (2008)
Ketterle and Zwierlein, Varenna lectures (2006)

$: NSF

Mysore, India, Dec 2010

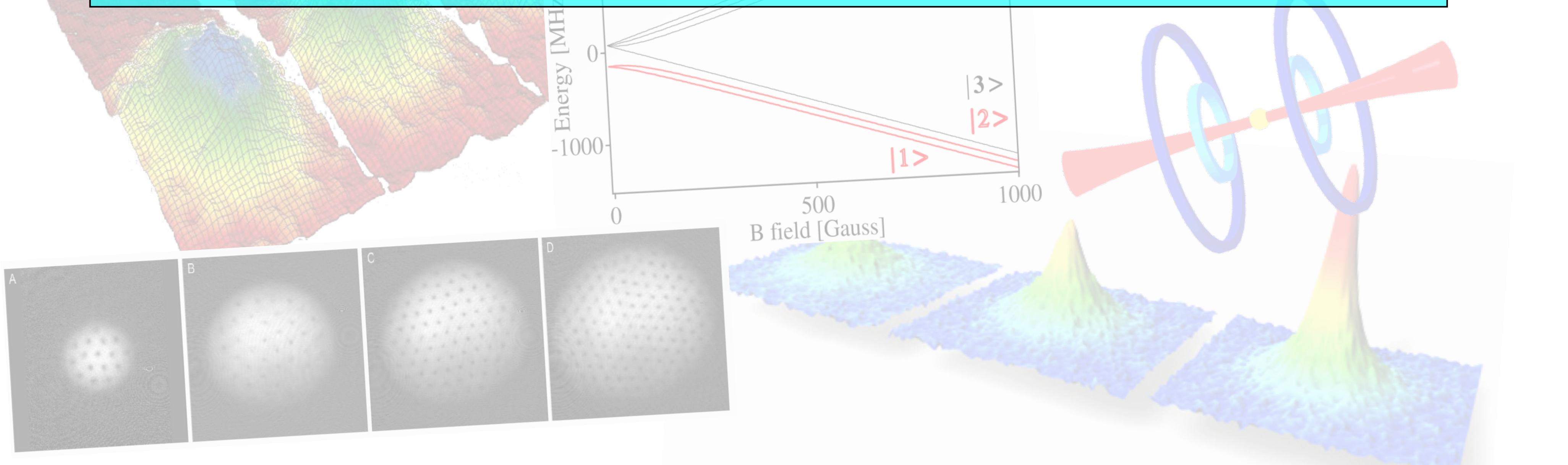
Course outline

- L1: AMO renaissance overview
- L2: Feshbach two-atom scattering
- L3: S-wave Feshbach resonant superfluidity
- L4: Imbalanced s-wave resonant Fermi gases
- L5: P-wave Feshbach resonant superfluidity



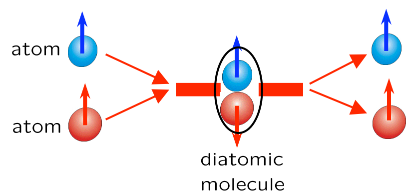
Lecture 3: s-wave Feshbach resonant superfluidity

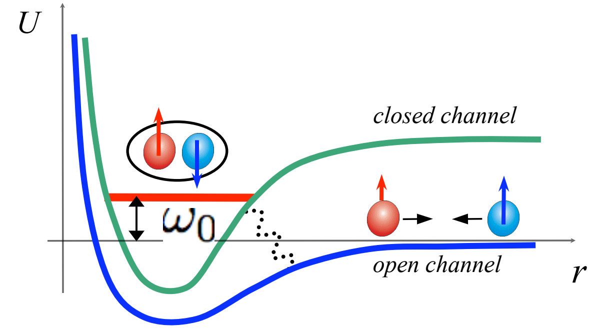
- two-channel resonant pairing model: narrow resonance
- one-channel model: broad resonance
- large-N Sp(2N) theory
- universality
- resonant bosonic superfluidity



S-wave Feshbach resonant scattering

- tunability (strength and sign) of interactions (sudden and adiabatic)

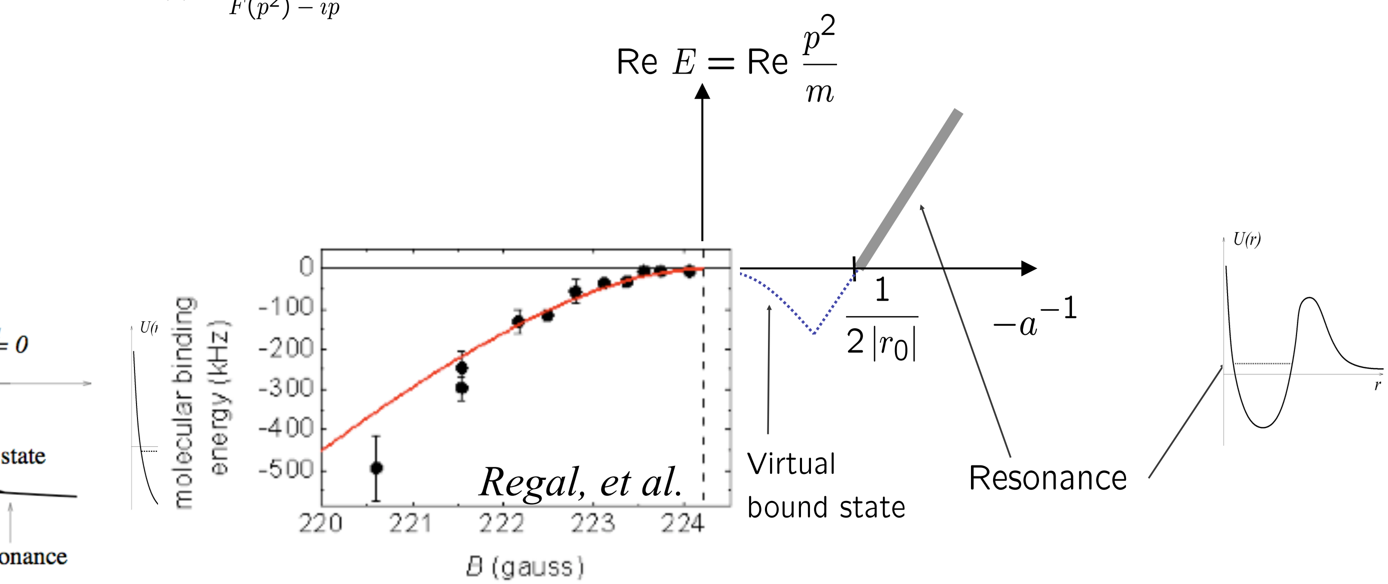


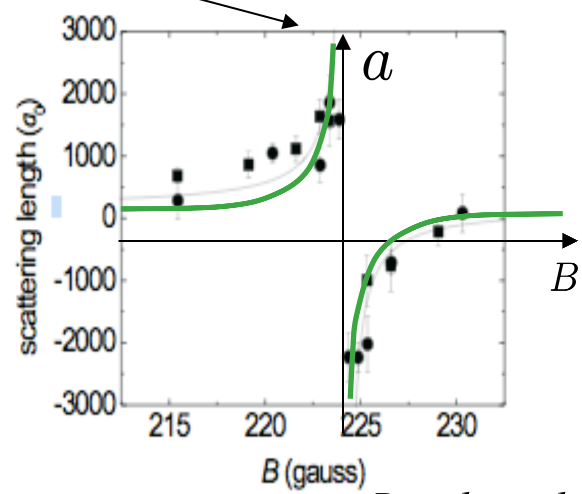


$$\mathcal{H}_{2ch} = \psi^\dagger_\sigma \frac{\hat{p}^2}{2m}\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} + \epsilon_0\right)\phi - g\phi\psi^\dagger_\uparrow\psi^\dagger_\downarrow + \text{h.c.}$$

