Theory of Groups (MATH 6270)

HOMEWORK ASSIGNMENT 7
(March 5, 2008)

Read Sections 4.2, 3.1, and 3.3 (from subsection 'Direct Products of Simple groups' through 'Structure of the Automorphism Group of a Centerless CR-group')

Problems:
Section 4.2: Exercises 9, 10, 12, 13, 14
Section 3.1: Exercises 1, 2, 3, 7
Section 3.3: Exercises 10, <u>11</u>, 12, 13
Additional Problems <u>1</u>, <u>2</u>, 3 (see below)

Turn in underlined problems
Due Date: March 12, 2008

Additional Problems:

1. Let H and K be subnormal subgroups of G.
(1) Use Zassenhaus's Lemma to show that $H \cap K$ is a subnormal subgroup of G.
(2) Prove that if G is finite and H is a Sylow subgroup of G, then $H \triangleleft G$.

3.3.17 in Robinson's text states that every group G has a largest normal subgroup T that is a direct product of nonabelian simple groups. Part (1) of the problem below asks you prove a variant of this statement which also shows how to locate T in G. Do not use 3.3.17 in your proof.

2. Let G be an arbitrary (finite or infinite) group.
(1) Prove that a normal subgroup N of G is a direct product of nonabelian simple groups if and only if $N \leq T$ where T is the subgroup of G generated by all nonabelian minimal normal subgroups of G that are direct products of simple groups. (T is the trivial subgroup if G has no such nonabelian minimal normal subgroups.)
(2) Use the example $G = D_8$ to show that the statement in (1) fails if the word "nonabelian" is omitted.

3. Prove that each factor of a principal series of a finite group is a direct product of isomorphic simple groups.