Appendices to "Learning from Peers in Signaling Game Experiments"
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A The Likelihood Function

This section derives the likelihood function for the generalized model (1). In addition to Y_{nt}, let $Y_{nt}^* = (y_{1t}^*, \cdots, y_{nt}^*)'$ be the n-dimensional vector of the latent dependent variables. Let X_t^∞, Y_t^∞ and $Y_t^{*\infty}$ denote the sequences $\{X_{n,t-s}\}_{s=0}^\infty, \{Y_{n,t-s}\}_{s=0}^\infty$ and $\{Y_{n,t-s}^*\}_{s=0}^\infty$ respectively. Conditional on initial conditions $(Y_0^{*\infty}, Y_0^\infty)$, exogenous variables X_T^∞ and individual components $\xi = (\xi_1, \cdots, \xi_n)'$, the joint density function of $Y_T = (Y_{n1}', \cdots, Y_{nT}')'$ and $Y_T^* = (Y_{n1}^{*\prime}, \cdots, Y_{nT}^{*\prime})'$ is given by

$$f(Y_T^*, Y_T | Y_0^{*\infty}, Y_0^\infty, X_T^\infty, \xi) = \prod_{t=1}^T f(Y_{nt}^*, Y_{nt} | Y_{t-1}^{*\infty}, Y_{t-1}^\infty, X_t^\infty, \xi).$$

Let $I_{y_{it}}(y_{it}^*)$ be the dichotomous indicator with $I_{y_{it}}(y_{it}^*) = 1$ if y_{it}^* determines the observed value of y_{it} and $I_{y_{it}}(y_{it}^*) = 0$ otherwise. Because v_{it} are mutually independent for $i = 1, \cdots, n$, the conditional densities of (Y_{nt}^*, Y_{nt}) can be further decomposed as $f(Y_{nt}^*, Y_{nt} | Y_{t-1}^{*\infty}, Y_{t-1}^\infty, X_t^\infty, \xi) = \prod_{i=1}^n f(y_{it}^*, y_{it} | Y_{t-1}^{*\infty}, Y_{t-1}^\infty, X_t^\infty, \xi_i) = \prod_{i=1}^n I_{y_{it}}(y_{it}^*) g(y_{it}^* | Y_{t-1}^{*\infty}, Y_{t-1}^\infty, X_t^\infty, \xi_i)$ for $t = 1, \cdots T$, where $g(\cdot|\cdot)$ is the conditional density of y_{it}^*. Therefore, the conditional probability of Y_T is

$$\begin{aligned} & P(Y_T | Y_0^{*\infty}, Y_0^\infty, X_T^\infty, \xi) = \int \cdots \int f(Y_T^*, Y_T | Y_0^{*\infty}, Y_0^\infty, X_T^\infty, \xi) dY_T^* \\ = \quad & \int \cdots \int \left[\prod_{t=1}^T \prod_{i=1}^n I_{y_{it}}(y_{it}^*) g(y_{it}^* | Y_{t-1}^{*\infty}, Y_{t-1}^\infty, X_t^\infty, \xi_i) dy_{it}^* \right]. \end{aligned} \quad (1)$$

For (1), $g(y_{it}^* | Y_{t-1}^{*\infty}, Y_{t-1}^\infty, X_t^\infty, \xi_i) = g(y_{it}^* | (\{y_{i,t-s}^*\}_{s=1}^\infty, Y_{t-1}^\infty, X_t^\infty, \xi_i)$, as interactions among cross-sectional units are going through the observed $Y_{n,s-1}$ and X_{ns} but not $Y_{n,s-1}^*$ for $s \leq t$. Under the normality assumption of v_{it}, it follows that $g(y_{it}^* | (\{y_{i,t-s}^*\}_{s=1}^\infty, Y_{t-1}^\infty, X_t^\infty, \xi_i) = \phi(y_{it}^* - h_{it})$, where $h_{it} = h_{it}(\{y_{i,t-s}^*\}_{s=1}^\infty, Y_{t-1}^\infty, X_t^\infty, \xi_i)$. Define the integral limits L_{it} and U_{it}, where $L_{it} = -h_{it}$ and $U_{it} = \infty$ if $y_{it} = 1$, and $L_{it} = -\infty$ and $U_{it} = -h_{it}$ otherwise. By transformations of variables, it follows that

