CONTESTS WITH THREE OR MORE HETEROGENEOUS AGENTS

SÉRGIO O. PARREIRAS$^\sharp$ AND ANNA RUBINCHIK$^\natural$

Abstract. We characterize equilibrium behavior in contests with observable effort (bid) where participants perceive their rivals as having distinct risk attitudes and distributions of the value from winning. These differences can either lead a player to drop out, i.e., always choose zero effort regardless of his valuation, or use "all-or-nothing" strategies with discontinuous effort choice. Neither of these behaviors, *complete drop-out* nor *discontinuous bidding*, is consistent with a Bayesian-Nash equilibrium (with monotone strategies) in a contest with either ex-ante identical players or with only two distinct participants.

Our model provides a framework to analyze the effects of group composition and shared beliefs on individual behavior in contests.

1. Introduction

1.1. **Motivation.**

Contests are all around us. Students striving to be the best in their class, employees awaiting promotion, sportsmen fighting for a gold medal, R&D firms racing to capture monopoly profits, researchers competing for grants — all can be viewed as players in games with a few winners. Typically, the rest of the participants are losers, who, in addition, have to absorb the cost of the invested effort.

Rarely do the contestants look alike: their background, previous experience, gender, age and other observable characteristics vary, and so, each can be perceived as being different from his opponents. Since a precise value attached to the prize by a rival is usually hard to determine, some residual uncertainty still remains, so we model a contest as a game of incomplete information. In-line with most of the literature on the subject, we assume that

---

We would like to thank the associate editor and the anonymous referees for their detailed comments, as well as participants of the International Conference of Game Theory; the World Game Theory Congress, seminar participants at Duke University, UNC at Chapel Hill, EPGE-FGV-RJ, Queen's University, Virginia Tech, University of CO at Boulder, Gary Biglaiser, Jim Friedman, Jennifer Lamping, Vijay Krishna, Eric Maskin, Jean-François Mertens, Claudio Mezzetti, Humberto Moreira, and Thomas Sjöström for their remarks. We are also grateful to Bernard Lebrun for sending us his working paper.

$^\sharp$Dept. of Economics, UNC at Chapel Hill. `sergiop@unc.edu`.

$^\natural$Dept. of Economics; University of Colorado; Boulder CO 80309; USA. `Anna.Rubinchik@Colorado.edu`.

contestants view the values of others as random variables, every two contestants agree on the distributions of values for the rest of the players (say, formed from the observable characteristics), and this is common knowledge.

The detectable differences might influence behavior of the contestants, most importantly, their choice of effort. Both one's view of the rivals and one's knowledge about their perceptions of oneself can be crucial.

However theoretical literature on all-pay auctions that we are aware of[1] so far has focused either on just two contestants or on ex-ante identical competitors.[2] In the latter case the effect of observable differences is impossible to analyze, while the former, we show, is rather special.

We introduce two sources of heterogeneity: individual risk attitudes and rival's beliefs about personal valuations of the prize, in addition, more than two participants are allowed. Now one can rationalize a wide range of behaviors that were inconsistent with an equilibrium where a contestant is facing only one rival or a group of 'clones' of oneself. So, the exercise goes beyond generalizing known results in all-pay auctions under incomplete information.

We demonstrate, for example, that an 'underdog', whose possible upper valuation is believed to be sufficiently low, might never choose a strictly positive effort, no matter what is his realized valuation. Moderately risk-averse avoid investing as much effort as their very-risk-averse rivals, while the latter might adopt an "all-or-nothing" strategy, never choosing an effort from an (open) interval adjacent to zero. None of this is consistent with a Bayesian-Nash equilibrium in a symmetric setting or with just two competitors.

As a consequence, we also provide a framework to analyze (1) the effect of *group composition* (i.e., distribution of risk preferences within a group) on elicited effort and participation; (2) changes in one's behavior induced by the *perception of the opponents* about one's abilities (i.e., beliefs about one's valuation for the prize).

1.2. **Outline.**

Section 2 describes the environment and the equilibrium bidding functions. The next two sections contain our main results: sufficient conditions for drop-out behavior are formulated in section 3, and those for discontinuous bidding are in section 4. Section 5 contains two possible reasons for an aggressive behavior in contests: differences in risk attitudes and dominance of distributions of valuations. We discuss the related literature and some implications of our findings in the conclusions, section 6. The proofs missing in the text are in the appendix.

2. The Model

There are $N \geq 3$ risk-averse individuals competing for a single prize ("bringing home the gold"). The prize is allocated to a contestant who

---

[1] We refer later to some relevant contributions when discussing our results.

[2] A notable exception is a recent work, Siegel (2007), analyzing the contests with participants that have different commonly observable cost functions.

demonstrates the top performance. In case of a tie the winner is selected randomly according to a non-degenerate distribution, p, describing the likelihood of getting the prize for all those at the top.[3] One's performance fully reflects the input, as is standard in the all-pay auction literature, so that the effort (bid) chosen by a contestant is observable.

The payoff to winner k is $u_k(v_k - b)$, where $b \geq 0$ is the (monetized) effort he exerts, and v_k is his monetary equivalent for the prize. A loser $j \neq k$ bidding b gets $u_j(-b)$, as the (investment of) effort is irreversible. We assume that all $u_i : \mathbb{R} \to \mathbb{R}$ are twice continuously differentiable, concave, strictly increasing, with $u_i' > 0$. Thus the contestants can have different risk attitudes.

Each contestant knows his value from winning and shares the same beliefs about the values of the common rivals with any other contestant. The values, they presume, are distributed independently, but not necessarily identically, $V_i \sim F_i$ on $[\underline{v}, \overline{v}_i]$, $\underline{v} \geq 0$, with the density f_i, continuous and bounded away from zero for all $v \in [\underline{v}, \overline{v}_i]$.

In case all are risk-neutral, F_i also reflects the shared belief about i's abilities, and so even in this case the players can be distinct at the outset of the game; this is the second source of ex-ante heterogeneity.

Contestant i can formulate beliefs about his rivals' behavior, that is, the likelihood of $j \neq i$, to exert effort x_j, and the resulting distribution of the maximal effort of the rivals, $\bar{x} = \max_{j \neq i}\{x_j\}$. His probability of winning then is $W_i(b) = \Pr(b > \bar{x}) + \Pr(b = \bar{x})\, p_i(b, x_{-i})$, and his expected *interim* payoff should be $\Pi_i(b|v_i) = W_i(b) u_i(v_i - b) + (1 - W_i(b))\, u_i(-b)$.

2.1. Equilibrium.

A strategy for individual i is a Lebesgue-measurable function that maps valuations into effort levels, $b_i : [\underline{v}, \overline{v}_i] \to \mathbb{R}_+$. We restrict attention to (pure-strategy) equilibria in which contestants with higher valuations for the prize expend weakly higher effort, or, simply, bid higher. Existence of a Bayes-Nash equilibrium in non-decreasing strategies follows from Athey (2001, thm. 7, p. 881).

