Online Appendices to

"Multivariate Choices and Identification of Social Interactions"

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A Model of Social Conformity

The usefulness of the proposed econometric model is not limited to the specific structural model considered in the main text. Here, we present another theoretical model to motivate the econometric model.

Patacchini and Zenou (2012) consider a social conformity model where the social norm is given by the average behavior of peers in a certain activity. We generalize their model by defining the social norm based on the weighted average behavior of two activities. Suppose a set of n individuals interact in a social network. Given the adjacency matrix $\mathbf{G} = [g_{ij}]$, individual i chooses effort levels y_{i1}, y_{i2} simultaneously to maximize her utility function

$$\begin{aligned} U_i(\mathbf{y}_1, \mathbf{y}_2) &= \textstyle\sum_{k=1}^{2} \varpi_{ik} y_{ik} - \frac{1}{2}(\varphi_{11} y_{i1}^2 + 2\varphi_{12} y_{i1} y_{i2} + \varphi_{22} y_{i2}^2) \\ &\quad - \frac{1}{2} \textstyle\sum_{k=1}^{2} \rho_k (y_{ik} - \sum_{l=1}^{2} \varrho_{lk} \sum_{j=1}^{n} g_{ij} y_{jl})^2. \end{aligned}$$

The first term of the utility captures the payoff from the efforts with the productivity of individual i in activity k given by ϖ_{ik}. The second term is the cost from the efforts with the substitution effect between efforts in different activities captured by φ_{12}. The last term reflects the influence of an individual's friends on her own behavior. It is such that each individual wants to minimize the social distance between her own behavior y_{ik} to the social norm of that activity. The social norm for activity k is given by the weighted average behavior of her friends in the two activities $\sum_{l=1}^{2} \varrho_{lk} \sum_{j=1}^{n} g_{ij} y_{jl}$ with the weights ϱ_{lk} such that $\varrho_{1k} + \varrho_{2k} = 1$. The coefficient ρ_k captures the taste for conformity.

Maximizing the utility function yields the best response function

$$y_{ik} = \phi_{lk} y_{ik} + \lambda_{kk} \sum_{j=1}^{n} g_{ij} y_{jk} + \lambda_{lk} \sum_{j=1}^{n} g_{ij} y_{jl} + \pi_{ik}, \qquad \text{for } k = 1, 2 \text{ and } l = 3 - k,$$

where $\phi_{lk} = -\varphi_{12}/(\varphi_{kk} + \rho_k)$, $\lambda_{kk} = \rho_k \varrho_{kk}/(\varphi_{kk} + \rho_k)$, $\lambda_{lk} = \rho_k \varrho_{lk}/(\varphi_{kk} + \rho_k)$, and $\pi_{ik} = \varpi_{ik}/(\varphi_{kk} + \rho_k)$. Let $\pi_{ik} = \mathbf{x}_i' \boldsymbol{\beta}_k + \sum_{j=1}^{n} g_{ij} \mathbf{x}_j' \boldsymbol{\gamma}_k + \alpha_k + \epsilon_{ik}$ (the network subscript r is suppressed for simplicity). Then, the best response function implies the econometric model considered in this paper.

B Proofs

Proof of Proposition 1. The reduced form of the model is

$$\begin{aligned}
\mathbf{y}_1 &= \mathbf{S}^{-1}[\mathbf{X}(\phi_{21}\boldsymbol{\beta}_2 + \boldsymbol{\beta}_1) + \mathbf{GX}(\lambda_{21}\boldsymbol{\beta}_2 - \lambda_{22}\boldsymbol{\beta}_1 + \phi_{21}\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_1) + \mathbf{G}^2\mathbf{X}(\lambda_{21}\boldsymbol{\gamma}_2 - \lambda_{22}\boldsymbol{\gamma}_1) \\
&\quad + (\phi_{21} + \lambda_{21})\mathbf{L}\boldsymbol{\alpha}_2 + (1 - \lambda_{22})\mathbf{L}\boldsymbol{\alpha}_1 + (\mathbf{I}_n - \lambda_{22}\mathbf{G})\boldsymbol{\epsilon}_1 + (\phi_{21}\mathbf{I}_n + \lambda_{21}\mathbf{G})\boldsymbol{\epsilon}_2] \\
\mathbf{y}_2 &= \mathbf{S}^{-1}[\mathbf{X}(\phi_{12}\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2) + \mathbf{GX}(\lambda_{12}\boldsymbol{\beta}_1 - \lambda_{11}\boldsymbol{\beta}_2 + \phi_{12}\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2) + \mathbf{G}^2\mathbf{X}(\lambda_{12}\boldsymbol{\gamma}_1 - \lambda_{11}\boldsymbol{\gamma}_2) \\
&\quad + (\phi_{12} + \lambda_{12})\mathbf{L}\boldsymbol{\alpha}_1 + (1 - \lambda_{11})\mathbf{L}\boldsymbol{\alpha}_2 + (\mathbf{I}_n - \lambda_{11}\mathbf{G})\boldsymbol{\epsilon}_2 + (\phi_{12}\mathbf{I}_n + \lambda_{12}\mathbf{G})\boldsymbol{\epsilon}_1], \qquad \text{(B.1)}
\end{aligned}$$