$$
\begin{aligned}
& P(Y_T|Y_0^{*\infty}, Y_0^{\infty}, X_T^{\infty}, \xi) = \prod_{i=1}^{n} \int_{L_{i1}}^{U_{i1}} \cdots \int_{L_{i,T-1}}^{U_{i,T-1}} \int_{L_{iT}}^{U_{iT}} \phi(v_{iT}) dv_{iT} \phi(v_{i,T-1}) dv_{i,T-1} \cdots \phi(v_{i1}) dv_{i1} \\
= \quad & \prod_{i=1}^{n} \int \cdots \int \left(\Phi(U_{iT}) - \Phi(L_{iT})\right) \prod_{s=1}^{T-1} \left(\Phi(U_{i,T-s}) - \Phi(L_{i,T-s})\right) \phi_{[L_{i,T-s}, U_{i,T-s}]}(v_{i,T-s}) dv_{i,T-s} \\
= \quad & \prod_{i=1}^{n} \int \cdots \int \Phi(d_{iT} h_{iT}) \prod_{s=1}^{T-1} \Phi(d_{i,T-s} h_{i,T-s}) \phi_{[L_{i,T-s}, U_{i,T-s}]}(v_{i,T-s}) dv_{i,T-s}, \qquad (2)
\end{aligned}
$$

where $d_{it} = 2y_{it} - 1$ and $\phi_{[L_t, U_t]}$ is a truncated standard normal density function on support $[L_t, U_t]$. Thus, the probability of Y_T conditional on $(Y_0^{*\infty}, Y_0^{\infty}, X_T^{\infty})$ can be obtained by integrating out the individual random component from (2).

B Some Monte Carlo Results on SMLEs

The finite sample performance of the simulated estimator is investigated by Monte Carlo experiments. We focus on the Markov and Polya models, which are used in the empirical studies. Special attention has been given to the finite sample bias due to simulation.

B.1 A Markov Model with Lagged Social Interactions

Suppose we have observations of G independent groups, with n subjects in each group. The Markov dynamic choice model for the Monte Carlo study in this section is

$$
y_{it}^* = \beta x_{i,t-1} + \lambda_1 y_{i,t-1} + \lambda_2 z_{i,t-1} + \sigma \xi_i + \varepsilon_{it}, \qquad (3)
$$

where $z_{i,t-1} = \sum_{j=1, j \neq i}^{n} y_{j,t-1} / (n-1)$, $\varepsilon_{it} = \rho \varepsilon_{i,t-1} + v_{it}$, and ξ_i and v_{it} are i.i.d. $N(0,1)$. The group subscript g is suppressed for simplicity. By replacing ε_{it} with $\rho(y_{i,t-1}^* - (\beta x_{i,t-2} + \lambda_1 y_{i,t-2} + \lambda_2 z_{i,t-2} + \sigma \xi_i)) + v_{it}$, (3) conforms to the general model (1).