To describe such an equilibrium, it is convenient to express the winning probability in terms of players' strategies.

Definition 1. For any player $i = 1, \ldots, N$ let $\phi_i : b \mapsto v$ be a (generalized) *inverse bid function* such that

$$\phi_i(b) \overset{\text{def}}{=} \max\left(\underline{v}, \sup\{v : b_i(v) \leq b\}\right) .$$

A generalized inverse bid function, ϕ_i, agrees with the inverse bid map b_i^{-1} wherever the latter is well defined, is constant at any point of discontinuity (jump) of the bid function, returns $\underline{v}$ for any bid b strictly below the lowest equilibrium bid, and is continuous at b, if and only if b_i is strictly increasing at $\phi_i(b)$.

---

[3] That is, $p(\mathbf{b})$ satisfies $p_i(\mathbf{b}) \geq 0$, $\sum p_i(\mathbf{b}) = 1$ and, $p_i(\mathbf{b}) > 0$ if and only if $b_i = \bar{b}$, where $\mathbf{b} = (b_1, \ldots, b_N)$ is the bid profile and, $\bar{b} = \max_i b_i$ is the top bid.

Lemma 1. *Equilibrium bid functions b_i are strictly increasing on $(\phi_i(0), \bar{v}_i]$.*

Proof. Assume to the contrary that b_i is not strictly increasing. Since b_i is non-decreasing, there must exist an interval of types where b_i assumes a constant value, say $\hat{b}$. This means that contestant i bids $\hat{b}$ with strictly positive probability and given the tie-breaking rule, the winning probability of any other contestant must jump discontinuously at $\hat{b}$. As a result, it is never optimal for any other contestant to place bids in the interval $(\hat{b} - \varepsilon, \hat{b}]$ for some $\varepsilon > 0$, which implies that contestant i should never bid $\hat{b}$. $\square$

Lemma 2. *The lowest bid in an equilibrium is zero.*

Proof. Assume to the contrary, that $b_i(\underline{v}) = \beta = \min_j b_j(\underline{v}) > 0$. Then no rival $j \neq i$ should place a bid in $(0, \beta]$ so, i's winning probability is constant on $(0, \beta]$ and, he would be better off by reducing his bid. $\square$

First, observe that by the two lemmas ties happen with strictly positive probability only at zero in an equilibrium.

Second, generalized inverse bid functions, ϕ_i, are continuous for $b > 0$ as follows from lemma 1, and at $b = 0$ they are continuous by construction; in addition they are differentiable almost everywhere as they are bounded and non-decreasing.

Given the first observation, we can express the equilibrium probability of winning for all the contestants. Let $G_i(b) \overset{\text{def}}{=} \text{Prob}\,[b_i(V_i) \leq b] = F_i(\phi_i(b))$ be the probability that contestant i bids at or below b. Then the probability of winning by contestant i who bids b can be expressed as the product of cumulative distributions of equilibrium bids of the rivals, $W_i(b) = \prod_{j \neq i} G_j(b)$.

Monotonicity of the bidding strategies implies that the winning probability W_i of player i is differentiable for almost every bid $b > 0$, allowing us to describe the best response of i using the first order conditions for maximization of the interim payoff, $\Pi_i(b|v_i)$, $v_i \in (\underline{v}, \overline{v}_i)$. Almost any choice of optimal bid $b > 0$, must satisfy[4]

$$\begin{aligned} MB_i(b) &= MC_i(b), \text{ where} \\ MB_i(b) &= [u_i(v_i - b) - u_i(-b)]\, W_i'(b) \text{ and} \\ MC_i(b) &= u_i'(-b)(1 - W_i(b)) + u_i'(v_i - b) W_i(b)\ . \end{aligned} \tag{2.1}$$

If the marginal benefit, MB_i, is below marginal cost, MC_i, for any choice of $b \in (0, v_i]$, then in an equilibrium the type v_i should 'drop out', $b_i(v_i) = 0$.

Recall, W_i is differentiable almost everywhere, so for those bids where W_i' is well-defined, the bid densities, g_j for $j \neq i$, are so too. Then we can describe equilibrium behavior using the equilibrium bid density functions, and the set of active participants for each effort $b > 0$, that is, the set of

---

[4] Since contestants are weakly risk-averse, $MB_i(b) - MC_i(b)$ is strictly increasing in v_i for $b > 0$, so $\Pi_i(b|v_i)$ satisfies the strict single-crossing property.

contestants, who choose b for at least one of their types:[5]

$$J(b) \overset{\text{def}}{=} \left\{ i \in \{1,..,N\} \,|g_i(b) > 0 \text{ or } \lim_{x \to b} g_i(x) = +\infty \right\}.$$

In a model with ex-ante identical or only two distinct participants, for almost all b, $J(b)$ either includes all the participants, or is empty: all the players bid in the same range in equilibria there. However, in the presence of more than two ex-ante distinct contestants this is no longer true, as we demonstrate in the next two sections. Although we are unable to provide an explicit algorithm to determine $J(b)$ from the primitives of the model, one can use the first order conditions to check whether a contestant with value $v > \underline{v}$ would place a bid $b > 0$ in equilibrium. The key test is verifying the sign of the implied rate of growth of G_i at b, which has to be non-negative.

This argument is used to derive our main results in the next two sections.

3. Participation, group composition and shared beliefs

Were the valuations commonly observed and costs of effort identical, as in Hillman and Riley (1989); Baye et al. (1993), generically, only the two individuals with the highest values for the prize enter the competition, while the rest drop out. Differences in the costs of effort, as in Siegel (2007), can induce higher participation. However, under incomplete information heterogeneity has a different effect: an ex-ante 'weaker' contestant might completely drop out of the competition, meaning, always exert zero effort, even if he has the top ex-post valuation for the prize.

Recall that under standard assumptions under incomplete information with two asymmetric players,[6] or with several symmetric ones,[7] all place their bids in the same range, so the behavior described by the next proposition can never arise in an equilibrium of these models.

Notation. The maximal equilibrium bid is $\overline{b} = \max_i b_i(\overline{v}_i)$. Also define the auxiliary function $\Phi(t,x) = t\frac{e^{tx}}{e^{tx}-1}$ and, let $r_i(x) = -u_i''(x)/u_i'(x)$ denote i's Arrow-Pratt coefficient of absolute risk-aversion with its respective bounds, $\overline{r}_i = \max_{x \in [-\overline{v}_i, \overline{v}_i]} r_i(x)$ and $\underline{r}_i = \min_{x \in [-\overline{v}_i, \overline{v}_i]} r_i(x)$, which are well-defined by our assumptions.