where

$$\mathbf{S} = (1 - \phi_{12}\phi_{21})\mathbf{I}_n - (\lambda_{11} + \lambda_{22} + \phi_{21}\lambda_{12} + \phi_{12}\lambda_{21})\mathbf{G} + (\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21})\mathbf{G}^2. \qquad \text{(B.2)}$$

$\mathrm{E}(\mathbf{JZ}_1|\mathbf{X}) = [\mathrm{E}(\mathbf{J}\mathbf{y}_2|\mathbf{X}), \mathrm{E}(\mathbf{JG}\mathbf{y}_1|\mathbf{X}), \mathrm{E}(\mathbf{JG}\mathbf{y}_2|\mathbf{X}), \mathbf{JX}, \mathbf{JGX}]$ has full column rank if and only if

$$\mathrm{E}(\mathbf{J}\mathbf{y}_2|\mathbf{X})d_1 + \mathrm{E}(\mathbf{JG}\mathbf{y}_1|\mathbf{X})d_2 + \mathrm{E}(\mathbf{JG}\mathbf{y}_2|\mathbf{X})d_3 + \mathbf{JX}\mathbf{d}_4 + \mathbf{JGX}\mathbf{d}_5 = \mathbf{0} \qquad \text{(B.3)}$$

implies that $d_1 = d_2 = d_3 = 0$ and $\mathbf{d}_4 = \mathbf{d}_5 = \mathbf{0}$. As $\mathbf{JGJ} = \mathbf{JG}$, $\mathbf{JSJ} = \mathbf{JS}$ and $\mathbf{SG} = \mathbf{GS}$, if we premultiply (B.3) by $\mathbf{JS}$, then it follows from the reduced form equations (B.1) that

$$\mathbf{JX}\boldsymbol{\eta}_1 + \mathbf{JGX}\boldsymbol{\eta}_2 + \mathbf{JG}^2\mathbf{X}\boldsymbol{\eta}_3 + \mathbf{JG}^3\mathbf{X}\boldsymbol{\eta}_4 = \mathbf{0}$$

where

$$\begin{aligned}
\boldsymbol{\eta}_1 &= (\phi_{12}\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2)d_1 + (1 - \phi_{12}\phi_{21})\mathbf{d}_4 \\
\boldsymbol{\eta}_2 &= (\lambda_{12}\boldsymbol{\beta}_1 - \lambda_{11}\boldsymbol{\beta}_2 + \phi_{12}\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2)d_1 + (\boldsymbol{\beta}_1 + \phi_{21}\boldsymbol{\beta}_2)d_2 + (\phi_{12}\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2)d_3 \\
&\quad -(\lambda_{11} + \lambda_{22} + \lambda_{12}\phi_{21} + \lambda_{21}\phi_{12})\mathbf{d}_4 + (1 - \phi_{12}\phi_{21})\mathbf{d}_5 \\
\boldsymbol{\eta}_3 &= (\lambda_{12}\boldsymbol{\gamma}_1 - \lambda_{11}\boldsymbol{\gamma}_2)d_1 + (\lambda_{21}\boldsymbol{\beta}_2 - \lambda_{22}\boldsymbol{\beta}_1 + \phi_{21}\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_1)d_2 + (\lambda_{12}\boldsymbol{\beta}_1 - \lambda_{11}\boldsymbol{\beta}_2 + \phi_{12}\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2)d_3 \\
&\quad +(\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21})\mathbf{d}_4 - (\lambda_{11} + \lambda_{22} + \lambda_{12}\phi_{21} + \lambda_{21}\phi_{12})\mathbf{d}_5 \\
\boldsymbol{\eta}_4 &= (\lambda_{21}\boldsymbol{\gamma}_2 - \lambda_{22}\boldsymbol{\gamma}_1)d_2 + (\lambda_{12}\boldsymbol{\gamma}_1 - \lambda_{11}\boldsymbol{\gamma}_2)d_3 + (\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21})\mathbf{d}_5.
\end{aligned}$$