The x_{it} are generated as $x_{it} = (1/\sqrt{2}) r_{it} + \sqrt{6} s_i$ where r_{it} are independent truncated standard normal variables on $[-2, 2]$ and s_i is a uniform variable on $[-0.5, 0.5]$, so that the variance of x_{it} is about 1 and its correlation coefficient over time is about 0.5. This process of generating exogenous variables is to allow the exogenous variables to correlate over time. It is used for all the Monte Carlo experiments in this article. The initial values of all variables for $t \leq 0$ are given as 0. Sample data are generated with $\beta = 1$, $\lambda_1 = 0.2$, $\lambda_2 = 0.4$, $\sigma^2 = 0.5$, and $\rho = 0.4$. The serial correlation of the total disturbance $\sigma\xi_i + \epsilon_{it}$ of two adjacent periods has a correlation coefficient about 0.6 and the fraction of variance due to the individual effect is about 0.3. The sample size is 200, with $G = 50$ and $n = 4$.[1] We have experimented with small, moderate and large numbers of random draws, namely $m = 15$, $m = 50$ and $m = 100$, for the construction of the GHK simulator. The number of periods for the panel data varies from 8 to 30. For each case, the number of replications is 200. For each replication, in addition to random disturbances in the model, the set of exogenous variables is also redrawn. The maximization algorithm used is the conjugate gradient method. For all cases and replications reported here, the algorithm converges without running into numerical problems. The initial estimate of σ is set to 1, and the initial estimates of the other parameters are set to 0. We have also tried some other starting values, with which the algorithm converges to similar solutions.

Table 5 reports the empirical means (Means), standard deviations (SDs) and root mean square errors (RMSEs) for both the bias-unadjusted SMLE and the bias-adjusted SMLE. For all panels with periods from 8 to 30, the bias-unadjusted SMLEs of β are biased downward. There are upward biases in the SMLEs of λ_1 and downward biases in those of λ_2, σ and ρ, so the true state dependence can be overstated but the peer group effect tends to be underestimated. The magnitude of bias increases with panel length, as the dimension of integration and the total number of choice alternatives are proportional to the number of periods. On the other hand, SDs of all the SMLEs decrease as panels become longer, since longer panel data provide more sample information about the stochastic process. If periods are not too long, RMSEs decrease. Biases of estimates are all substantially reduced when the number of simulated random variables m increases from 15 to 50. By increasing m to 100, biases become rather small and RMSEs can further be reduced, but the time cost is double. The issue of selecting m in practice has been addressed by Lee (1997). For small m, bias correction is valuable. Although SDs of bias-adjusted estimates are slightly larger, RMSEs of bias-adjusted estimates are smaller in general. The additional CPU cost for bias correction is negligible. However, as biases of estimates, especially for longer panels, are relatively large to begin with in this model, larger m is desirable for better improvement.[2]

Table 6 reports Means, SDs and RMSEs for alternative group sizes. For a given number of subjects $G \times n = 200$, biases, SDs and RMSEs of all the SMLEs increase when the group size n

[1] We pick $n = 4$ because in the experimental data used for the empirical studies, a reference group in each round typically has 3 or 4 monopolists of the same type.

[2] Results for the bias-adjusted estimates are omitted in subsequent tables to save space. The bias correction procedure for all the models in this article reduces bias and RMSE. The improvement is comparable with that from the bias correction procedure reported in Table 5.

increases from 4 to 8 (by comparing results in Tables 5 and 6). As the group size becomes even larger, biases, SDs and RMSEs of the SMLEs of λ_1, λ_2 and ρ further increase, while the estimates of β and σ are not much affected. As such, other things equal, more sessions with fewer subjects in each session are preferred to fewer sessions with more subjects to estimate the model.

To illustrate effects of omitted lagged social interactions on other SMLEs, we estimate (3) under the restriction $\lambda_2 = 0$ and report the results in the top panel of Table 8. With positive peer group interactions omitted, the SMLEs of β, σ and ρ are biased downward, and those of λ_1 are biased upward. The estimates of λ_1 are more than double in magnitude and the estimated ρ are reduced almost by half. Hence, the true state dependence can be overstated.

Misspecified disturbances, in general, would cause estimates to be inconsistent. First, we estimate the random component model with $\rho = 0$ using the data generated by the model (3). For the random component model, the multivariate probability function only involves a single integral, which can be effectively implemented by the Gaussian Quadrature method as suggested by Butler and Moffitt (1982). However, for the sake of easy comparison, here we report the SMLEs of the random component model.[3] The simulated log likelihood function for the random component model is

$$\mathcal{L} = \sum_{i=1}^{G\times n} \ln \left\{ \frac{1}{m} \sum_{j=1}^{m} \prod_{t=1}^{T} \Phi \left[(2y_{it} - 1)(\beta x_{i,t-1} + \lambda_1 y_{i,t-1} + \lambda_2 z_{i,t-1} + \sigma \xi_i^{(j)}) \right] \right\}.$$