Proposition 1. *Assume:*

(1) $b_j\left([\underline{v}, \overline{v}_j]\right) = [0, \overline{b}]$ *for all* $j \neq 1$;
(2) $\sum_{j \neq 1} \Phi(\underline{r}_j, \bar{v}_j) e^{-\underline{r}_j \bar{v}_j} < (N-2)\Phi(\bar{r}_1, \bar{v}_1) e^{-\bar{r}_1 \bar{v}_1}$;

Then, $b_1(\overline{v}_1) < \overline{b}$. *Moreover, if* $\underline{v} > 0$ *and we replace condition 2 by:*

(3) $\sum_{j \neq 1} \left[\Phi(\underline{r}_j, \underline{v}) e^{-\underline{r}_j \underline{v}} - \Phi(\bar{r}_j, \underline{v})\right] < (N-2) \left[\Phi\left(\bar{r}_1, \bar{v}_1\right) e^{-\bar{r}_1 \bar{v}_1} - \Phi\left(\underline{r}_1, \bar{v}_1\right)\right]$; *and*

---

[5] An alternative, but equivalent, formulation for the set of active bidders is $J(b) = \{i \,|\, b_i(v) = b$ for some $v \in [\underline{v}, \overline{v}_i]$ and b_i is differentiable at $v\}$.

[6] See Amann and Leininger (1996).

[7] See, e.g., Lizzeri and Persico (1998); Krishna and Morgan (1997); Gavious et al. (2002).

(4) *There are* $\lambda_i > 0$ *with* $\sum_{i=1}^{N} \lambda_i = 1$ *such that for all* $j \neq 1$,

$$F_j\left(v\right) \leq \lambda_j (N-2) \frac{\Phi\left(\underline{r}_1, \bar{v}_1\right)}{\Phi\left(\bar{r}_j, v\right)} \text{ for any } v \in \left[\underline{v}, \bar{v}_j\right];$$

Then, $b_1(v) = 0$ *for all* $v \in [\underline{v}, \overline{v}_1]$.

Remark 1. Proposition 1 can be applied to rule out participation of several contestants in a recursive manner.

The first part of the proposition establishes the *partial drop-out effect*: one of the contestants is never going to bid as high as some of his rivals do in an equilibrium. His lack of competitiveness has a rationale: he could have bid at the top and get the prize with certainty, by assumption 1, but he does not, since 2 implies that his marginal cost of bidding at the top is lower than the marginal benefit.

Note, condition 2 is likely to hold when one's top valuation is below the average of that of his rivals or when he is sufficiently less risk-averse than an 'average' rival.

Example 1. If all the contestants are risk-neutral, but the distributions of their values have distinct respective supports, $[\underline{v}, \overline{v}_i]$, condition 2 becomes:

$$\frac{1}{N-2} \sum_{j \neq 1} \frac{1}{\overline{v}_j} < \frac{1}{\overline{v}_1}.$$

Example 2. If the preferences of the contestants exhibit constant relative risk aversion, CARA, so $r_i(x) = \rho_i$, and their valuations have common support, $[\underline{v}, \overline{v}]$, condition 2 translates to:

$$\frac{1}{N-2} \sum_{j \neq 1} \frac{\rho_j}{e^{\rho_j \overline{v}} - 1} < \frac{\rho_i}{e^{\rho_i \overline{v}} - 1}.$$

We use the latter to construct a numerical example, see fig. 1. As one would expect, more risk-averse rivals are more aggressive in their bidding at the top since they want to insure themselves against losing. The figure shows that with such rivals the first contestant finds it more profitable to shade his bid and bear the risk of losing rather than play at the top.

Remarkably, further increasing the gap in risk aversion is not going to generate a complete drop-out. Indeed, given all the distributions have the same support, player 1 could have bid at the top and get the prize for sure, thus enjoying a positive payoff, as other players do. However, even when his valuation is the highest, he chooses a different bid, indicating his payoff must be even higher and thus, strictly positive. Hence so should be the bid, which means the player actively participates in the contest for at least some of his valuations.

Differences in contestants' highest possible valuations are crucial for the complete drop-out result:

Remark 2. Any two contestants i and j with identical risk attitudes and with common support of valuations, place the same bid at the top, $b_i(\overline{v}) = b_j(\overline{v})$.

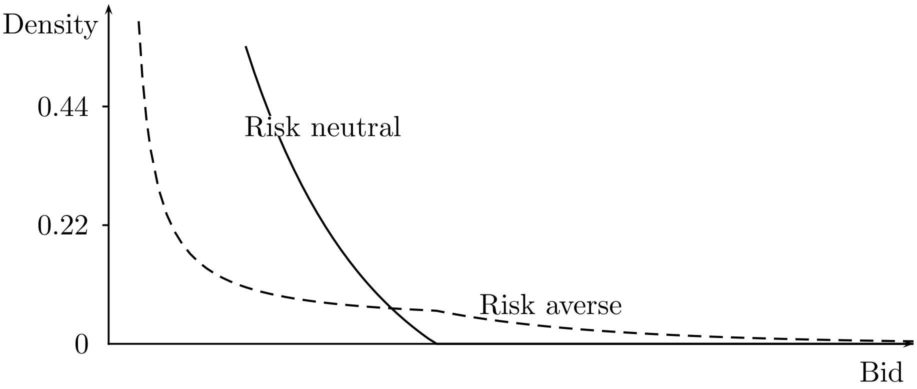

FIG. 1. Bid density of five identical contestants with $\rho_j = 1$ who face a risk neutral one. All have uniform valuations drawn from $[0, 1]$.

Having established the partial drop-out, we obtain the *complete drop-out* by imposing 4, which guarantees that reducing the bid will not cut the costs by more than the drop in benefits, and so the contestant should never choose a strictly positive bid (note assumptions 4 and 3 imply 2).

Assumption 4, which is a first-order stochastic dominance condition, requires that valuations of 1's rivals are not very likely to be low, thereby guaranteeing bid cuts do not increase the marginal benefit of winning. This is easily seen in case all the contestants are risk-neutral, as then the marginal benefit of winning is proportional to the marginal winning probability while the marginal cost is constant. By assumption 4 then, the marginal probability of winning at lower bids is lower than the marginal probability of winning at the top, implying player 1 can not do better by reducing the bid.

Proposition 1 can be used to generate predictions about the effect of group composition on participation in contests with more than two players.

Example 3. Risk-neutral contestants are either 'strong' or 'weak', with upper valuations $\overline{v}_s$ and $\overline{v}_w$ respectively, such that $\overline{v}_s > \overline{v}_w$. Valuations are uniformly distributed. Then, according to proposition 1, weak contestants participate only when the number of strong contestants is smaller than a measure of the relative advantage, $\frac{\overline{v}_s}{\overline{v}_s - \overline{v}_w}$. Otherwise, all weak contestants shy away from competition, bidding zero regardless of their valuation.

Using the proposition, one can also study the effect of contestants' beliefs about the ability (i.e., the reciprocal of the valuation in case all are risk-neutral) of a rival on the elicited effort.

