Theory of Groups (MATH 6270)

HOMEWORK ASSIGNMENT 10

(April 9, 2008)

Read Sections 7.2 (pp. 199–202), 7.3 (pp. 203–204), and Handout 6.

Problems:

Section 7.2: Exercises 9, 10, 11, 12, 16
Section 7.3: Exercises 1,
Additional Problems 1, 2 (see below)

Turn in underlined problems
Due Date: April 16, 2008

Additional Problems:

1. Let $H \leq \operatorname{Sym} X$ be a permutation group acting primitively on a finite set X, and let N, L be nontrivial normal subgroups of H such that $N \cap L = 1$. Prove that

(1) N and L are isomorphic centerless groups that act regularly on X, and the action of NL on X is similar to the following action of $N \times N$ on N:

$$N \times N \to \operatorname{Sym} N, \quad (\lambda, \nu) \mapsto \rho_{\lambda,\nu} \colon x \mapsto \lambda^{-1} x \nu = x^{\lambda}(\lambda^{-1}\nu).$$

(2) N and L are minimal normal subgroups of H, H has no other minimal normal subgroups, and NL acts primitively on X if and only if N is a nonabelian simple group.

Hint: (1) First prove that N acts regularly on X. To set up the similarity use the same bijection $\Phi \colon N \to X$ as in Lemma 3 on Handout 6.
(2) To prove the claim on primitivity, replace NL by the similar permutation group on N obtained via the similarity in part (1). (See the first paragraph of the proof of Theorem 5 (3) on Handout 6.)

2. Let $M = \mathrm{PSL}(n+1, q)$ with its usual permutation action on the set X of points of the projective geometry $\mathcal{PG}(n, q)$. Prove the equality $N_{\operatorname{Sym} X}(M) = \mathrm{P\Gamma L}(n+1, q)$ for $n \geq 2$.

Hint: Show that every $\pi \in N_{\operatorname{Sym} X}(M)$ preserves collinearity, and use the Fundamental Theorem of Projective Geometry (see Handout 5).