The results are reported in the second block of Table 8. There are substantial downward biases in the SMLEs of β and λ_2 and upward biases in those of λ_1. Biases are more severe for longer panels. Even with $m = 100$, the estimates of λ_1 are three times larger than the true value; and the estimates of λ_2 are reduced by 2/3. Hence, the true state dependence tends to be overestimated and the peer group effect tends to be underestimated when serial correlation in ϵ_{it} is ignored. Biases of the estimated σ are not uniform. The third block of Table 8 reports the restricted SMLEs under $\sigma = 0$, i.e., with random component ξ ignored. With this error specification, serially correlated disturbances $\varepsilon_{it} = \rho\varepsilon_{i,t-1} + v_{it}$ capture all the spurious state dependence. Ignoring random individual components biases the SMLEs of β and λ_1 downward, but λ_2 and ρ upward. Biases of estimated λ_1 and λ_2 are more severe for longer panels. The upward bias of λ_2 are not really large. The biases of ρ are upward by 50%. But the biases of λ_1 towards zero are much more severe.

[3] See Table 5 for a comparison of the performance between the numerical Gaussian quadrature procedure and the GHK simulator for the estimation of the random component model.

B.2 A Polya Model with Lagged Social Interactions

In the Polya model, the entire history of the dynamic process is relevant to current decision making. The Polya model with a depreciation factor δ is given as follows[4]:

$$y_{it}^{*} = \beta x_{i,t-1} + \lambda_1 \textstyle\sum_{s=1}^{t} \delta^{s-1} y_{i,t-s} + \lambda_2 \sum_{s=1}^{t} \delta^{s-1} z_{i,t-s} / (\sum_{s=1}^{t} \delta^{s-1}) + \sigma \xi_i + \varepsilon_{it}, \tag{4}$$

where $z_{i,t-s} = \sum_{j=1, j\neq i}^{n} y_{j,t-s} / (n-1)$ and $\varepsilon_{it} = \rho \varepsilon_{i,t-1} + v_{it}$ with ξ_i and v_{it} i.i.d. $N(0,1)$. The group subscript g has been suppressed for simplicity. The initial values of all variables for $t \leq 0$ are given as 0. Trivial transformation of (4) shows it conforms to the general model (1). For comparison purpose, the discount factor δ is assumed to be a known constant and is set at 0.7. Sample data are generated with $\beta = 1$, $\lambda_1 = 0.2$, $\lambda_2 = 0.4$, $\sigma^2 = 0.5$, and $\rho = 0.4$.

The SMLEs are reported in Table 7. There are some downward biases in the SMLEs of β, λ_2, σ and ρ and upward bias in λ_1. Compared to estimates of the Markov model in Table 5, λ_1 and ρ in the Polya model can be estimated more accurately. They not only have small biases but also have much smaller SDs, due to an apparently stronger state dependence property of the Polya model. On the other hand, since we specified lagged social interactions as a weighted average of the past history instead of a weighted sum, variation in this term is reduced. So with such specification, λ_2 in the Polya model is much more difficult to estimate than in the Markov model. For small m and long panels, biases in the SMLEs of λ_2 is quite severe. By increasing m, biases in the estimates of λ_2 can be substantially reduced. For $T = 8$ or 15, the biases reduce by 90% as m increases from 15 to 50. By comparison with the Markov model, SDs and RMSEs of the estimates of λ_2 here are much larger.

We also investigate effects of misspecification in dynamic structures on SMLEs. The bottom block of Table 8 reports the SMLEs of the Markov model (3) when the data generating process follows the Polya model (4). The SMLEs of λ_1 and σ are biased upward. And the SMLEs of λ_2 and ρ are biased downward. Hence, with this kind of misspecification in the dynamic structure, the true state dependence tends to be overestimated but the peer group effect tends to be underestimated. The SMLEs of β are not affected very much by this model misspecification and their biases are not large.