Example 4. Two risk-neutral players face a different rival in different contests, see fig. 2. As the upper support of the distribution of the rival grows (in the eyes of the others), his reluctance to participate gives way to an increasingly aggressive behavior in the equilibrium.

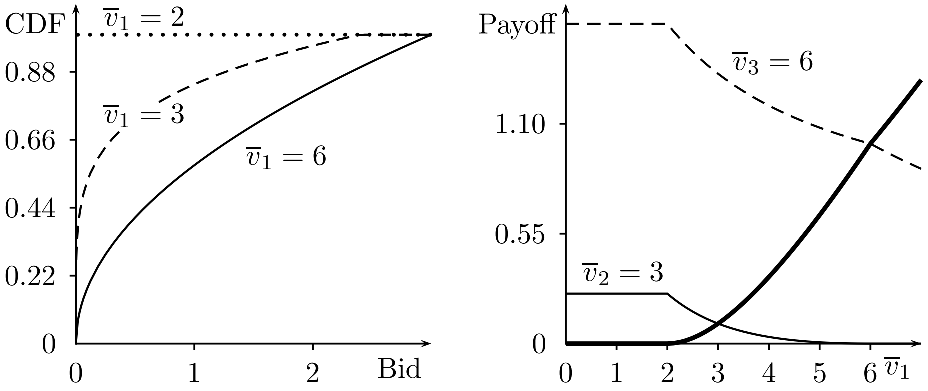

FIG. 2. Complete drop-out. Left: bid density of the first player facing the same two rivals ($\overline{v}_2 = 3$, $\overline{v}_3 = 6$) in three different contests. Right: the equilibrium payoff of players as $\overline{v}_1$ varies from 0 to 7.

4. DISCONTINUOUS BIDDING

Recall that in a standard all-pay auction with incomplete information,[8] if there are only two players, then they bid in the same interval; the same is true if there are more bidders, but all of them are ex-ante identical. Moreover, since the distribution of the maximal rivals' bids faced by a player can not exhibit gaps, the equilibria of these models rule out discontinuous bidding.[9]

However, as we show in the next proposition, sufficient heterogeneity in risk attitudes (including that of one's rivals) makes continuous bidding for all *inconsistent* with an equilibrium.

Proposition 2. *There are at least four contestants; contestants* 1 *and* 2 *have identical preferences and distributions of valuations. All the distributions have common support,* $[\underline{v}, \overline{v}]$*, with* $\underline{v} > 0$*. If contestants* 1 *and* 2 *are sufficiently risk averse, there is no equilibrium in which all bid continuously.*

The crucial argument behind this proposition is the following: as a contestant becomes more risk averse, he should never place positive bids that bring him the victory with probability below certain threshold. Provided the support of the distribution of the maximal rival bid is an interval, this prescription is equivalent to never picking low positive bids.

Naturally, then, a candidate for an equilibrium (which exists) will be the one in which the risk averse are bidding discontinuously, in accord with a *bid bifurcation* scenario, where only bids at zero or bids above a threshold are observed. Such an example, where the equilibrium was computed numerically, is in fig. 3.

---

[8] See, e.g., Krishna and Morgan (1997); Amann and Leininger (1996), Fibich et al. (2006).

[9] However, an anonymous referee pointed that in his experimental data set, discontinuous bidding occurred even in two-player tournaments.

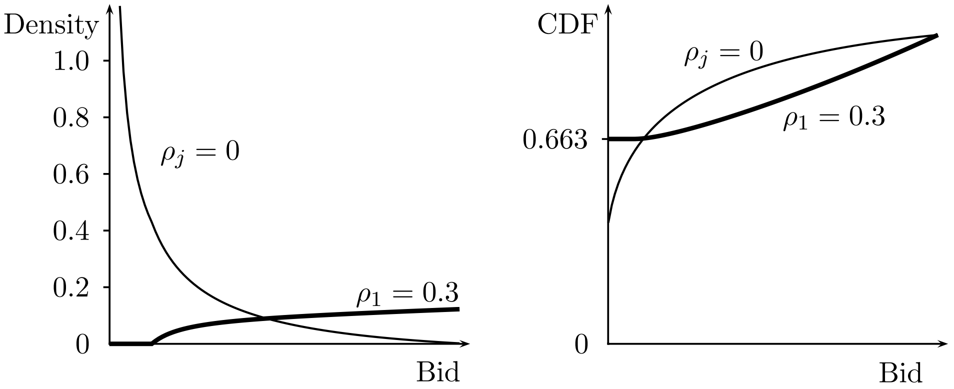

FIG. 3. 4 risk-neutral and 2 CARA competitors with $\rho = 0.3$. Valuations are uniform on $[1/250, 4]$. Note the flat part of the CDF of the risk-averse player 1, corresponding to a 'jump' in his bidding function. Also his CDF indicates an 'atom' at zero.

It is easy to see why risk aversion leads to bid bifurcation. Bidding at the top brings the prize for sure, and this is the premium for the "full insurance" against losing. However, when a contestant places low bids, this analogy breaks down due to the "all-pay" feature of the auction. A bid still can be seen as a "premium" that the contestant must pay, but low bids fail to deliver the prize with certainty. If the probability of winning is not very high, (say, v_i is low), as the contestant becomes more risk-averse, he would be better off to get "insurance" by bidding zero and this is his best response.Hence increasing risk aversion of the agent creates a tension leading to the discontinuity: a mass of low valuation types drops out from the contest, high valuation types place high bids, but no types place low bids.

It is important to remember though that if all agents become *equally* more risk-averse, despite the tension outlined above, the bidding will remain continuous, thus, ruling out bifurcation of effort, and this is due to the constraints imposed by the symmetric environment. A similar reasoning applies to the two bidder case.

The effect of risk aversion in this model is not the same as in the first-price auction. Though in both models as a contestant becomes more risk-averse, his willingness to bid at the top increases, however, in contrast with the first-price auction, where an increase in risk-aversion *always* raises bids, the "all-pay feature" of a contest motivates low valuation contestants to bid zero.

Finally, notice that the *aggregate* distribution of bids should not have gaps in equilibrium. The bifurcation (discontinuity) result highlights the importance of looking at the *individual* behavior in actual contests.

5. Aggressive Bidding

Here we offer two reasons for a player to be more aggressive in contests: risk aversion and stronger ex-ante distribution of valuations.

Recall, proposition 1 shows that a *sufficiently* risk-averse rival chooses a range of bids at the top that are never used by his less risk-averse competitor and proposition 2 implies that in some equilibria a middle range of bids will be foregone by sufficiently risk-averse. Our next result, proposition 3, demonstrates that the more risk-averse contestants are less likely to choose low *strictly positive* effort levels and are more likely to choose high effort levels as compared to their less risk-averse rivals.

Note the following proposition is 'in the same spirit' as prop. 2 by Fibich et al. (2006), who compare behavior of more and less risk-averse players in two different symmetric equilibria, though in our case the contestants with distinct risk attitudes face each other in the same game.

