Chisholm’s Paradox Revisited

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Chisholm’s Paradox

The central principle:

(a) □(s made from p and q only slightly different from p

→ ◊(s made from q))

The form of (a): □(A → ◊B)
The fallacy?

\[ \begin{align*}
&w_1 & w_2 & w_3 \\
A & \leftrightarrow \bot & A & \leftrightarrow \top & A & \leftrightarrow \bot \\
B & \leftrightarrow \bot & B & \leftrightarrow \bot & B & \leftrightarrow \top \\
\end{align*} \]

\[ \Diamond A, \Box (A \rightarrow \Diamond B) \neq \Diamond B \]
The Sorites formulation

◇ (s is made from $p_1$)

◇ (s is made from $p_1$) $→$ ◇ (s is made from $p_2$)

◇ (s is made from $p_2$)

◇ (s is made from $p_2$) $→$ ◇ (s is made from $p_3$)

◇ (s is made from $p_3$)

and so on
A Sorites Progression
The *many conditionals* Sorites

1) \( g_1 \) is short (for an adult male giraffe)
2) \( g_1 \) is short \( \rightarrow \) \( g_2 \) is short
   \[ \downarrow \quad \downarrow \]
3) \( g_999 \) is short \( \rightarrow \) \( g_{1000} \) is short

\[ \therefore g_{1000} \text{ is short} \]
The *tolerance* Sorites

1) $g_1$ is short (for an adult male giraffe)

2) (Tolerance) Any giraffe that is not perceptibly taller than a short giraffe is itself short

∴ $g_{1000}$ is short
Constraints

(C1) Conditionals of the form $A \rightarrow A$ are as true as any statement can be.

(C2) Whatever status is ascribed to the premises of a Sorites argument should make the force of the argument intelligible. (This rules out Fine’s supervaluations, on which no conditional in the many-conditionals Sorites is true, and Edgington’s verity-semantics, on which the tolerance principle of the Tolerance Sorites is wholly false.)
Two translations of a Chisholm conditional

\[ \Diamond (s \text{ is made from } p_2) \rightarrow \Diamond (s \text{ is made from } p_3) \]

*Accessibility:*

\[(\exists u)(s \text{ is made from } p_2 \text{ at } u \& u \text{ is accessible from } @) \rightarrow (\exists v)(s \text{ is made from } p_3 \text{ at } v \& v \text{ is accessible from } @)\]

*Counterparts:*

\[(\exists u)(\exists x)(x \text{ is made from } p_2 \text{ at } u \& x \text{ is a counterpart of } s \text{ at } u) \rightarrow (\exists v)(\exists y)(y \text{ is made from } p_3 \text{ at } v \& y \text{ is a counterpart of } s \text{ at } v)\]
Fuzzy Logic

Both “is accessible from” and “is a counterpart of” may be taken to be *predicates of degree*. Let the real interval [0,1] be the set of degrees of truth, 0 = definite falsity, 1 = definite truth. Given an assignment of degrees of truth to atomic wffs, evaluate complex wffs by the following canonical clauses:

\[
\begin{align*}
(\sim): \ & \text{deg}(\sim A) = 1 - \text{deg}(A) \\
(&): \ & \text{deg}(A \& B) = \min\{\text{deg}(A), \text{deg}(B)\} \\
(\lor): \ & \text{deg}(A \lor B) = \max\{\text{deg}(A), \text{deg}(B)\} \\
(\rightarrow): \ & \text{deg}(A \rightarrow B) = 1 - (\text{deg}(A) - \text{deg}(B)) \text{ if } \text{deg}(A) > \text{deg}(B), =1 \text{ o.t.} \\
(\forall): \ & \text{deg}(\forall x A) = \min\{\text{deg}(A(x/t))\} \\
(\exists): \ & \text{deg}(\exists x A) = \max\{\text{deg}(A(x/t))\}
\end{align*}
\]
Objections to (&): 1

Edgington: if \( \text{deg}(x \text{ is red}) = 1 \) and \( \text{deg}(x \text{ is small}) = .5 \), then by (&), \( \text{deg}(x \text{ is small and red}) = .5 \); and if \( \text{deg}(y \text{ is red}) = \text{deg}(y \text{ is small}) = .5 \), then \( \text{deg}(y \text{ is small and red}) = .5 \) also. But \( x \) is a better case for “red and small” than \( y \), because \( x \) is redder than \( y \) and just as big as \( y \).