[4] Here we specify the lagged social interactions term as the (weighted) average of peers' lagged choices over the entire history, so that it is not affected by the number of total observations.

Table 1: The Markov model (sample size: G=50, n=4)

T	m	$\beta = 1$	$\lambda_1 = 0.2$	$\lambda_2 = 0.4$	$\sigma = \sqrt{0.5}$	$\rho = 0.4$
Bias unadjusted SMLE						
8	15	.969 (.067) [.073]	.246 (.132) [.139]	.349 (.141) [.150]	.628 (.136) [.157]	.379 (.123) [.125]
15	15	.965 (.052) [.063]	.296 (.105) [.142]	.318 (.103) [.132]	.672 (.077) [.084]	.323 (.079) [.111]
30	15	.957 (.035) [.056]	.345 (.071) [.161]	.292 (.068) [.127]	.686 (.058) [.061]	.286 (.045) [.123]
8	50	.991 (.066) [.067]	.207 (.124) [.123]	.387 (.137) [.137]	.675 (.111) [.115]	.398 (.108) [.108]
15	50	.988 (.052) [.053]	.236 (.091) [.097]	.371 (.102) [.106]	.692 (.071) [.073]	.369 (.061) [.069]
30	50	.982 (.034) [.039]	.264 (.068) [.093]	.354 (.066) [.080]	.698 (.053) [.053]	.349 (.042) [.066]
8	100	.996 (.065) [.065]	.203 (.124) [.124]	.393 (.135) [.135]	.684 (.110) [.112]	.397 (.104) [.104]
15	100	.994 (.052) [.052]	.219 (.088) [.090]	.384 (.098) [.099]	.697 (.070) [.070]	.382 (.059) [.061]
30	100	.990 (.035) [.037]	.235 (.065) [.074]	.370 (.065) [.072]	.702 (.052) [.052]	.372 (.040) [.049]
Bias adjusted SMLE						
8	15	.991 (.069) [.069]	.217 (.131) [.132]	.380 (.143) [.144]	.681 (.135) [.138]	.385 (.127) [.127]
15	15	.982 (.053) [.056]	.259 (.107) [.122]	.352 (.106) [.116]	.699 (.078) [.078]	.347 (.082) [.098]
30	15	.970 (.035) [.046]	.307 (.072) [.129]	.326 (.069) [.101]	.704 (.060) [.060]	.314 (.046) [.097]
8	50	1.000 (.067) [.067]	.194 (.123) [.123]	.400 (.138) [.137]	.692 (.107) [.108]	.403 (.109) [.109]
15	50	.996 (.052) [.052]	.214 (.092) [.092]	.390 (.103) [.104]	.701 (.072) [.072]	.386 (.062) [.063]
30	50	.991 (.035) [.036]	.232 (.068) [.075]	.377 (.067) [.071]	.708 (.054) [.054]	.374 (.043) [.050]
8	100	1.001 (.066) [.066]	.195 (.124) [.124]	.400 (.135) [.135]	.692 (.109) [.109]	.401 (.104) [.104]
15	100	.999 (.052) [.052]	.205 (.088) [.088]	.395 (.098) [.098]	.701 (.070) [.070]	.394 (.059) [.060]
30	100	.996 (.036) [.036]	.212 (.066) [.067]	.386 (.066) [.067]	.708 (.053) [.053]	.391 (.040) [.041]

The values are given as Means (SDs) [RMSEs]

