A many particle generalization of the Landau-Zener problem

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- system
- rate equations/semiclassical analysis
- adiabacity

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Landau-Zener (LZ) problem

- two coupled levels subject to weakly time-dependent driving

Formalize by

\[ \hat{H} = \begin{pmatrix} \lambda t & g \\ g & -\lambda t \end{pmatrix} \]

Instantaneous levels

\[ \epsilon_{\pm} = \pm \sqrt{(\lambda t)^2 + g^2} \]

Excitation probability

\[ P_{\text{excite}} = e^{-\pi g^2 / \lambda} \]

\[ P = 1 - e^{-\pi g^2 / \lambda} \]

Landau/Zener 1932
The model: 1. Fermi-Bose
(Time-dependent Dicke model)

\[ \hat{H} = -\gamma \hat{b}^\dagger \hat{b} + \frac{\gamma}{2} \sum_{i=1}^{N} \left( \hat{a}_{i\uparrow}^\dagger \hat{a}_{i\uparrow} + \hat{a}_{i\downarrow}^\dagger \hat{a}_{i\downarrow} \right) + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left( \hat{b}^\dagger \hat{a}_{i\downarrow} \hat{a}_{i\uparrow} + \hat{b} \hat{a}_{i\uparrow}^\dagger \hat{a}_{i\downarrow}^\dagger \right) \]

\[ \gamma = \lambda t \]

\{ N \text{ levels} \}

Particle conservation:

\[ \left\langle \hat{b}^\dagger \hat{b} \right\rangle + \frac{1}{2} \sum_{i=1}^{N} \left\langle \hat{a}_{i\uparrow}^\dagger \hat{a}_{i\uparrow} + \hat{a}_{i\downarrow}^\dagger \hat{a}_{i\downarrow} \right\rangle = N \]

Initially, at \( t \rightarrow -\infty \)

\[ \frac{1}{2} \sum_{i=1}^{N} \left\langle \hat{a}_{i\uparrow}^\dagger \hat{a}_{i\uparrow} + \hat{a}_{i\downarrow}^\dagger \hat{a}_{i\downarrow} \right\rangle = N \]

\[ \left\langle \hat{b}^\dagger \hat{b} \right\rangle = 0 \]

Would like to find, at \( t \rightarrow +\infty \)

\[ n_{b} = \left\langle \hat{b}^\dagger \hat{b} \right\rangle = ? \]

How many bosons did we create?
2. Cavity QED representation

- fermionic level $i$ is represented by two spin configurations
  (pseudo-) spin up
  (pseudo-) spin down

\[
\hat{H} = -\gamma \hat{b}^{\dagger} \hat{b} + \frac{\gamma}{2} \sum_{i=1}^{N} \sigma_i^z + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left( \hat{b}^{\dagger} \sigma_i^- + \hat{b} \sigma_i^+ \right) \]

- introduce spin operators as:
  \[
  \hat{S}^a = \frac{1}{2} \sum_{i} \sigma_i^a, \quad (a = 1, 2, 3)
  \]

\[
\hat{H} = -\gamma \hat{b}^{\dagger} \hat{b} + \gamma \hat{S}^z + \frac{g}{\sqrt{N}} \left( \hat{b}^{\dagger} \hat{S}^- + \hat{b} \hat{S}^+ \right)
\]

Initially, at $t \to -\infty$
\[
\langle \hat{S}_z \rangle = \frac{N}{2}
\]
\[
\langle \hat{b}^{\dagger} \hat{b} \rangle = 0
\]

Would like to find, at $t \to +\infty$
\[
n_b = \langle \hat{b}^{\dagger} \hat{b} \rangle = ?
\]

How many bosons did we create?
3. Atomic/molecular Bose condensates

\[ H = -\gamma \hat{b}_a^\dagger \hat{b}_a + \gamma \hat{b}_m^\dagger \hat{b}_m + \frac{g}{\sqrt{N}} \left( \hat{b}_a^\dagger \hat{b}_a \hat{b}_m + \hat{b}_a \hat{b}_a \hat{b}_m^\dagger \right) \]

\[ \gamma = \lambda t \]

Atoms \quad Molecules

Actually realized in Carl Wieman’s group, 2005.

Particle conservation:
\[ \langle \hat{b}_m^\dagger \hat{b}_m \rangle + \frac{1}{2} \langle \hat{b}_a^\dagger \hat{b}_a \rangle = N \]

Initially, at \( t \to -\infty \)
\[ \langle \hat{b}_m^\dagger \hat{b}_m \rangle = N \]
\[ \langle \hat{b}_a^\dagger \hat{b}_a \rangle = 0 \]

Would like to find, at \( t \to +\infty \)
\[ n_b = \langle \hat{b}_a^\dagger \hat{b}_a \rangle = ? \]

How many atoms did we create?
Simplest case: $N=1$

$$\hat{H} = -\gamma \hat{b}^\dagger \hat{b} + \frac{\gamma}{2} \sigma^z + g \left( \hat{b}^\dagger \sigma^- + \hat{b} \sigma^+ \right)$$

$\gamma = \lambda t$  \hspace{1cm} Time-dependent standard cavity QED

Simple Hilbert space

$|\uparrow, 0\rangle$ Spin up, no bosons

$|\downarrow, 1\rangle$ Spin down, 1 boson

Two level system (an atom)

Maps into a Landau-Zener Hamiltonian

Thus: if initially $|\uparrow, 0\rangle$

Then the end

Yet: see J. Keeling, VG, PRL (2008) for interesting new physics even in this situation
Case of interest: large $N$

\[
\hat{H} = -\gamma \hat{b}^\dagger \hat{b} + \frac{\gamma}{2} \sum_{i=1}^{N} \sigma_i^z + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left( \hat{b}^\dagger \sigma_i^- + \hat{b} \sigma_i^+ \right)
\]

\[
\gamma = \lambda t
\]

Initially:

\[
\langle \hat{b}^\dagger \hat{b} \rangle = 0
\]

\[
\langle \sigma_i^z \rangle = 1
\]

Conjecture: finally

\[
n_b = N \left( 1 - e^{-\frac{\lambda^2}{\gamma^2}} \right)
\]

Wrong!!