If $d_2 = (\lambda_{12} + \lambda_{11}\phi_{12})d_1/(\phi_{12}\phi_{21} - 1)$, $d_3 = (\lambda_{22} + \lambda_{21}\phi_{12})d_1/(\phi_{12}\phi_{21} - 1)$, $\mathbf{d}_4 = (\phi_{12}\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2)d_1/(\phi_{12}\phi_{21} - 1)$ and $\mathbf{d}_5 = (\phi_{12}\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2)d_1/(\phi_{12}\phi_{21} - 1)$, then (B.3) holds. Therefore, $\mathrm{E}(\mathbf{JZ}_1|\mathbf{X})$ does not have full column rank. Similarly, $\mathrm{E}(\mathbf{JZ}_2|\mathbf{X})$ does not have full column rank. $\blacksquare$

Proof of Proposition 2. The identification of the structural parameters takes two steps. In the first step, we show that the pseudo reduced form parameters can be identified under Assumption 1. In the second step, we show that the structural parameters can be identified from the pseudo reduced form parameters under Assumption 2.

Step 1. The proof follows a similar argument as in Bramoullé, Djebbari and Fortin (2009). We first show that, under Assumption 1 (ii), $\mu_0\mathbf{I}_n + \mu_1\mathbf{G} + \mu_2\mathbf{G}^2 + \mu_3\mathbf{G}^3$ has identical rows implies $\mu_0 = \mu_1 = \mu_2 = \mu_3 = 0$. If $\mu_0\mathbf{I}_n + \mu_1\mathbf{G} + \mu_2\mathbf{G}^2 + \mu_3\mathbf{G}^3$ has identical rows, then

$$\mu_0\boldsymbol{\iota}_n + \mu_1\mathbf{G}\boldsymbol{\iota}_n + \mu_2\mathbf{G}^2\boldsymbol{\iota}_n + \mu_3\mathbf{G}^3\boldsymbol{\iota}_n = c_0\boldsymbol{\iota}_n, \tag{B.4}$$

for some constant c_0. As $\mathbf{G}\boldsymbol{\iota}_n = \boldsymbol{\iota}_n$, multiplying both sides of (B.4) by $\mathbf{G}$ gives

$$\mu_0\mathbf{G}\boldsymbol{\iota}_n + \mu_1\mathbf{G}^2\boldsymbol{\iota}_n + \mu_2\mathbf{G}^3\boldsymbol{\iota}_n + \mu_3\mathbf{G}^4\boldsymbol{\iota}_n = c_0\boldsymbol{\iota}_n. \tag{B.5}$$

Subtracting (B.4) from (B.5) gives $\mu_0\boldsymbol{\iota}_n + (\mu_1 - \mu_0)\mathbf{G}\boldsymbol{\iota}_n + (\mu_2 - \mu_1)\mathbf{G}^2\boldsymbol{\iota}_n + (\mu_3 - \mu_2)\mathbf{G}^3\boldsymbol{\iota}_n - \mu_3\mathbf{G}^4\boldsymbol{\iota}_n = \mathbf{0}$, which implies $\mu_0 = \mu_1 = \mu_2 = \mu_3 = 0$ under Assumption 1 (ii).

The moment conditions $\mathrm{E}(\mathbf{J}\boldsymbol{\epsilon}_1|\mathbf{X}) = \mathrm{E}(\mathbf{J}\boldsymbol{\epsilon}_2|\mathbf{X}) = \mathbf{0}$ imply that

$$\begin{aligned}
\mathrm{E}(\mathbf{J}\mathbf{y}_1|\mathbf{X}) &= \lambda_{11}^*\mathrm{E}(\mathbf{J}\mathbf{G}\mathbf{y}_1|\mathbf{X}) + \lambda_{21}^*\mathrm{E}(\mathbf{J}\mathbf{G}\mathbf{y}_2|\mathbf{X}) + \mathbf{J}\mathbf{X}\boldsymbol{\beta}_1^* + \mathbf{J}\mathbf{G}\mathbf{X}\boldsymbol{\gamma}_1^* \\
\mathrm{E}(\mathbf{J}\mathbf{y}_2|\mathbf{X}) &= \lambda_{22}^*\mathrm{E}(\mathbf{J}\mathbf{G}\mathbf{y}_2|\mathbf{X}) + \lambda_{12}^*\mathrm{E}(\mathbf{J}\mathbf{G}\mathbf{y}_1|\mathbf{X}) + \mathbf{J}\mathbf{X}\boldsymbol{\beta}_2^* + \mathbf{J}\mathbf{G}\mathbf{X}\boldsymbol{\gamma}_2^*.
\end{aligned}$$