Table 2: The Markov model with alternative group sizes

T	m	$\beta = 1$	$\lambda_1 = 0.2$	$\lambda_2 = 0.4$	$\sigma = \sqrt{0.5}$	$\rho = 0.4$
$G = 25, n = 8$						
8	15	.961 (.067) [.078]	.261 (.157) [.168]	.321 (.180) [.197]	.604 (.174) [.202]	.372 (.140) [.142]
8	50	.986 (.066) [.067]	.215 (.141) [.142]	.374 (.168) [.169]	.665 (.126) [.133]	.392 (.119) [.119]
8	100	.993 (.063) [.063]	.195 (.125) [.124]	.388 (.166) [.165]	.675 (.100) [.105]	.398 (.106) [.106]
$G = 10, n = 20$						
8	15	.958 (.069) [.080]	.285 (.182) [.200]	.288 (.218) [.245]	.605 (.155) [.185]	.356 (.154) [.160]
8	50	.983 (.068) [.070]	.227 (.160) [.162]	.357 (.203) [.207]	.663 (.119) [.126]	.384 (.131) [.131]
8	100	.989 (.067) [.068]	.218 (.160) [.161]	.370 (.199) [.200]	.676 (.107) [.111]	.386 (.126) [.127]
$G = 4, n = 50$						
8	15	.957 (.069) [.081]	.300 (.193) [.217]	.271 (.238) [.270]	.606 (.150) [.181]	.345 (.158) [.167]
8	50	.985 (.068) [.070]	.231 (.161) [.164]	.354 (.209) [.213]	.666 (.113) [.120]	.381 (.130) [.131]
8	100	.990 (.068) [.068]	.221 (.164) [.165]	.368 (.208) [.210]	.675 (.107) [.112]	.386 (.127) [.127]
$G = 1, n = 200$						
8	15	.958 (.071) [.082]	.302 (.200) [.224]	.268 (.252) [.284]	.606 (.152) [.182]	.345 (.163) [.172]
8	50	.985 (.070) [.071]	.234 (.171) [.174]	.352 (.221) [.226]	.665 (.116) [.123]	.380 (.136) [.137]
8	100	.991 (.069) [.070]	.220 (.170) [.171]	.370 (.217) [.219]	.674 (.110) [.114]	.388 (.130) [.131]

The values are given as Means (SDs) [RMSEs]

Table 3: The Polya model (sample size: G=50, n=4)

T	m	$\beta = 1$	$\lambda_1 = 0.2$	$\lambda_2 = 0.4$	$\sigma = \sqrt{0.5}$	$\rho = 0.4$
8	15	.981 (.071) [.073]	.218 (.085) [.087]	.337 (.249) [.256]	.647 (.177) [.187]	.399 (.102) [.101]
15	15	.977 (.056) [.060]	.228 (.050) [.057]	.301 (.179) [.204]	.679 (.088) [.092]	.366 (.057) [.066]
30	15	.964 (.036) [.051]	.255 (.038) [.067]	.201 (.134) [.240]	.676 (.067) [.074]	.339 (.033) [.069]
8	50	.995 (.070) [.070]	.200 (.083) [.082]	.394 (.245) [.245]	.690 (.134) [.135]	.399 (.083) [.083]
15	50	.995 (.055) [.055]	.204 (.051) [.051]	.389 (.181) [.181]	.697 (.079) [.080]	.390 (.054) [.055]
30	50	.988 (.037) [.038]	.219 (.040) [.044]	.329 (.132) [.149]	.699 (.062) [.062]	.381 (.037) [.041]
8	100	.997 (.068) [.068]	.198 (.082) [.082]	.400 (.246) [.245]	.691 (.118) [.119]	.402 (.083) [.083]
15	100	.997 (.056) [.056]	.201 (.052) [.052]	.398 (.185) [.185]	.696 (.081) [.081]	.393 (.053) [.054]
30	100	.994 (.037) [.037]	.211 (.042) [.043]	.358 (.133) [.139]	.703 (.064) [.064]	.391 (.036) [.037]

The values are given as Means (SDs) [RMSEs]

Table 4: Model misspecification (sample size: G=50, n=4)