$$\longrightarrow \quad f_s(p) = \frac{1}{-a^{-1} + \frac{r_0}{2}p^2 - ip}, \quad \text{with } a \sim -\frac{g^2}{\omega_0} \sim a_{bg}\frac{\Delta B}{B_0 - B}, \quad r_0 \sim -\frac{1}{g^2}$$

$$f_s(p) = \frac{1}{F(p^2) - ip}$$

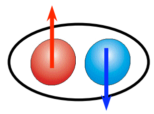




Regal, et al.

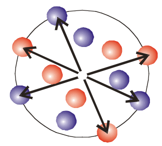
S-wave resonant fermionic superfluidity

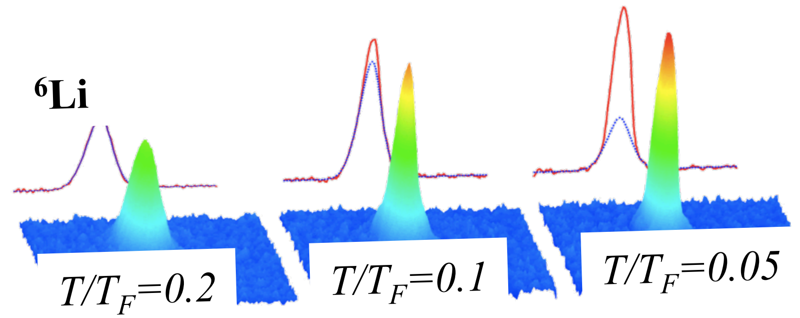
- molecular BEC *(Regal, Jin '03)*



K_2 , Li_2

- BCS superfluid *(Regal, Jin 04 Zwierlein, Ketterle '04)*





- BCS-BEC crossover:

$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger \Big(\frac{\hat{p}^2}{2m} - \mu\Big)\psi_\sigma + \phi^\dagger \Big(\frac{\hat{p}^2}{4m} + \epsilon_0 - 2\mu\Big)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \text{h.c.}$$

$$(\phi = B + \delta\phi)$$

$$F(B,\mu) = -T\log[\mathrm{Tr}\big(e^{-H/T}\big)]$$

$\longrightarrow$ gap equation: $\dfrac{\delta F}{\delta B} = 0,$

number equation: $-\dfrac{\delta F}{\delta \mu} = n = n_a + 2n_m$

S-wave resonant superfluidity

$$H = \sum_k \left[\left(\frac{k^2}{2m} - \mu\right) a^\dagger_{k\sigma} a_{k\sigma} + \left(\frac{k^2}{4m} + \epsilon_0 - 2\mu\right) b^\dagger_k b_k - gBa^\dagger_{-k\downarrow} a^\dagger_{k\uparrow} \right]$$

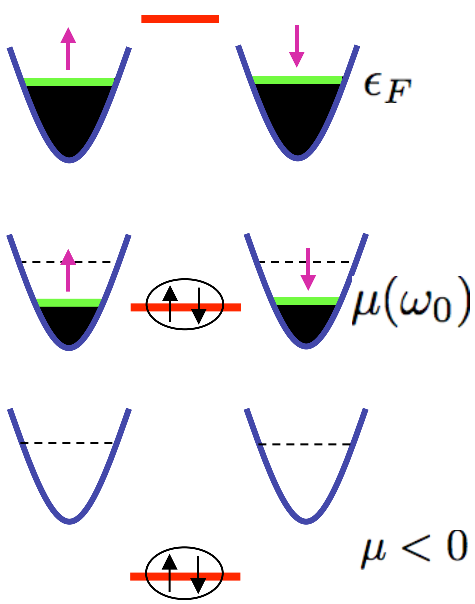
exactly solvable g → 0⁺ limit: *free Fermi- and molecular Bose gases in chemical equilbrium*

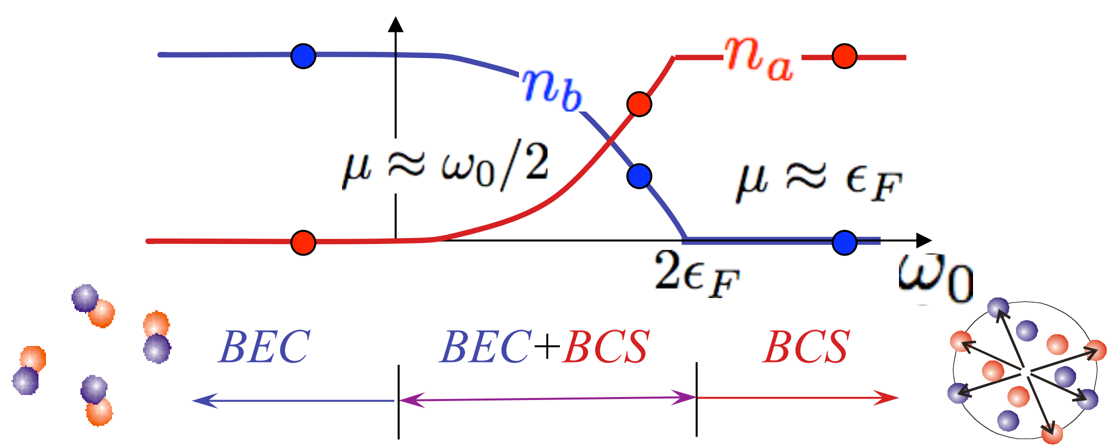
$$H^{g=0} = \sum_k \left(\frac{k^2}{2m} - \mu\right) n^a_k + (\epsilon_0 - 2\mu) n^m_0$$

T=0:

$$n_a = \frac{(2m)^{3/2}}{3\pi^2}\mu^{3/2}, \text{ for } \mu > 0 \quad \Longrightarrow n^m_0 = |B|^2 = \frac{n}{2}\left[1 - \left(\frac{\mu}{\epsilon_F}\right)^{3/2}\right]$$

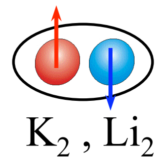
$$\begin{aligned} \mu(\epsilon_0) &= \epsilon_F, \text{ for } \epsilon_0 > 2\epsilon_F \rightarrow \text{no molecules} \\ &= \epsilon_0/2, \text{ for } 0 < \epsilon_0 < 2\epsilon_F \rightarrow \text{atomic Fermi sea \& molecular BEC} \\ &= \epsilon_0/2, \text{ for } \epsilon_0 < 0 \rightarrow \text{molecular BEC, no free atoms} \end{aligned}$$