Let $\boldsymbol{\theta}^* = (\boldsymbol{\theta}_1^{*\prime}, \boldsymbol{\theta}_2^{*\prime})'$ with $\boldsymbol{\theta}_k^* = (\lambda_{kk}^*, \lambda_{lk}^*, \boldsymbol{\beta}_k^{*\prime}, \boldsymbol{\gamma}_k^{*\prime})'$, for $k = 1, 2$ and $l = 3 - k$. If $[\mathrm{E}(\mathbf{J}\mathbf{G}\mathbf{y}_1|\mathbf{X}), \mathrm{E}(\mathbf{J}\mathbf{G}\mathbf{y}_2|\mathbf{X}), \mathbf{J}\mathbf{X}, \mathbf{J}\mathbf{G}\mathbf{X}]$ has full column rank, then $\boldsymbol{\theta}^*$ and $\widetilde{\boldsymbol{\theta}}^*$ leading to the same $\mathrm{E}(\mathbf{J}\mathbf{y}_1|\mathbf{X})$ and $\mathrm{E}(\mathbf{J}\mathbf{y}_2|\mathbf{X})$ implies $\boldsymbol{\theta}^* = \widetilde{\boldsymbol{\theta}}^*$, i.e. $\boldsymbol{\theta}^*$ is identified. $[\mathrm{E}(\mathbf{J}\mathbf{G}\mathbf{y}_1|\mathbf{X}), \mathrm{E}(\mathbf{J}\mathbf{G}\mathbf{y}_2|\mathbf{X}), \mathbf{J}\mathbf{X}, \mathbf{J}\mathbf{G}\mathbf{X}]$ has full column rank if

$$\mathrm{E}(\mathbf{J}_r\mathbf{G}_r\mathbf{y}_{1,r}|\mathbf{X}_r)d_1 + \mathrm{E}(\mathbf{J}_r\mathbf{G}_r\mathbf{y}_{2,r}|\mathbf{X}_r)d_2 + \mathbf{J}_r\mathbf{X}_r\mathbf{d}_3 + \mathbf{J}_r\mathbf{G}_r\mathbf{X}_r\mathbf{d}_4 = \mathbf{0} \tag{B.6}$$

implies that $d_1 = d_2 = 0$ and $\mathbf{d}_3 = \mathbf{d}_4 = \mathbf{0}$, for some network r. The pseudo reduced form equations imply

$$\begin{aligned}
\mathrm{E}(\mathbf{J}_r\mathbf{y}_{1,r}|\mathbf{X}_r) &= \mathbf{J}_r\mathbf{S}_r^{*-1}[\mathbf{X}_r\boldsymbol{\beta}_1^* + \mathbf{G}_r\mathbf{X}_r(\lambda_{21}^*\boldsymbol{\beta}_2^* - \lambda_{22}^*\boldsymbol{\beta}_1^* + \boldsymbol{\gamma}_1^*) + \mathbf{G}_r^2\mathbf{X}_r(\lambda_{21}^*\boldsymbol{\gamma}_2^* - \lambda_{22}^*\boldsymbol{\gamma}_1^*)] \\
\mathrm{E}(\mathbf{J}_r\mathbf{y}_{2,r}|\mathbf{X}_r) &= \mathbf{J}_r\mathbf{S}_r^{*-1}[\mathbf{X}_r\boldsymbol{\beta}_2^* + \mathbf{G}_r\mathbf{X}_r(\lambda_{12}^*\boldsymbol{\beta}_1^* - \lambda_{11}^*\boldsymbol{\beta}_2^* + \boldsymbol{\gamma}_2^*) + \mathbf{G}_r^2\mathbf{X}_r(\lambda_{12}^*\boldsymbol{\gamma}_1^* - \lambda_{11}^*\boldsymbol{\gamma}_2^*)]
\end{aligned}$$