	T	m	$\beta = 1$	$\lambda_1 = 0.2$	$\lambda_2 = 0.4$	$\sigma = \sqrt{0.5}$	$\rho = 0.4$
	True model: Markov; estimated model: misspecified Markov						
(a)	8	15	.939 (.065)	.478 (.116)	-	.617 (.118)	.236 (.121)
	15	15	.941 (.054)	.493 (.097)	-	.643 (.074)	.214 (.080)
	8	50	.961 (.066)	.469 (.111)	-	.662 (.090)	.228 (.109)
	15	50	.962 (.054)	.459 (.097)	-	.657 (.068)	.241 (.075)
	8	100	.966 (.064)	.466 (.114)	-	.665 (.096)	.229 (.110)
	15	100	.967 (.054)	.444 (.100)	-	.661 (.068)	.254 (.079)
(b)	8	15	.935 (.066)	.592 (.090)	.091 (.122)	.685 (.085)	-
	15	15	.919 (.051)	.639 (.058)	.075 (.090)	.652 (.069)	-
	8	50	.961 (.067)	.567 (.091)	.123 (.120)	.713 (.081)	-
	15	50	.939 (.051)	.626 (.057)	.101 (.084)	.655 (.063)	-
	8	100	.965 (.068)	.561 (.090)	.132 (.118)	.717 (.079)	-
	15	100	.943 (.052)	.623 (.057)	.105 (.085)	.654 (.058)	-
(c)	8	15	.932 (.061)	.103 (.103)	.434 (.124)	-	.624 (.055)
	15	15	.940 (.051)	.102 (.081)	.436 (.095)	-	.612 (.043)
	8	50	.945 (.060)	.072 (.100)	.462 (.122)	-	.649 (.050)
	15	50	.962 (.052)	.056 (.078)	.474 (.096)	-	.649 (.039)
	8	100	.948 (.061)	.065 (.100)	.468 (.122)	-	.655 (.049)
	15	100	.968 (.051)	.044 (.076)	.484 (.096)	-	.658 (.038)
	True model: Polya; estimated model: Markov						
(d)	8	15	.978 (.069)	.429 (.198)	.289 (.176)	.710 (.142)	.250 (.179)
	15	15	.968 (.057)	.515 (.149)	.304 (.128)	.733 (.080)	.196 (.116)
	8	50	1.004 (.069)	.408 (.188)	.321 (.170)	.767 (.099)	.242 (.161)
	15	50	.993 (.057)	.436 (.157)	.369 (.133)	.751 (.077)	.252 (.120)
	8	100	1.009 (.068)	.392 (.192)	.334 (.168)	.770 (.096)	.253 (.166)
	15	100	.998 (.058)	.416 (.169)	.382 (.137)	.754 (.079)	.267 (.131)

The values are given as Means (SDs)

Table 5: The random component model (sample size: G=50, n=4)

	m	$\beta = 1$	$\lambda_1 = 0.2$	$\lambda_2 = 0.4$	$\sigma = \sqrt{0.5}$
$T = 8$					
Gaussian Quadrature		0.998(0.068)[0.068]	0.201(0.091)[0.091]	0.398(0.103)[0.103]	0.699(0.071)[0.071]
GHK	15	0.968(0.065)[0.072]	0.241(0.089)[0.098]	0.356(0.104)[0.113]	0.669(0.074)[0.083]
	50	0.992(0.066)[0.066]	0.209(0.088)[0.088]	0.389(0.103)[0.103]	0.698(0.070)[0.070]
	100	0.995(0.066)[0.066]	0.203(0.088)[0.088]	0.396(0.101)[0.100]	0.701(0.067)[0.067]
$T = 15$					
Gaussian Quadrature		1.002(0.054)[0.054]	0.202(0.060)[0.060]	0.397(0.083)[0.083]	0.701(0.061)[0.062]
GHK	15	0.977(0.052)[0.057]	0.228(0.062)[0.067]	0.370(0.083)[0.088]	0.710(0.070)[0.070]
	50	0.997(0.053)[0.053]	0.207(0.060)[0.060]	0.390(0.081)[0.081]	0.708(0.061)[0.061]
	100	1.000(0.053)[0.053]	0.204(0.060)[0.060]	0.394(0.083)[0.083]	0.707(0.059)[0.059]

The values are given as Means (SDs) [RMSEs]