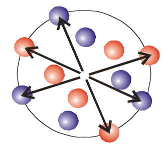


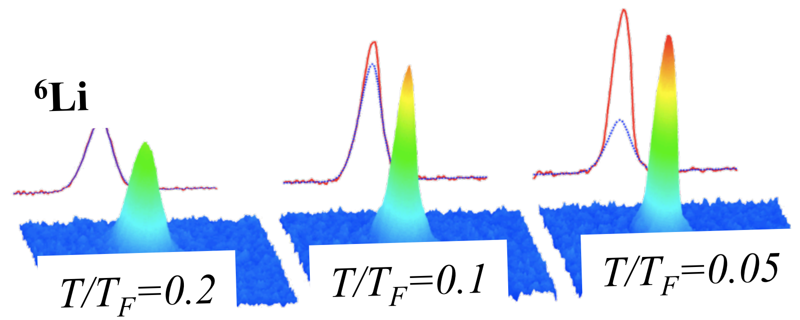
S-wave resonant fermionic superfluidity

- molecular BEC *(Regal, Jin '03)*



- BCS superfluid *(Regal, Jin 04 Zwierlein, Ketterle '04)*





- BCS-BEC crossover:

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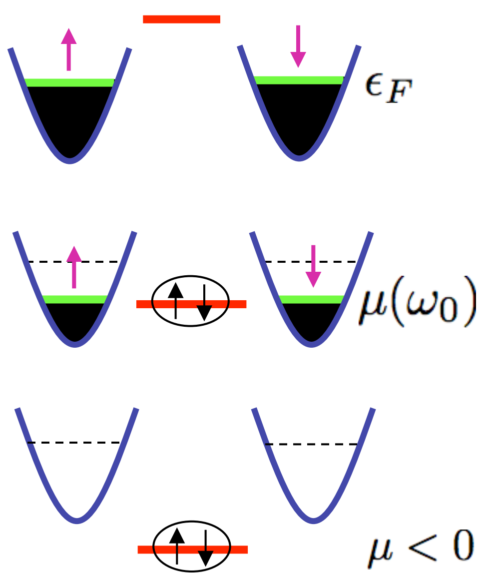

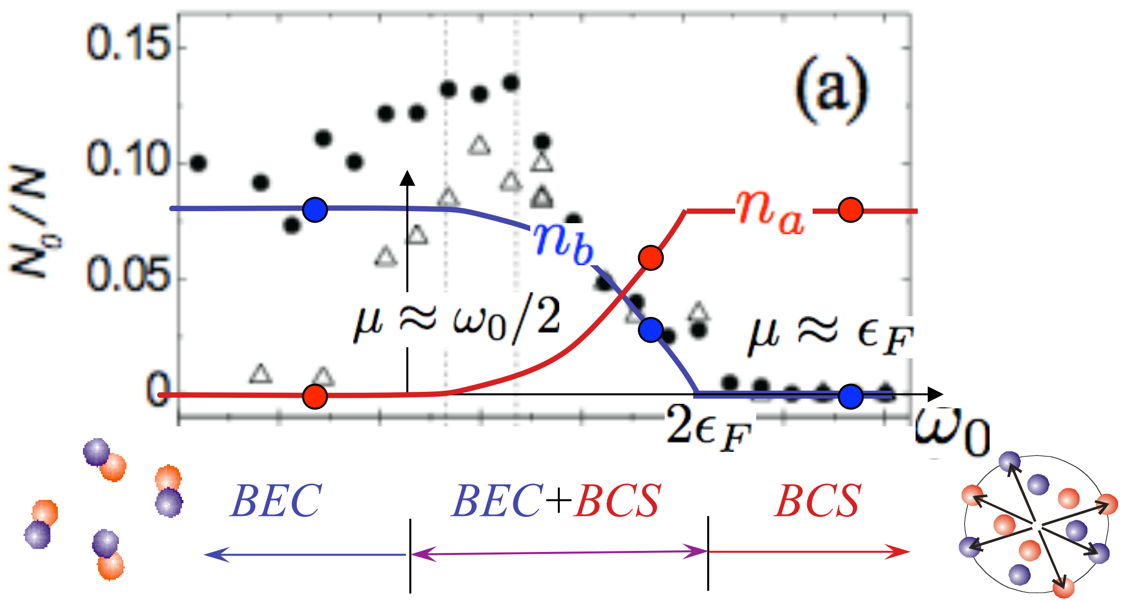

S-wave resonant fermionic superfluidity

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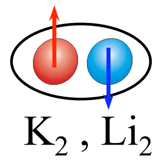