Proposition 3. *Assume valuations share a common support,* $[\underline{v}, \overline{v}]$*, contestants* i *and* j *bid continuously, and contestant* i *is strictly more risk averse than* j*. Then in a neighborhood of zero and in a neighborhood of the top bid,* $\overline{b}$*, bid distributions are ordered,* $G_i(b) < G_j(b)$*.*

So far we have been silent about the likelihood of a more risk-averse agent to choose zero effort. While the probability of him picking low *positive* effort levels declines with risk-aversion, he might start choosing zero effort more often as a result. Our discontinuity result, proposition 2, is consistent with this implication. Unfortunately, the system of differential equations that characterizes the equilibrium is extremely 'ill behaved' at zero, thus we are unable to confirm this conjecture at this stage.

We conclude our analysis by demonstrating that contestants who are perceived as being 'more willing to win' ex-ante are also expected to be more aggressive, i.e., bid higher in the sense of first-order stochastic dominance. See Lebrun (1999), Maskin and Riley (2000) and Milgrom (2004, thm 4.25) for a similar exercise in the context of first-price auctions.

Proposition 4. *Assume all contestants have identical thrice differentiable utility functions, their valuations have a common support,* $[\underline{v}, \overline{v}]$*, and the set of active bidders* $J(b)$ *is constant. If* $F_j(v) < F_i(v)$ *for all* $v \in (\underline{v}, \overline{v})$*, then* $G_j(b) < G_i(b)$ *for all* $b \in \big(0, \overline{b}\big)$*.*

6. Conclusions

With the exception of models that allow for affiliated signals (Krishna and Morgan, 1997; Lizzeri and Persico, 1998) all the previous incomplete information all-pay auctions models are nested within the one offered in this paper. Moreover, the mixed strategy equilibrium of complete information models (Hillman and Riley, 1989; Baye et al., 1993, 1996; Che and Gale, 1998) can also be described by our necessary conditions for any strictly positive effort level.

This generalization allows us to rationalize two important phenomena: (1) in a traditional winner-take-all contest "underdogs" (viewed as weak by their rivals) might be discouraged from participating; (2) relatively more risk-averse agents might choose "all-or-nothing" strategies in such contests.

Although not conclusive, there is some empirical support for both: (1) Gneezy et al. (2003) show that women's performance is higher in single-sex than in co-ed pool of contestants, see also Niederle and Vesterlund (2007); (2) Muller and Schotter (2007) and Noussair and Silver (2006) show that a subset of subjects in their all-pay auctions experiments display discontinuous bidding behavior.[10]

Key insight used in the derivation of both results is the "participation test": if an equilibrium strategy prescribes contestant i to place a strictly positive bid b_0 then the beliefs of his rivals must be consistent, i.e., i's bidding density can not be negative at b_0. The test is easy to use: given equilibrium winning probability of the active bidders, the equilibrium density can be calculated from the primitives of the model. We also conjecture that the same method can be applied to obtain similar results for an auction with M homogenous prizes and $M+2$ or more heterogeneous participants.

Let us stress the behavior described in our results is micro in its nature. With Bayesian-Nash equilibrium, all-or-nothing strategies or 'partial drop-out' behavior can only be detected at the individual level: aggregation of the equilibrium bidding behavior eliminates the gaps in the distribution of effort. With this caveat, each of the examples (1, 2, 3) provided in the second section can be easily turned into a testable hypothesis, and so is the discontinuity result (proposition 2) requiring "sufficient" heterogeneity in risk attitudes of the contestants.

Previous literature on labor tournaments has established that heterogenous contests are likely to generate inefficient allocations (Lazear and Rosen, 1981) and that handicaps may reduce the efficiency loss (Schotter and Weigelt, 1992). However, as attention was restricted to just two-player tournaments, participation was not analyzed. Our work complements these contributions by demonstrating that the effect of beliefs and group composition on participation can be non-trivial with several distinct agents.

Clearly, we have only looked at some determinants for (non-)participation for a *fixed* contest format. If one is to address, say, the question of inducing high participation rates from all qualified groups, which is often stated as an objective of affirmative action programs, then an optimal design framework would be the most appropriate. However, our current contribution, devoted to the analysis of a particular contest, provides a first step in that direction.

---

[10] However, for a critical analysis of the approach, see Kirchkamp (2006).

Appendix A. Proofs

Lemma 3. *For almost all bids,*[11] $b > 0$*, the system of first order conditions* (2.1) *can be represented as*

(A.1)

$$\frac{g_i(b)}{G_i(b)} = \begin{cases} \frac{1}{K(b)-1}\left(\sum\limits_{j\in J(b)\setminus\{i\}} S_j(b) - (K(b)-2)S_i(b)\right), & i \in J(b) \\ 0, & \textit{otherwise} \end{cases}$$

where $K(b) = \#J(b)$*,*[12] *and* $S_i(b) = \frac{u_i'(\phi_i(b)-b)+\left(\frac{1}{W_i(b)}-1\right)u_i'(-b)}{u_i(\phi_i(b)-b)-u_i(-b)}$.

Proof. For $i \in J(b)$, differentiating the identity $W_i(b) = \prod_{j\neq i} G_j(b)$, yields

$$\sum_{j\neq i} \frac{g_j(b)}{G_j(b)} = \frac{W_i'(b)}{W_i(b)}, \quad i \in J(b) \tag{A.2}$$

The system (A.2) is linear in the rate of growth of $G_j(b)$, implying

(A.3)

$$\left(\frac{g_j(b)}{G_j(b)}\right)_j = M^{-1}\left(\frac{W_i'(b)}{W_i(b)}\right)_i$$

$$M^{-1} = \frac{1}{K(b)-1}\begin{pmatrix} -(K(b)-2) & 1 & \cdots & 1 \\ 1 & -(K(b)-2) & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \dots & 1 & -(K(b)-2) \end{pmatrix}$$

For $i \in J(b)$ this proves lemma 3 and for $i \notin J(b)$ we have that $g_i(b) = 0$ provided g_i exists, which is the case for almost every b. $\square$

Remark 3. (1) For $i \in J(b)$ with $b > 0$, the first order conditions (2.1) must be satisfied with equality:

$$\frac{W_i'(b)}{W_i(b)} = S_i(b) = \frac{u_i'(\phi_i(b)-b)+\left(\frac{1}{W_i(b)}-1\right)u_i'(-b)}{u_i(\phi_i(b)-b)-u_i(-b)} > 0 \tag{A.4}$$

(2) Since inverse bidding functions are continuous, $S_i(b)$ is continuous as well.

---

[11] More precisely, (A.1) holds for all $b > 0$ where the inverse bid functions are differentiable and the set of active contestants, $J(b)$, is constant in some neighborhood of b.

[12] #X denotes the cardinality of X.