where $\mathbf{S}_r^* = \mathbf{I}_{n_r} - (\lambda_{11}^* + \lambda_{22}^*)\mathbf{G}_r + (\lambda_{11}^*\lambda_{22}^* - \lambda_{12}^*\lambda_{21}^*)\mathbf{G}_r^2$. As $\mathbf{J}_r\mathbf{G}_r\mathbf{J}_r = \mathbf{J}_r\mathbf{G}_r$, $\mathbf{J}_r\mathbf{S}_r^*\mathbf{J}_r = \mathbf{J}_r\mathbf{S}_r^*$ and $\mathbf{S}_r^*\mathbf{G}_r = \mathbf{G}_r\mathbf{S}_r^*$, premultiplying (B.6) by $\mathbf{J}_r\mathbf{S}_r^*$ gives

$$\sum_{h=1}^{p}(\eta_{0,h}\mathbf{I}_{n_r} + \eta_{1,h}\mathbf{G}_r + \eta_{2,h}\mathbf{G}_r^2 + \eta_{3,h}\mathbf{G}_r^3)\mathbf{x}_{r,h} = c_1\boldsymbol{\iota}_{n_r} \tag{B.7}$$

where $\mathbf{x}_{r,h}$ is the h-th column of $\mathbf{X}_r$,

$$\begin{aligned}
\boldsymbol{\eta}_0 &= (\eta_{0,1}, \cdots, \eta_{0,p})' = \mathbf{d}_3 \\
\boldsymbol{\eta}_1 &= (\eta_{1,1}, \cdots, \eta_{1,p})' = \boldsymbol{\beta}_1^* d_1 + \boldsymbol{\beta}_2^* d_2 - (\lambda_{11}^* + \lambda_{22}^*)\mathbf{d}_3 + \mathbf{d}_4 \\
\boldsymbol{\eta}_2 &= (\eta_{2,1}, \cdots, \eta_{2,p})' = (\lambda_{21}^*\boldsymbol{\beta}_2^* - \lambda_{22}^*\boldsymbol{\beta}_1^* + \boldsymbol{\gamma}_1^*)d_1 + (\lambda_{12}^*\boldsymbol{\beta}_1^* - \lambda_{11}^*\boldsymbol{\beta}_2^* + \boldsymbol{\gamma}_2^*)d_2 \\
&\quad +(\lambda_{11}^*\lambda_{22}^* - \lambda_{12}^*\lambda_{21}^*)\mathbf{d}_3 - (\lambda_{11}^* + \lambda_{22}^*)\mathbf{d}_4 \\
\boldsymbol{\eta}_3 &= (\eta_{3,1}, \cdots, \eta_{3,p})' = (\lambda_{21}^*\boldsymbol{\gamma}_2^* - \lambda_{22}^*\boldsymbol{\gamma}_1^*)d_1 + (\lambda_{12}^*\boldsymbol{\gamma}_1^* - \lambda_{11}^*\boldsymbol{\gamma}_2^*)d_2 + (\lambda_{11}^*\lambda_{22}^* - \lambda_{12}^*\lambda_{21}^*)\mathbf{d}_4
\end{aligned}$$

and $c_1 = n_r^{-1}\boldsymbol{\iota}_{n_r}'(\mathbf{X}_r\boldsymbol{\eta}_0 + \mathbf{G}_r\mathbf{X}_r\boldsymbol{\eta}_1 + \mathbf{G}_r^2\mathbf{X}_r\boldsymbol{\eta}_2 + \mathbf{G}_r^3\mathbf{X}_r\boldsymbol{\eta}_3)$. As (B.7) holds for all possible realizations of $\mathbf{x}_{r,h}$, $\eta_{0,h}\mathbf{I}_{n_r} + \eta_{1,h}\mathbf{G}_r + \eta_{2,h}\mathbf{G}_r^2 + \eta_{3,h}\mathbf{G}_r^3$ has identical rows. Therefore, $\boldsymbol{\eta}_0 = \boldsymbol{\eta}_1 = \boldsymbol{\eta}_2 = \boldsymbol{\eta}_3 = \mathbf{0}$, which implies that $d_1 = d_2 = 0$ and $\mathbf{d}_3 = \mathbf{d}_4 = \mathbf{0}$ under Assumption 1 (i). Hence, $[\mathrm{E}(\mathbf{JGy}_1|\mathbf{X}), \mathrm{E}(\mathbf{JGy}_2|\mathbf{X}), \mathbf{JX}, \mathbf{JGX}]$ has full column rank and thus $\boldsymbol{\theta}^*$ is identified.

Step 2. Under Assumption 2, the identification of the structural parameters from the pseudo reduced form parameters follows the same argument as in a classical simultaneous-equation model (see, e.g., Schmidt, 1976), and thus the proof is omitted here. ■

References

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