- BCS superfluid *(Regal, Jin 04*
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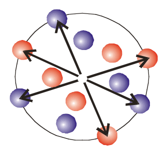
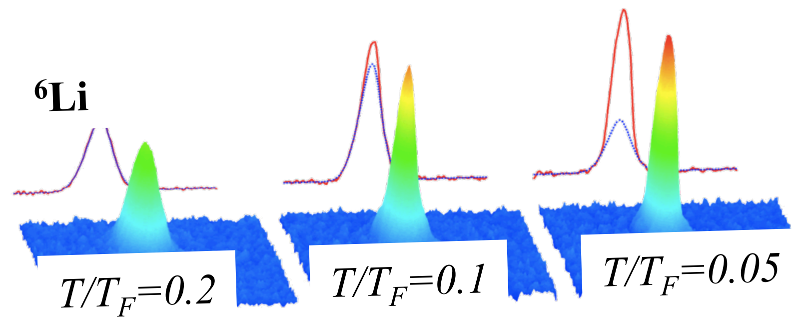
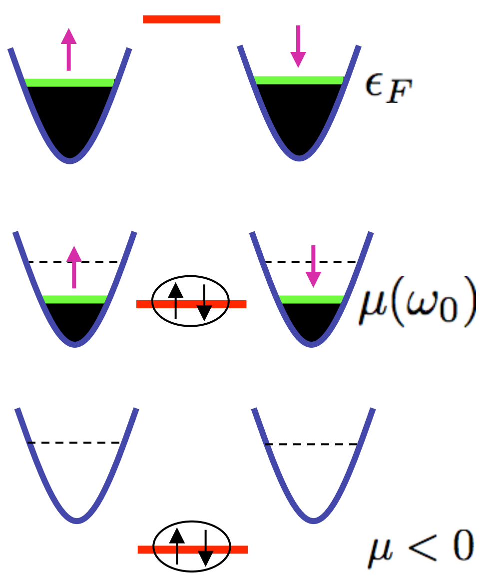
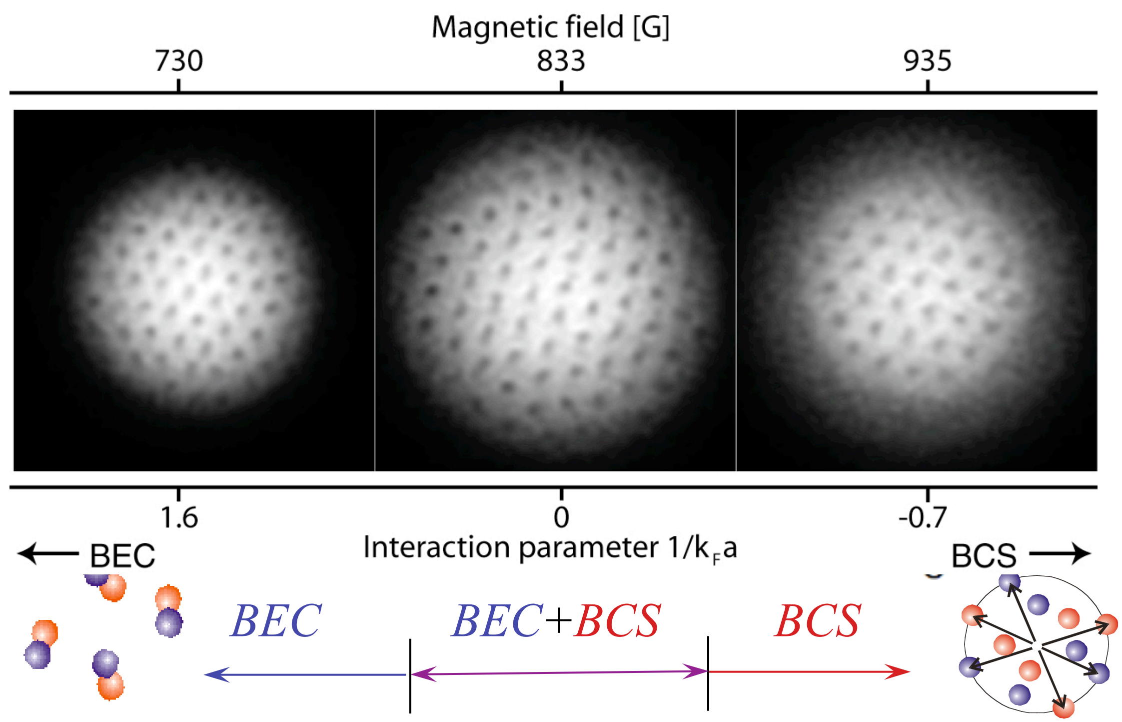




- BCS-BEC crossover:

$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger\left(\frac{\hat{p}^2}{2m}-\mu\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m}+\epsilon_0-2\mu\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \text{h.c.}$$





S-wave resonant superfluidity: details

$$H = \sum_k \left[\left(\frac{k^2}{2m} - \mu\right) a^\dagger_{k\sigma} a_{k\sigma} + \left(\frac{k^2}{4m} + \epsilon_0 - 2\mu\right) b^\dagger_k b_k - gB a^\dagger_{-k\downarrow} a^\dagger_{k\uparrow} + h.c. \right]$$

small g: *hybridized paired Fermi-sea (Cooper pairs) and molecular BEC*

no UV cutoff

T=0:

$$E_{gs}(B,\mu) = (\omega_0 - 2\mu)|B|^2 - \sum_k \left(E_k - \varepsilon_k - \frac{g^2}{2\epsilon_k}|B|^2\right)$$

$$= \frac{\omega_0 - 2\mu}{g^2}|\Delta|^2 - E^a_{\text{condense}}$$

$E_k n^a_k$

$$\begin{cases} -\Delta^2 \ln \dfrac{8e^{-3/2}\mu}{\Delta}, & \text{for } \mu > 0, \\ \alpha\Delta^2 + \beta\Delta^4, & \text{for } \mu < 0, \end{cases}$$

gap eqn: $$\frac{\omega_0 - 2\mu}{g^2} = \frac{1}{2}\int_k \left[\frac{1}{\sqrt{(\epsilon_k - \mu)^2 + \Delta^2}} - \frac{1}{\epsilon_k}\right]$$

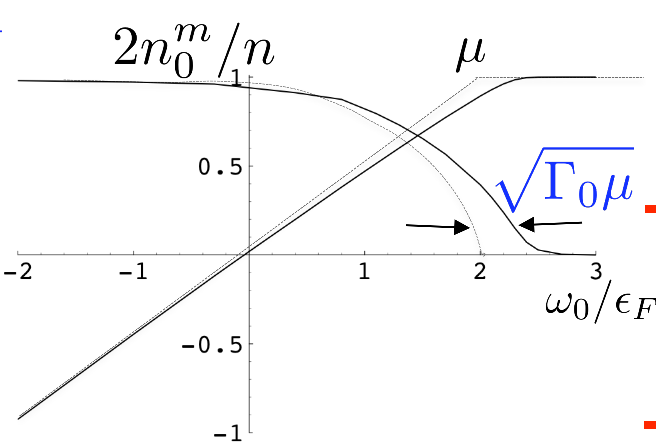
N eqn: $$n = \int_k \left[1 - \frac{\epsilon_k - \mu}{\sqrt{(\epsilon_k - \mu)^2 + \Delta^2}}\right] + 2|B|^2$$

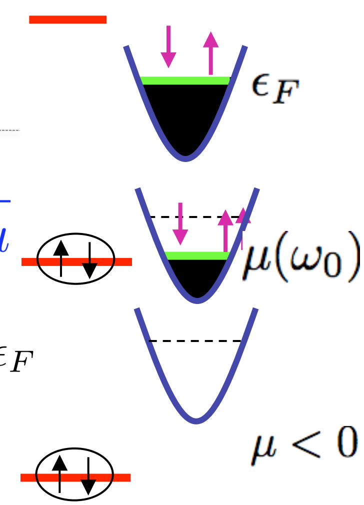
μ > 0 BCS: $\Delta \approx gB = 8e^{-2}\mu e^{-c\frac{\omega_0 - 2\mu}{\sqrt{\Gamma_0 \mu}}}$

$n \approx c(m\mu)^{3/2} + 2B^2$

μ < 0 BEC: $\omega_0 - 2\mu \approx \sqrt{\Gamma_0 \mu}$

$$n \approx \left(\sqrt{\frac{\Gamma_0}{|\mu|}} + 2\right) B^2$$





S-wave resonant SF: small parameter

$$\mathcal{H}_{2ch} = \psi^\dagger_\sigma\left(\frac{\hat{p}^2}{2m} - \mu\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} - 2\mu + \epsilon_0\right)\phi - g\phi\psi^\dagger_\uparrow\psi^\dagger_\downarrow$$

dimensionless coupling:

$$\gamma \sim \frac{g^2\nu(\epsilon_F)}{\epsilon_F} \sim \left(\frac{g\sqrt{n}}{\epsilon_F}\right)^2 \sim \frac{1}{k_F r_0} \sim \sqrt{\frac{\Gamma_0}{\epsilon_F}}$$

$$\gamma^{^{40}K}_{202G} \approx 5,\ \Delta B \sim 1G \sim 100\mu K$$

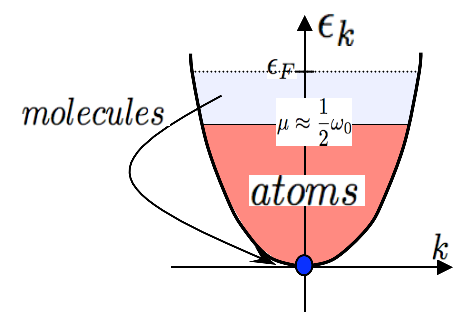
$$\gamma^{^{6}Li}_{544G} \approx 0.1,\ \Delta B \sim 0.1G \sim 10\mu K$$

