Theory of Groups (MATH 6270)

HOMEWORK ASSIGNMENT 5
(February 13, 2008)

Read Sections 2.2 (subsections 'Finitely Presented Groups' and 'The Word Problem') and 2.3.

Problems:
Section 2.2: Exercises <u>6</u>, 7,
Section 2.3: Exercises 4, 5, 6, 7, 8, <u>9</u>, 10
Additional Problems <u>1</u>, 2, 3 (see below)

Turn in underlined problems
Due Date: February 20, 2008

Additional Problems:

1. Let G be a fixed group. For each normal subgroup $N \triangleleft G$ let Σ_N denote the set of all equations $xg \approx x$ ($g \in N$) in the language of G-sets, and let $\mathcal{V}_N$ denote the variety of G-sets defined by Σ_N.
(1) For each $N \triangleleft G$ find a G-set that is free in $\mathcal{V}_N$ with 1 free generator.
(2) Show that $\mathcal{V}_M \neq \mathcal{V}_N$ if M, N are distinct normal subgroups of G.
(3) Prove that every variety of G-sets that contains a G-set with two or more elements is equal to $\mathcal{V}_N$ for some $N \triangleleft G$.
(4) Given a G-set $(X; G)$, what is the smallest variety of G-sets that contains $(X; G)$?

2. Show that the following conditions on an abelian group G are equivalent:
(a) G is subdirectly irreducible;
(b) for some prime p, G is either a cyclic p-group or is isomorphic to the infinite group $P = \{\varepsilon \in \mathbb{C} : \varepsilon^{p^k} = 1 \text{ for some } k \geq 1\}$ (i.e., P is the multiplicative group of all p^k-th roots of unity, $k = 1, 2, \ldots$).

3. Let G be a finitely presented group, and let $\langle X \mid R\rangle$ and $\langle Y \mid S\rangle$ be two finite presentations of G. Prove that the word problem for $\langle X \mid R\rangle$ is solvable if and only if the word problem for $\langle Y \mid S\rangle$ is solvable.