Corollary 1 (**Participation Test**). *In an equilibrium, player i with valuation v does not place bids in a neighborhood of $b > 0$ if*

$$\sum_{j \in J(b) \setminus \{i\}} S_j(b) - (K(b) - 2) Z_i(b, v) < 0, \quad \textit{where}$$

$$Z_i(b,v) \overset{\text{def}}{=} \frac{\exp\left(-\int_{-b}^{v-b} r_i\left(z\right)\, dz\right) + W_i^{-1}\left(b\right) - 1}{\int_{-b}^{v-b} \exp\left(-\int_{-b}^{y} r_i\left(z\right)\, dz\right) dy}. \tag{A.5}$$

Proof of corollary 1. First, note that the definition of Z, using Pratt's (1964) representation of utilities,[13] is equivalent to $Z(b) = \frac{u_i'(v-b) + u_i'(-b)\left(\frac{1}{\prod_{j \neq i} G_j(b)} - 1\right)}{u_i(v-b) - u_i(-b)}$. Next, if $b_i(v) = b$, then $Z_i(b,v) = S_i(b)$, which along with the inequality implies (by lemma 3 and the continuity of S_j) that the growth rate of winning probability is either negative or zero in a neighborhood of b. But since the former is not possible, we must have $g_i = 0$ in a neighborhood of b. $\square$

Remark 4. In practice, if no previous knowledge of the set $J(b)$ is provided, we can still obtain the result in the corollary by using the stronger 'participation test':

$$\sum_{j \in X \setminus \{i\}} S_j(b) - (\#X - 2) Z_i(b,v) < 0 \textbf{ for all } X \subset \{1, \ldots, N\} \setminus \{i\}.$$

Lemma 4. *For any $p > 0$, define $b_p \overset{\text{def}}{=} \inf\{b : W_1(b) \geq p\}$. Fix some $q > 0$ with $b_q > 0$. Contestant 1 should not bid in the neighborhood of b_q if the two following condition hold true:*

(1) $\exists\, c > 0$ *such that* $\forall j \neq 1$, $W_j\left(b_q\right) \geq c$ *and*

(2) $$\frac{\sum_{j \neq 1} \Phi\left(\underline{r}_j, \phi_j\left(b_q\right)\right) e^{-\underline{r}_j \phi_j(b_q)} + (1/c - 1)\,\Phi\left(\bar{r}_j, \phi_j\left(b_q\right)\right)}{(N-2)} <$$
$$< \Phi\left(\bar{r}_1, \bar{v}_1\right) e^{-\bar{r}_1 \bar{v}_1} + (1/q - 1)\,\Phi\left(\underline{r}_1, \bar{v}_1\right).$$

Remark 5. Provided $q > 1$ and $\underline{v} > 0$, condition (2) in lemma 4 can be replaced by:

$$\frac{1}{c(N-2)} \sum_{j \neq 1} \Phi\left(\underline{v}, \bar{r}_j\right) < \left(\frac{1}{q} - 1\right) \Phi\left(\bar{v}_1, \underline{r}_1\right).$$

Proof. First, using notation of cor. 1,

$$\begin{aligned} Z_1\left(b,v\right) \quad &> \quad \frac{1}{\int_{-b}^{\bar{v}_1 - b} \exp\left(\int_{y}^{\bar{v}_1 - b} \bar{r}_1\, dz\right) dy} + \frac{\left(W_1^{-1}\left(b\right) - 1\right)}{\int_{-b}^{\bar{v}_1 - b} \exp\left(-\int_{-b}^{y} \underline{r}_1\, dz\right) dy} \\ &= \quad \Phi\left(\bar{r}_1, \bar{v}_1\right) e^{-\bar{r}_1 \bar{v}_1} + (1/q - 1)\,\Phi\left(\underline{r}_1, \bar{v}_1\right). \end{aligned}$$

---

[13] That is, $u_i(x) = \int_0^x \exp\left(-\int_0^y r_i(z) dz\right) dy$.

Next,

$$\begin{aligned} S_j\left(b\right) &< \frac{1}{\int_{-b}^{\phi_j(b)-b} \exp\left(\int_y^{\phi_j(b)-b} \underline{r}\; dz\right)\, dy} + \frac{\left(W^{-1}\left(b\right)-1\right)}{\int_{-b}^{\phi_j(b)-b} \exp\left(-\int_{-b}^{y} \bar{r} dz\right) dy} \\ &= \Phi\left(\underline{r}_j, \phi_j\left(b\right)\right) e^{-\underline{r}_j \phi_j(b)} + \left(1/c-1\right)\Phi\left(\bar{r}_j, \phi_j\left(b\right)\right). \end{aligned}$$

The result then follows by cor. 1. □

Proof of proposition 1. First part of the statement, the partial drop-out, follows from applying lemma 4, with $c = 1$, $b_q = \overline{b} > 0$ and $q = 1$.

Next we show that if assumptions 3-4 of the proposition hold, the contestant one with the highest valuation should never bid any other positive bid. Assume to the contrary contestant one with the highest valuation, $\bar{v}_1$, is bidding some $b > 0$. For any strictly positive b the winning probability of any active player should be strictly positive, so $\Pi_{i=1}^{N} G_i\left(b\right) = W_1\left(b\right) > 0$, in the view of cor. 1 it is sufficient to show $\frac{1}{(N-2)}\sum_{j\neq 1} W_1\left(b\right) S_j\left(b\right) < Z_1\left(b,v\right) W_1\left(b\right)$.

First, using notation of cor. 1,

$$\begin{aligned} Z_1\left(b,v\right) W_1\left(b\right) &= \frac{W_1\left(b\right) + \exp\left(\int_{-b}^{v-b} r_1\left(z\right)\, dz\right)\left(1-W_1\left(b\right)\right)}{\int_{-b}^{v-b} \exp\left(\int_y^{v-b} r_1\left(z\right)\, dz\right)\, dy} \\ &> \frac{W_1\left(b\right)}{\int_{-b}^{\bar{v}_1-b} \exp\left(\int_y^{\bar{v}_1-b} \bar{r}_1\, dz\right)\, dy} + \frac{\left(1-W_1\left(b\right)\right)}{\int_{-b}^{\bar{v}_1-b} \exp\left(-\int_{-b}^{y} \underline{r}_1\, dz\right) dy} \\ &= W_1\left(b\right)\Phi\left(\bar{r}_1, \bar{v}_1\right) e^{-\bar{r}_1 \bar{v}_1} + \left(1-W_1\left(b\right)\right)\Phi\left(\underline{r}_1, \bar{v}_1\right) \end{aligned}$$

Next,

$$\begin{aligned} S_j\left(b\right) &< \frac{1}{\int_{-b}^{\underline{v}-b} \exp\left(\int_y^{\underline{v}-b} \underline{r}\; dz\right)\, dy} + \frac{\left(W^{-1}\left(b\right)-1\right)}{\int_{-b}^{\underline{v}-b} \exp\left(-\int_{-b}^{y} \bar{r} dz\right) dy} \\ &= \Phi\left(\underline{r}_j, \underline{v}\right) e^{-\underline{r}_j \underline{v}} + \left(W_j^{-1}\left(b\right)-1\right)\Phi\left(\bar{r}_j, \underline{v}\right) \end{aligned}$$