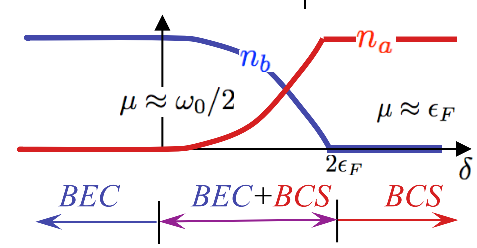
$$\epsilon_F \sim 1\mu K$$

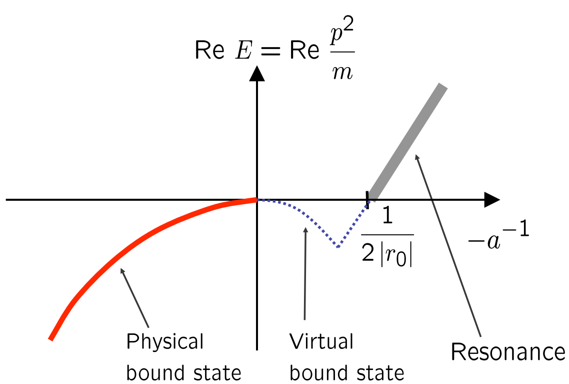
- **narrow** resonance $\gamma \ll 1 \ \rightarrow \ \mathrm{MFT}: \phi(x) = B$
- **broad** resonance $\gamma \gg 1$

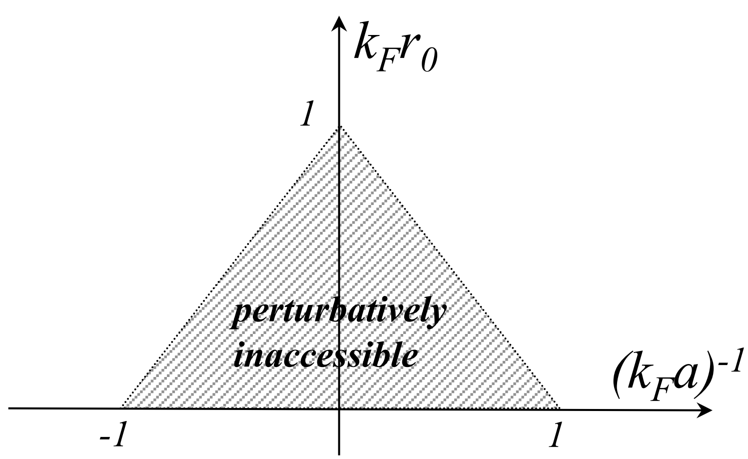
Strongly coupled ϕ and ψ

⇨ *MFT quantitatively uncontrolled*









$$\gamma \approx |T_{k_F}|n/\epsilon_F \approx \frac{1}{(k_F a)^{-1} - k_F r_0 + 1}$$

$\gamma >> 1$

Broad resonance scattering

$$\mathcal{H}_{2ch} \longrightarrow \mathcal{H}_{1ch} = \psi^\dagger_\sigma\left(\frac{p^2}{2m} - \mu_\sigma\right)\psi_\sigma + \lambda\psi^\dagger_\uparrow\psi^\dagger_\downarrow\psi_\downarrow\psi_\uparrow$$

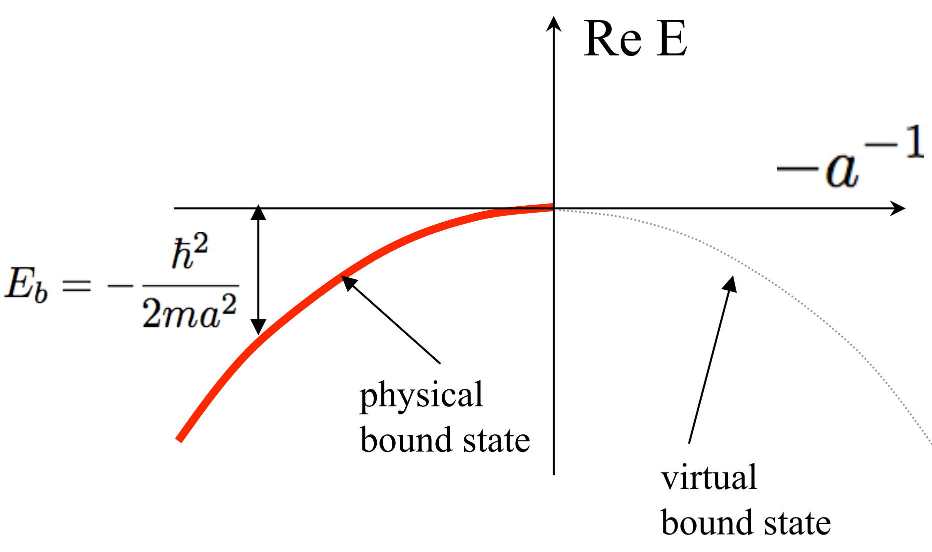
- scattering T-matrix relates λ to a:

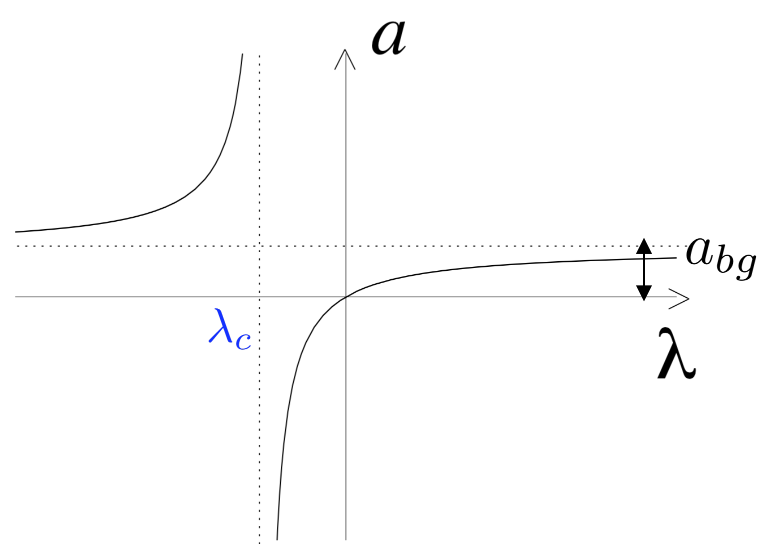
$T_{k\,k'} =$

$$= \frac{\lambda}{1-\lambda\Pi}$$

$$= -\frac{4\pi\hbar^2}{m} f_{kk'} \approx \frac{4\pi\hbar^2}{m}\frac{1}{a^{-1}+ik} \longrightarrow \boxed{a = \frac{m}{4\pi\hbar^2}\frac{\lambda}{1+\lambda/\lambda_c}}$$

