Theory of Groups (MATH 6270)

HOMEWORK ASSIGNMENT 3
(January 30, 2008)

Read Sections 1.4 (subsections 'Direct Products' and 'Internal Direct Products'), 1.5 (subsection 'Semidirect Products'), and 1.6 (subsections 'The Holomorph', 'The Wreath Product of Permutation groups', and 'Standard Wreath Products and Sylow Subgroups of the Symmetric Group').

Problems:
Section 1.4: Exercises 9, 10
Section 1.5: Exercises 10, <u>11</u>, 13
Section 1.6: Exercises 9, 14, <u>15</u>, 16
Additional Problems 1, <u>2</u> (see below)

Turn in underlined problems
Due Date: February 6, 2008

Additional Problems:

1. Let G be a group. Prove that G is abelian if and only if $D = \{(g,g) : g \in G\}$ is a normal subgroup of $G \times G$.

2. Prove that the following conditions on a group G are equivalent:
(a) $\zeta G = 1$ and $\operatorname{Aut} G = \operatorname{Inn} G$;
(b) whenever $G \triangleleft K$ for some group K, then K is an internal direct product of G and another normal subgroup of K.

(*Hint:* (a) $\Rightarrow$ (b): Show that $K = G \times C_K(G)$.
(b) $\Rightarrow$ (a): $G \triangleleft \operatorname{Hol} G$, therefore $\operatorname{Hol} G = M \times G$ for some $M \triangleleft \operatorname{Hol} G$. Deduce that
(i) $\operatorname{Aut} G = \operatorname{Inn} G$,
(ii) $G = G_0 \times \zeta G$ for some $G_0 \triangleleft G$, and hence
(iii) $\operatorname{Aut}(\zeta G) = 1$ and
(iv) $|\zeta G| \leq 2$.
Finally, use the assumption (b) again to eliminate the case $|\zeta G| = 2$.)