Note the latter is bounded, as $\underline{v} > 0$. Then

$$\begin{aligned} \Pi_{i=1}^{N} G_i\left(b\right) S_j\left(b\right) &< W_1\left(b\right)\Phi\left(\underline{r}_j, \underline{v}\right) e^{-\underline{r}_j \underline{v}} - W_1\left(b\right)\Phi\left(\bar{r}_j, \underline{v}\right) \\ &\quad + G_j\left(b\right)\Phi\left(\bar{r}_j, \phi_j\left(b\right)\right) \end{aligned}$$

By cor. 1 to assure that player 1 of type $\overline{v}_1$ does not bid any positive bid, it is sufficient to have

$$\begin{aligned} &\frac{1}{(N-2)}\sum_{j\neq 1} W_1\left(b\right)\left[\Phi\left(\underline{r}_j, \underline{v}\right) e^{-\underline{r}_j \underline{v}} - \Phi\left(\bar{r}_j, \underline{v}\right)\right] + G_j\left(b\right)\Phi\left(\bar{r}_j, \phi_j\left(b\right)\right) \\ &< W_1\left(b\right)\left[\Phi\left(\bar{r}_1, \bar{v}_1\right) e^{-\bar{r}_1 \bar{v}_1} - \Phi\left(\underline{r}_1, \bar{v}_1\right)\right] + \Phi\left(\underline{r}_1, \bar{v}_1\right) \end{aligned}$$

For which it is sufficient then to have the two inequalities hold with at least one of them being strict:

$$\begin{aligned} \frac{1}{(N-2)} \sum_{j \neq 1} \left[\Phi \left(\underline{r}_j, \underline{v} \right) e^{-\underline{r}_j \underline{v}} - \Phi \left(\bar{r}_j, \underline{v} \right) \right] &\leq \left[\Phi \left(\bar{r}_1, \bar{v}_1 \right) e^{-\bar{r}_1 \bar{v}_1} - \Phi \left(\underline{r}_1, \bar{v}_1 \right) \right] \\ \frac{1}{(N-2)} \sum_{j \neq 1} G_j \left(b \right) \Phi \left(\bar{r}_j, \phi_j \left(b \right) \right) &\leq \Phi \left(\underline{r}_1, \bar{v}_1 \right) \end{aligned}$$

And these conditions are implied by the assumptions 3-4. Note that given we have showed the highest type of player 1 will never place any $b > 0$, it is then never optimal for any type of player one to bid above zero in equilibrium. □

Proof of remark 2. Assume to the contrary, there exists $b_j(\overline{v}) = \beta < b_i(\overline{v}) = \tilde{\beta}$, so $G_i(\beta) < G_j(\beta) = 1$. Given the identity, $W_i(b)G_i(b) = W_j(b)G_j(b)$, this implies $W_i(\beta) > W_j(\beta)$ and $W_j(\tilde{\beta}) = W_i(\tilde{\beta})$. Thus, $\Pi_i(\beta|\overline{v}) > \Pi_j(\beta|\overline{v})$ and $\Pi_i(\tilde{\beta}|\overline{v}) = \Pi_j(\tilde{\beta}|\overline{v})$ which contradict the revealed preferences, $\Pi_i(\tilde{\beta}|\overline{v}) \geq \Pi_i(\beta|\overline{v})$ and $\Pi_j(\beta|\overline{v}) \geq \Pi_j(\tilde{\beta}|\overline{v})$. □

Proof of Proposition 2. Assume all players bid continuously. It must be that $b_q > 0$ since $b_q = 0$ would imply that $G_j(0) > q$ for all $j \neq 1$ and moreover since 1 and 2 must choose identical strategies[14], $G_1(0) = G_2(0) > q$. Thus, all players have an atom at zero, which is not possible. Thus, given that $b_q > 0$, we can apply lemma 4 with $c = q^N$: provided players 1 and 2 choose the same strategies the inequality in the remark following the lemma follows from $\frac{1}{q^N (N-3)} \sum_{j \neq 1,2} \Phi \left(\underline{v}, \overline{r}_j \right) < \left(\frac{1}{q} - 1 \right) \Phi \left(\bar{v}_1, \underline{r}_1 \right)$ which is satisfied if $\underline{r}_1 > K$ for sufficiently high K. Clearly, condition 2 could have been used as well (and it provides an alternative bound for K). □

Proof of proposition 3. To prove the claim for b near the top bid, note that by continuity of g_i (lemma 3), it is sufficient to demonstrate that $g_i(\overline{b}) > g_j(\overline{b})$. All the players are bidding the top bid, so, $W_i \left(\overline{b} \right) = 1$ for all i, but then the inequality follows from $S_i(\overline{b}) < S_j \left(\overline{b} \right)$ by lemma 3. The last inequality follows from definition (A.5) in cor. 1, as $r_i > r_j$.
To prove the claim for b near zero, note that by continuity of bidding functions, given $\phi_i(0) = \underline{v}$ and the identity $W_i G_j = W_j G_i$, L'Hôspital's Rule implies $\lim_{b \searrow 0} \frac{G_j(b)}{G_i(b)} = \frac{W_i'(0)}{W_j'(0)}$. Using Pratt's representation of utilities again, by the first order conditions, (2.1), $W_i'(0) = (\int_0^{\underline{v}} \exp \left(-\int_0^y r_i(z)dz \right) dy)^{-1}$ implying $W_i'(0) > W_j'(0)$, as $r_i > r_j$. □

Proof of proposition 4. We shall use the following lemma in this proof.

---

[14] If strategies are continuous, the system of differential equations given by the first-order conditions has a unique solution for any set of initial conditions with $b > 0$. Moreover, the solution must be the same for players 1 and 2 since they have identical preferences and, by remark 2, they bid the same at the top.

Lemma 5. *If utilities are k-differentiable and the set of active contestants $J(b)$ is constant for b in an open interval not containing the origin then the distribution of bids G_i are $(k-1)$-differentiable in that interval.*

To establish the lemma, notice that if its conditions hold true then the system of differential equations A.1 is $(k-1)$-differentiable and so its solution is $(k-1)$-differentiable. So by the lemma and assumptions of the prop. 4, winning probabilities W_i are twice-differentiable.

Since F_j first-order stochastically dominates F_i, we have the weak inequality $f_i(\overline{v}) \leq f_j(\overline{v})$. For simplicity, we will prove the prop. 4 first for the case where the strict inequality, $f_i(\overline{v}) < f_j(\overline{v})$, holds.