$$(\lambda_c = \pi\hbar^2 d/m)$$





$\gamma >> 1$ **Broad resonance superfluidity: Large N**

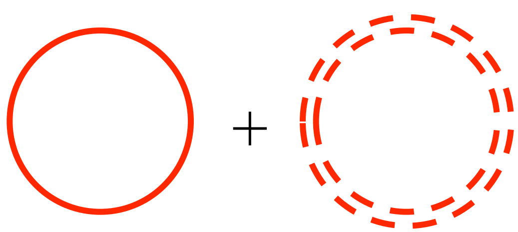
- no small parameter for $k_F a \sim n^{1/3} a >> 1 \rightarrow$ *introduce 1/N*

$$\mathcal{H}_{1ch} \xrightarrow{Sp(2N)} \mathcal{H}_N = \psi^\dagger_{\sigma\alpha}\left(\frac{p^2}{2m} - \mu_\sigma\right)\psi_{\sigma\alpha} + \frac{\lambda}{N}\psi^\dagger_{\uparrow\alpha}\psi^\dagger_{\downarrow\alpha}\psi_{\downarrow\beta}\psi_{\uparrow\beta}$$

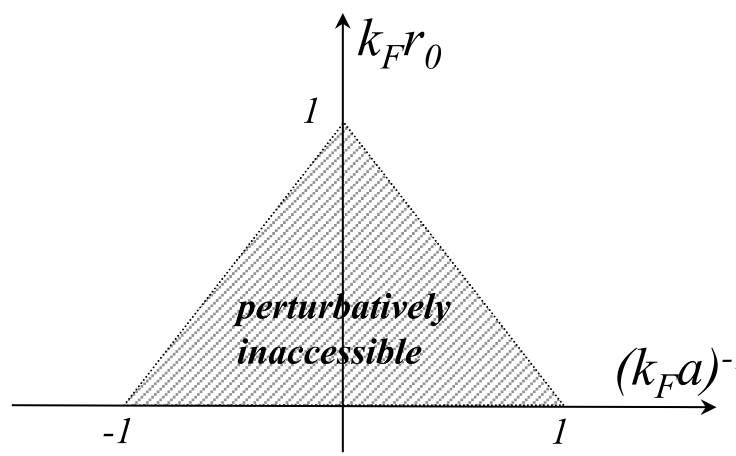
$$S[\phi] = -\frac{N}{\lambda}\int_0^\beta d\tau d^3r|\phi|^2 - N\mathrm{Tr}\log\left[-G_\phi^{-1}\right]$$

$$G_\phi^{-1} = \begin{pmatrix} -\partial_\tau + \frac{\nabla^2}{2m} + \mu_\uparrow & \phi_x \\ \phi_x^* & -\partial_\tau - \frac{\nabla^2}{2m} - \mu_\downarrow \end{pmatrix}$$

$$\begin{aligned} f &= -\frac{1}{\beta V}\log\int D\phi e^{-S[\phi]}, \\ &= N f^{(0)} + f^{(1/N)} + \ldots \end{aligned}$$



MFT



$$f^{(0)} = -\frac{|\Delta|^2}{\lambda} - \int_k (E_k - \xi_k) - \sum_{\sigma=\pm}\int_k \log\left[1 + e^{-\beta(E_k+\sigma h)}\right]$$

Veillette, Sheehy, LR
Nikolic, Sachdev
also Nishida, Son
ε-expansion

$\gamma >> 1$ **Broad resonance superfluidity: $N \rightarrow \infty$**

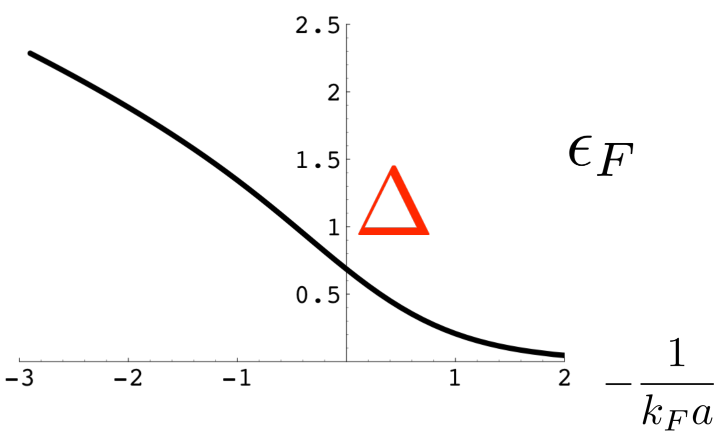
$$f^{(0)} = -\frac{|\Delta|^2}{\lambda} - \int_k (E_k - \xi_k) - \sum_{\sigma=\pm} \int_k \log\left[1 + e^{-\beta(E_k + \sigma h)}\right]$$

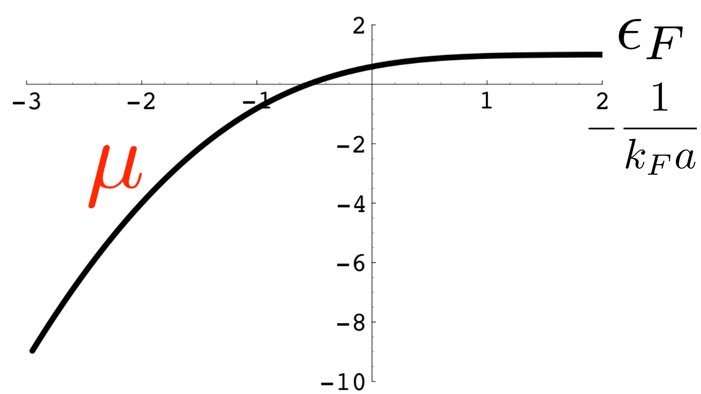
T=0:

- $$E_{gs}(\Delta,\mu) = -\frac{m}{4\pi\hbar^2}\frac{|\Delta|^2}{a} - \sum_k \left(E_k - \varepsilon_k - \frac{1}{2\epsilon_k}|\Delta|^2\right)$$
- $$-\frac{m}{4\pi\hbar^2 a} = \frac{1}{2}\int_k \left[\frac{1}{\sqrt{(\epsilon_k-\mu)^2+\Delta^2}} - \frac{1}{\epsilon_k}\right] \qquad n = \int_k \left[1 - \frac{\epsilon_k - \mu}{\sqrt{(\epsilon_k-\mu)^2+\Delta^2}}\right]$$

for a < 0 (BCS): $\Delta \sim \mu e^{\frac{\pi}{2k_F a}\sqrt{\frac{\mu}{\epsilon_F}}}$ $\qquad \mu \approx \epsilon_F\left(1 - c\frac{\Delta^2}{\epsilon_F^2}\frac{1}{k_F|a|}\right)$

for a > 0 (BEC): $\Delta \sim \epsilon_F\sqrt{\frac{1}{k_F a}}$ $\qquad \mu \approx -\frac{\hbar^2}{2ma^2}$