Reply: by the classical truth table for &, \( \text{deg}(x \text{ is red}) = 1 \) and \( \text{deg}(x \text{ is small}) = 0 \) implies \( \text{deg}(x \text{ is small and red}) = 0 \), and \( \text{deg}(y \text{ is red}) = \text{deg}(y \text{ is small}) = 0 \) implies \( \text{deg}(y \text{ is small and red}) = 0 \) also. But by reasoning of the objection, \( x \) is a better case for “red and small”, because \( x \) is redder than \( y \) and just as big as \( y \).

Hence if (&) gets conjunction wrong, so does the classical truth-table.
Objections to (&): 2

Williamson: if \( \text{deg}(A) = .5 \) then \( \text{deg}(A \& \sim A) = .5 \). But \( \text{deg}(A \& \sim A) \) should always be 0.

Reply: suppose \( x \) is a certain shade \( s \) of bluish green (equally, greenish blue). Because \( x \) is \( s \), both “\( x \) is green” and “\( x \) is blue” are somewhat true. But if \( x \) is blue then \( x \) is not green, so both “\( x \) is green” and “\( x \) is not green” are made somewhat true by \( x \)’s being \( s \). If a single state of affairs makes each of two propositions partially true, it is difficult to see how it could make their conjunction anything but partially true.

Contrast probability: it may be that the probability of \( E \) occurring is .5, and the probability of \( E \) not occurring is .5, but the probability of \( E \) occurring and not occurring is 0. This simply shows that we treat occurrence as all or nothing. Color is not all or nothing.
Objections to (&): 3

Williamson, Keefe & Smith: suppose $\deg(x \text{ is tall}) = .4$, and $\deg(y \text{ is tall}) = .5$ So $y$ is taller than $x$. By (&), $\deg(x \text{ is tall and } y \text{ is tall}) = .4$, but also, $\deg(x \text{ is tall and } y \text{ is not tall}) = .4$ However, $\deg(x \text{ is tall and } y \text{ is not tall})$ should be 0, since $y$ is taller than $x$.

*Reply:* the objection is based on the universal conditional *anyone taller than a tall person is tall*. This is wholly true, but that only means that $\deg(x \text{ is tall and } y \text{ is not tall})$ should be 0 if $\deg(x \text{ is tall}) = 1$. More generally, if $y$ is taller than $x$, $\deg(y \text{ is tall})$ cannot fall below $\deg(x \text{ is tall})$, but $\deg(y \text{ is not tall})$ may not fall below it either.
Objection to (→)

Edgington: suppose \( \text{deg}(x \text{ is tall}) = .4 \), and \( \text{deg}(y \text{ is tall}) = .5 \) So \( y \) is taller than \( x \). By (→), \( \text{deg}(x \text{ is tall} \rightarrow y \text{ is tall}) = 1 \), but also, \( \text{deg}(x \text{ is tall} \rightarrow y \text{ is not tall}) = 1 \). But how can it be wholly true that \( (x \text{ is tall} \rightarrow y \text{ is not tall}) \), if \( y \) is taller than \( x \)?

Reply: the problem is with the underlying material conditional. Suppose \( \text{deg}(x \text{ is tall}) = 0 \), and \( \text{deg}(y \text{ is tall}) = 1 \) (so \( y \) is a whole lot taller than \( x \)). \( \text{deg}(x \text{ is tall} \supset y \text{ is tall}) = 1 \), but also, \( \text{deg}(x \text{ is tall} \supset y \text{ is not tall}) = 1 \). How can this last conditional be wholly true if \( y \) is taller than \( x \)?
Two Solutions to Chisholm’s Paradox (I)

The accessibility solution:

Among the worlds where $s$ is made from $p_2$ let $w$ be one of the most accessible from $\Diamond$. Among the worlds where $s$ is made from $p_3$ let $w^*$ be one of the most accessible from $\Diamond$. Since $s$ in $w^*$ is less like it is in $\Diamond$ than it is in $w$, $\deg(w^* \text{ is accessible from } \Diamond)$ may be less than $\deg(w \text{ is accessible from } \Diamond)$. Then in Accessibility (slide 9) we can get a slightly higher degree of truth for the antecedent than we can for the consequent. So the conditional is slightly less than wholly true.
Two Solutions to Chisholm’s Paradox (II)

The counterpart solution:

Among the worlds where some $x$ is made of $p_2$ choose one where such an $x$ is a strongest counterpart of $s$. Among the worlds where some $y$ is made of $p_3$ choose one where such a $y$ is a strongest counterpart of $s$. We may assume the only intrinsic difference between $x$ and $y$ is composition. Then $\deg(x \text{ is a counterpart of } s)$ may be slightly higher than $\deg(y \text{ is a counterpart of } s)$. So Counterparts may be slightly less than wholly true.
Against the accessibility solution:

There are temporal versions of Chisholm’s Paradox, for example, the simple Ship of Theseus puzzle, to which the accessibility solution has no applicability, because there is no relevant sense in which one future time is less accessible to the present than another.