Suppose that either: G_i and G_j cross at some point in the interior of support of equilibrium effort levels, $b^* \in (0, \overline{b})$, or that they are tangent at b^*. In this case, it follows that $G_i(b^*) = G_j(b^*)$, which, coupled with the assumption $F_j(v) < F_i(v) \, \forall v \in (\underline{v}, \overline{v})$, implies $\phi_i(b^*) < \phi_j(b^*)$. Moreover, from $G_i(b^*) = G_j(b^*)$, $\phi_i(b^*) < \phi_j(b^*)$, we obtain that $S_i(b^*) > S_j(b^*)$. Using the characterization of the effort densities (lemma 3), we get $g_i(b^*) < g_j(b^*)$. In sum, we have:

> (A) G_i and G_j can not be tangent at any $b \in (0, \overline{b})$ and moreover, if G_i and G_j cross then G_j must intersect G_i from below.

At the boundaries of the support of the equilibrium effort levels, the distributions of effort may be tangent. In particular since $S_i(\overline{b}) = S_j(\overline{b})$, they are tangent at $\overline{b}$, that is, $G_i(\overline{b}) = G_j(\overline{b}) = 1$ and $g_i(\overline{b}) = g_j(\overline{b})$, where $\overline{b} = b_i(\overline{v}) = b_j(\overline{v})$ as established in remark 2.

We have that if $G_i(b) = G_j(b)$, $\phi_i(b) = \phi_j(b)$, and $g_i(b) = g_j(b)$ for some b then the following statements are equivalent: $g_i'(b) \geq g_j'(b) \Leftrightarrow W_i''(b) \leq W_j''(b) \Leftrightarrow \phi_i'(b) \geq \phi_j'(b) \Leftrightarrow f_i(\phi_i(b)) \leq f_j(\phi_j(b))$. These statements follow respectively from the identities listed below:

(1) $$\frac{g_i' G_i - g_i^2}{G_i^2} = \frac{\sum_{k \in J} \left[\frac{W_k''}{W_k} - \left(\frac{W_k'}{W_k} \right)^2 \right] - (K-1) \left[\frac{W_i''}{W_i} - \left(\frac{W_i'}{W_i} \right)^2 \right]}{K-1}.$$

(2) $$0 = W_i'' \left(u_i(\phi_i - b) - u_i(-b) \right) + (\phi_i'(b) - 1) \left(W_i' \, u_i'(\phi_i - b) - u_i''(\phi_i - b) \right) - W_i' \left(u_i'(\phi_i - b) - u_i'(-b) \right) + (1 - W_i) \, u_i''(-b)$$

(3) $$\phi_i'(b) = \frac{g_i(b)}{f_i(\phi_i(b))}.$$

To obtain (1) we differentiate the expression of the growth rate of the bid distribution; as for (2), we differentiate the first-order conditions having previously substituted $\phi_i(b)$ for v_i; and finally for (3), we differentiate the definition of the cumulative distribution of bids, $G_i(b) = F_i(\phi_i(b))$. It follows $g_i'(\overline{b}) \geq g_j'(\overline{b})$, if and only if, $f_i(\overline{v}) \leq f_j(\overline{v})$. But, F_j first-order dominates F_i implies $f_i(\overline{v}) \leq f_j(\overline{v})$.Moreover, by assumption $f_i(\overline{v}) < f_j(\overline{v})$ and therefore $g_i'(\overline{b}) > g_j'(\overline{b})$. Therefore, we have:

> (B) at the top, G_j must intersect G_i from below.

The conclusions (A) and (B) above imply that G_i and G_j can never intersect at a point in the interior of their support and, G_i is always above G_j.

Finally we consider the case where $f_i(\overline{v}) = f_j(\overline{v})$, which can be seen as a limiting case of the original case. Clearly (A) also holds for this case. Moreover, since for any pair of points in the sequences of continuous distributions G_i^n and G_j^n, G_n^i, we have that at the top G_j^n intersects G_i^n from below then at the limit we also have that at $\overline{b}$, G_j intersects G_i from below. □

References

Amann, E. and W. Leininger (1996). Asymmetric all-pay auctions with incomplete information: The two player case. *Games and Economic Behavior 14*, 1–18.

Athey, S. (2001). Single crossing properties and the existence of pure strategy equilibria in games of incomplete information. *Econometrica 60*, 861–889.

Baye, M. R., D. Kovenock, and C. G. de Vries (1993). Rigging the lobbying process: An application of the all-pay auction. *The American Economic Review 83*(1), 289–294.

Baye, M. R., D. Kovenock, and C. G. de Vries (1996). The all-pay auction with complete information. *Economic Theory 8*, 291–305.

Che, Y. K. and I. L. Gale (1998, June). Caps on political lobbying. *American Economic Review 88*(3), 643–51.

Fibich, G., A. Gavious, and A. Sela (2006). All-pay auctions with risk-averse players. *International Journal of Game Theory 34*, 583–599.

Gavious, A., B. Moldovanu, and A. Sela (2002). Bid costs and endogenous bid caps. *Rand Journal of Economics 33*(4), 709–722.

Gneezy, U., M. Niederle, and A. Rustichini (2003). Performance in competitive environments: gender differences. *The Quarterly Journal of Economics 118*(3), 1049–1074.

Hillman, A. L. and J. G. Riley (1989). Politically contestable rents and transfers. *Economics and Politics 1*, 17–39.

Kirchkamp, O. (2006, November). Less fighting than expected – experiments with wars of attrition and all-pay auctions. Manheim University.

Krishna, V. and J. Morgan (1997). An analysis of the war of attrition and the all-pay auction. *Journal of Economic Theory 72*, 343–362.

Lazear, E. P. and S. Rosen (1981). Rank-order tournaments as optimal contracts. *Quarterly Journal of Economics 89*, 841–864.

Lebrun, B. (1999). First price auctions in the asymmetric n bidder case. *International Economic Review 40*(1), 125–142.

Lizzeri, A. and N. Persico (1998). Uniqueness and existence of equilibrium in auctions with a reserve price. *Games and Economic Behavior 30*, 83–114.

Maskin, E. and J. Riley (2000). Asymmetric auctions. *Review of Economic Studies 67*, 413–438.

Milgrom, P. (2004). *Putting Auction Theory to Work*. Cambridge University Press.

Muller, W. and A. Schotter (2007). Workaholics and drop outs in optimal organizations. Tilburg University.

Niederle, M. and L. Vesterlund (2007). Do women shy away from competition? Do men compete too much? *Technology 1067*.

Noussair, C. and J. Silver (2006). Behavior in all-pay auctions with incomplete information. *Games and Economic Behavior 55*, 189–206.

Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica 32*(1/2), 122–136.

Schotter, A. and K. Weigelt (1992). Asymmetric tournaments, equal opportunity laws and affirmative action: Some experimental results. *Quarterly Journal of Economics 107*(2), 511–539.

Siegel, R. (2007). All-Pay Contests. Technical report, mimeo, Northwestern University.