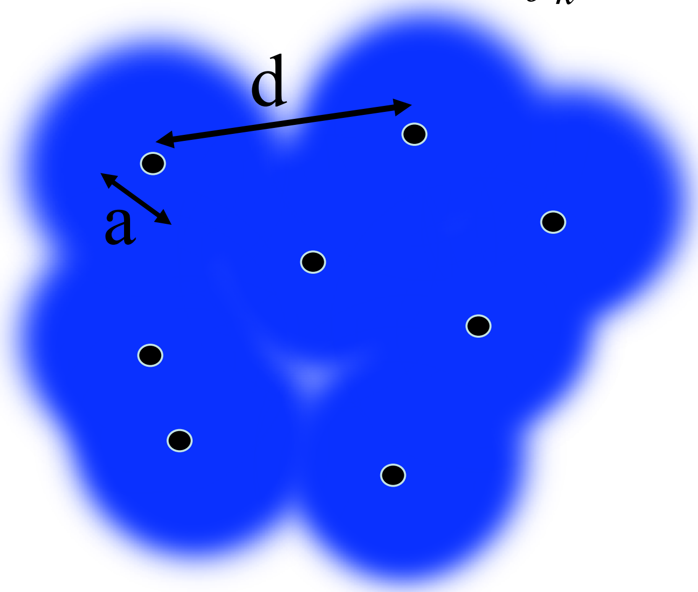


$\gamma >> 1,\ k_F a \to \infty$

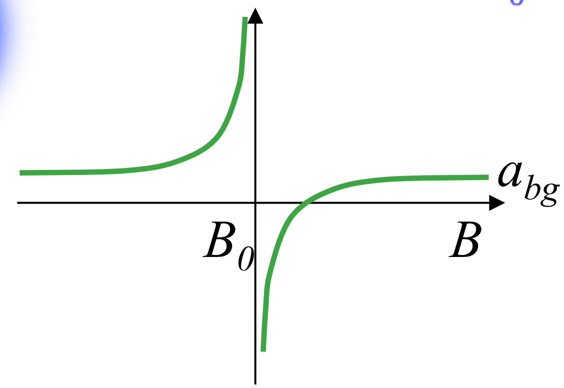
Universality at unitary point

T.L. Ho '04

$f_k = -1/(a^{-1} + i\,k) \to i/k,\ \Rightarrow$ $d = 1/k_F$ is the only scale



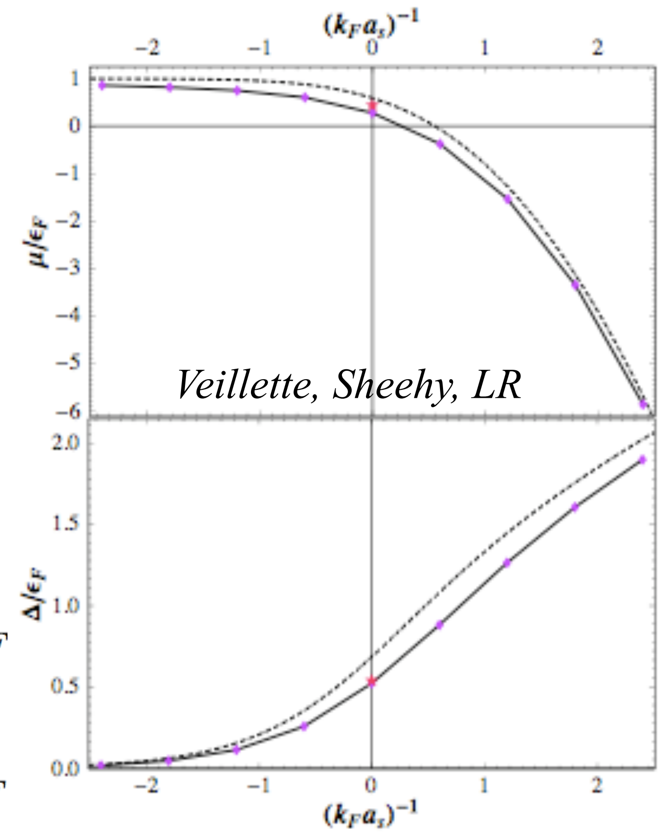
$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$



check in $N \to \infty$ (BCS) limit:

$$\frac{m}{2\pi\hbar^2 a} \to 0 = \int_k \left(\frac{1}{E_k} - \frac{1}{\epsilon_k}\right)$$

$$0 = \ln(\Delta/\alpha\epsilon_F)$$

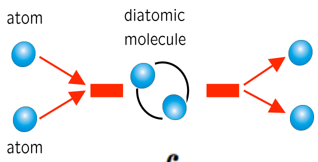


$$\boxed{f(T, n) = n\epsilon_F \hat{f}(k_B T/\epsilon_F)}$$

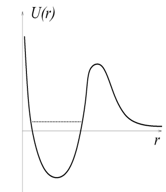
$$\begin{aligned}
\epsilon &= \xi \frac{3}{5}\epsilon_F \\
\mu &= \xi \epsilon_F \\
\Delta &= \alpha \epsilon_F \\
\Delta_{exc} &= \alpha_{exc}\epsilon_F \\
k_B T_c &= \gamma \epsilon_F \\
B &= \xi \frac{2}{3} n \epsilon_F
\end{aligned}$$

1/N theory $\xi = 0.5906 - 0.312/N + \cdots$

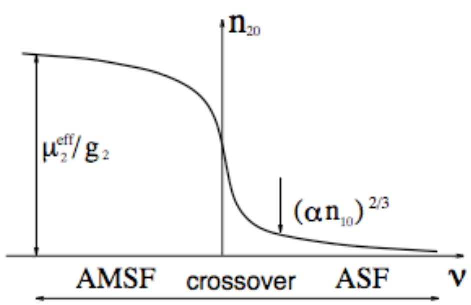
Exp with ^{40}K $\xi = 0.46^{+0.05}_{-0.12}$