Against counterparts:

When applied to temporal puzzles, the counterpart-theoretic approach forces us to adopt a controversial ontology, on which ordinary persisting objects are viewed as sequences of instantaneous object-stages.

Against both:

Neither can adequately account for Salmon’s disassembly/reassembly paradox.
The Disassembly/Reassembly Puzzle

Let $a_0,...,a_n$ be $n + 1$ distinct objects, each with $n + 1$ parts, and each capable of being disassembled and reassembled. For each $a_i$, let $b_i$ be the object which results when $a_i$ is disassembled, then reassembled with replacement of $i$-many parts. We have

$$1) \ a_0 = b_0$$

since $b_0$ is just $a_0$ over again, 0 parts having been replaced.

We also have

$$2) \ a_n \neq b_n$$

since entirely different parts were used in the “reassembly” of $a_n$.

But we also have tolerance: if $x$ can be reassembled with $m$ new parts, $y$ can be reassembled with $m + 1$ new parts.
Indeterminate Identity?

The disassembly/reassembly puzzle cannot be solved by any analogue of counterpart theory or accessibility relations, because ‘$a_i = b_i$’ contains no operators that invoke such apparatus.

At face value, if ‘$a_i = b_i$’ has some status that is neither wholly (determinately) true nor wholly (determinately) false, that must be because indeterminateness is located in

(a) one or other singular term, or
(b) the identity symbol.

(a) can be ruled out, so we are left with (b), indeterminate identity.
Application to CP

◇ (s is made from \( p_2 \)) \rightarrow \ ◇ (s is made from \( p_3 \))

We may have a true antecedent and a truth-valueless consequent.

(∃v)(s is made from \( p_3 \) at v)

is truth-valueless because

(∃v)(∃x)(x is made from \( p_3 \) at v and s = x)

is truth-valueless, because there is no fact of the matter about whether or not the x made of \( p_3 \) at v is identical to s.
Conditionals

Are Chisholm conditionals true or truth-valueless?

_The orthodox view:_ some are truth-valueless, but, because of higher-order vagueness, there is no fact of the matter about which is the “first” truth-valueless one.

_‘if-true’ evaluation:_ perhaps $p \rightarrow q$ is evaluated with respect to a shift or recalibration, on which the antecedent $p$ is securely true. Arguably, then, all Sorites conditionals are true. The fallacy is in repeated applications of _modus ponens_ over conditionals which are evaluated with respect to _different_ shifts.
Semantics for identity (I)

In the semantics of pure identity, we treat ‘=’ as a non-logical binary relation subject to certain constraints.

Given a domain $D$ of objects, ‘=’ is assigned an extension $\text{Ext}(=) \subseteq D \times D$, and a disjoint counter-extension $\text{Cxt}(=) \subseteq D \times D$. $\text{Ext}(=)$ is closed under reflexivity, symmetry and transitivity. $\text{Cxt}(=)$ is closed under symmetry. If $\langle x, y \rangle$ is in neither $\text{Ext}(=)$ nor $\text{Cxt}(=)$, there is no fact of the matter about whether or not $x = y$.

Thus the “real” identity relation on $D$ is ignored.
Semantics for identity (II)

In the modal pure semantics of identity, the interpretation of ‘=’ is given for the domain D of possible objects (it is not relative a world). Hence we have

\[ a = b \rightarrow \Box(a = b) \]

assuming the indeterminacy of ‘a = b’ at every world implies the indeterminacy of ‘\( \Box(a = b) \)’ at every world.

In interpreting ‘=’ in a language with other non-logical predicates, we generalize Parsons’ Principle:
\[ \langle v(a), v(b) \rangle \] is in neither Ext(=) nor Cxt(=) iff (i) there is no world where some other non-logical predicate has one of \( \{v(a), v(b)\} \) in its extension and the other in its counterextension; and (ii) there is some world where exactly one of them is in the extension of an n-lp.
The new solution to CP

We assume $T \rightarrow I$ and $I \rightarrow F$ are both I.

Hence in a many-conditionals Sorites, we will have one $T \rightarrow I$ conditional and one $I \rightarrow F$ one.

In the Chisholm’s Paradox case, we will have a world $v$ where the ‘made of’ relation has, say, $\langle x, p_3 \rangle$ in its extension and $\langle s, p_3 \rangle$ neither in its extension nor its counterextension.