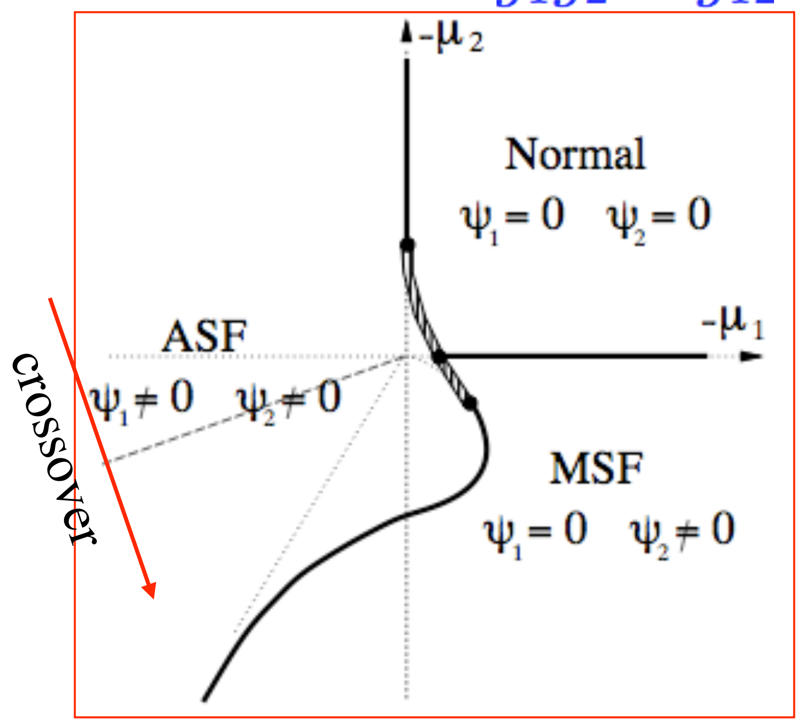
Resonant bosonic superfluidity



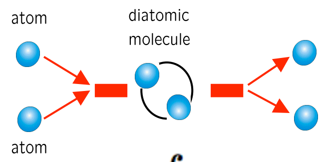
$$f_{\rm mf} = -\mu_1|\Psi_{10}|^2 + \frac{g_1}{2}|\Psi_{10}|^4 - \mu_2|\Psi_{20}|^2 + \frac{g_2}{2}|\Psi_{20}|^4 + g_{12}|\Psi_{10}|^2|\Psi_{20}|^2$$
$$-\alpha \mathrm{Re}[\Psi_{20}^*\Psi_{10}^2]$$



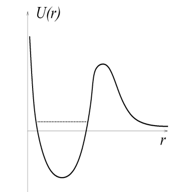
$g_1 g_2 > g_{12}$



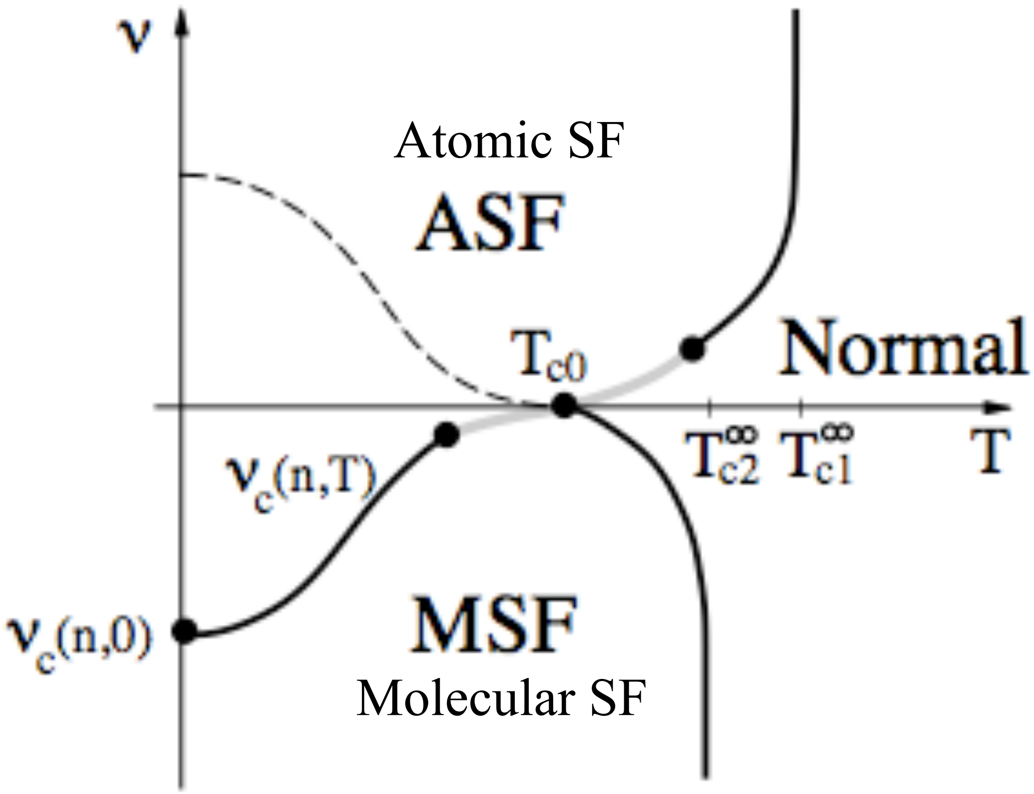
LR, Park, Weichman, PRL ‘04, Annals of Physics ’08
Sachdev, et al. PRL ‘04

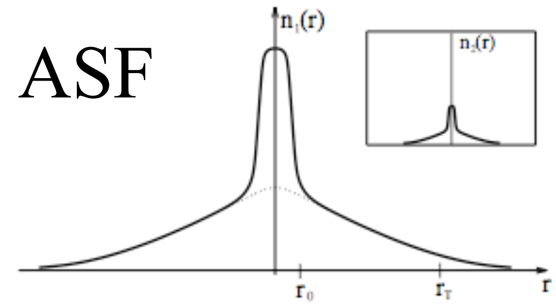


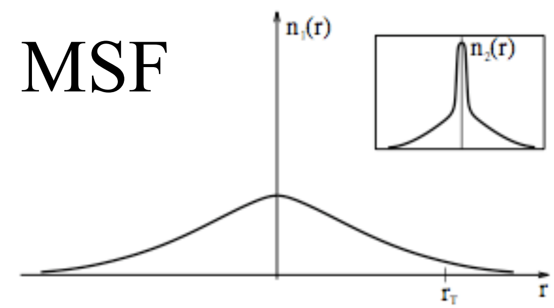
Resonant bosonic superfluidity



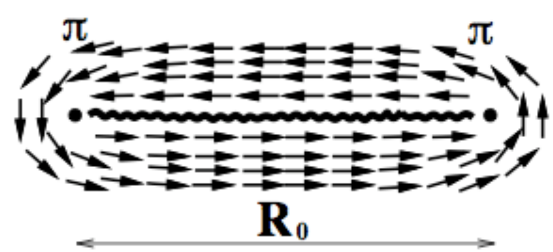
$$f_{\mathrm{mf}} = -\mu_1|\Psi_{10}|^2 + \frac{g_1}{2}|\Psi_{10}|^4 - \mu_2|\Psi_{20}|^2 + \frac{g_2}{2}|\Psi_{20}|^4 + g_{12}|\Psi_{10}|^2|\Psi_{20}|^2 - \alpha \mathrm{Re}[\Psi_{20}^*\Psi_{10}^2]$$







- atomic (ASF) and molecular (MSF) superfluids
- quantum Ising transition
- π vortex deconfinement



Next time

- L1: AMO renaissance overview
- L2: Feshbach two-atom scattering
- L3: S-wave Feshbach resonant superfluidity
- L4: Imbalanced s-wave resonant Fermi gases
- L5: P-wave Feshbach resonant superfluidity