Correct formula at $\frac{\lambda}{g^2} \ll 1$: $n_b \approx N \left( 1 - \frac{\lambda}{\pi g^2} \log N \right)$

Quantum Phase Transition

\[ \hat{H} = -\gamma \hat{b}^{\dagger} \hat{b} + \gamma \hat{S}^z + \frac{g}{\sqrt{N}} \left( \hat{b}^{\dagger} \hat{S}^- + \hat{b} \hat{S}^+ \right) \]

\[ \langle \hat{S}^z \rangle \in \left[ -\frac{N}{2}, \frac{N}{2} \right] \]

N+1 - dim Hilbert space:

\[ |N/2, 0\rangle \]
\[ |N/2 - 1, 1\rangle \]
\[ |N/2 - 2, 2\rangle \]
\[ \ldots \]
\[ |-N/2, N\rangle \]

Matrix Hamiltonian:

\[ |N/2 - n, n\rangle \equiv |n\rangle \]

\[ \hat{H} = -2\gamma n \delta_{n', n} + \frac{g}{\sqrt{N}} \left( n \sqrt{N-n'} \delta_{n'+1, n} + n' \sqrt{N-n} \delta_{n'-1, n} \right) \]

Eigenvalues:

N=10, g=1

\[ E = -4N \left[ 9\gamma - \gamma^3 + 4(\gamma^2 + 3)^{3/2} \right] / 27 \]

Large N: tuning through a phase transition
Tuning through a quantum phase transition

A field explored in the literature recently:
Polkovnikov, arxiv:0312144; PRB (2005)

A scaling argument due to Polkovnikov gives:
\[ n_{\text{exc}} \sim \lambda \frac{z \nu}{z \nu + 1} ; \]  
z and \( \nu \) are the critical exponents

However, many exceptions

see Polkovnikov, Gritsev, Nat. Phys. (2008)

Bottom line: Landau-Zener exponential approach to the adiabatic limit typically does not work
Back to the model: three regimes

\[ \hat{H} = -\gamma \hat{b}^\dagger \hat{b} + \gamma \hat{S}^z + \frac{g}{\sqrt{N}} \left( \hat{b}^\dagger \hat{S}^- + \hat{b} \hat{S}^+ \right) \]
\[ \gamma = \lambda t \]

Regimes:

1. Holstein-Primakoff \[ \lambda \gg \frac{\pi g^2}{\log(N)} \]

2. Intermediate \[ \lambda \sim \frac{\pi g^2}{\log(N)} \]

3. Semiclassical (adiabatic) \[ \lambda \ll \frac{\pi g^2}{\log(N)} \]

The crossover from HP to semiclassical regimes occur at very slow driving rates!
Holstein-Primakoff regime

\[ \hat{H} = -\gamma \hat{b}^\dagger \hat{b} + \gamma \hat{S}^z + \frac{g}{\sqrt{N}} \left( \hat{b}^\dagger \hat{S}^- + \hat{b} \hat{S}^+ \right) \]
\[ \gamma = \lambda t \]

Fast driving rate \( \lambda \gg \frac{\pi g^2}{\log(N)} \)

Spin points almost up \( \langle \hat{S}^z \rangle \approx \frac{N}{2} \)

HP bosons:
\[ \hat{S}^+ \approx \sqrt{N} \hat{b}_{HP}, \hat{S}^- \approx \sqrt{N} \hat{b}_{HP}^\dagger \]

Hamiltonian is now quadratic and solvable
\[ \hat{H} = -\lambda t \hat{b}^\dagger \hat{b} - \lambda t \hat{b}_{HP}^\dagger \hat{b}_{HP} + g \left( \hat{b}^\dagger \hat{b}_{HP}^\dagger + \hat{b} \hat{b}_{HP} \right) \]

Answer: \( n_b = e^{\frac{\pi g^2}{\lambda}} - 1 \) \quad Of course, works only if \( n_b \ll N \)

Yurovsky, Ben-Reuven, Julienne, PRA (2002)
Intermediate regime: rate equations

\[ \hat{H} = -\gamma \hat{b}^\dagger \hat{b} + \frac{\gamma}{2} \sum_{i=1}^{N} \left( \hat{a}_{i\uparrow}^\dagger \hat{a}_{i\uparrow} + \hat{a}_{i\downarrow}^\dagger \hat{a}_{i\downarrow} \right) + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left( \hat{b}^\dagger \hat{a}_{i\downarrow} \hat{a}_{i\uparrow} + \hat{b} \hat{a}_{i\uparrow}^\dagger \hat{a}_{i\downarrow}^\dagger \right) \]

\[ \text{rate equation} \]

\[ d_t n_b = 2\pi g^2 \delta(2\lambda t) \left( n_f^2 (1 + n_b) - n_b (1 - n_f)^2 \right) \]

where \( n_b + N n_f = N \)

Rate equation can be justified within Keldysh RPA approximation

Answer:

\[ n_b = \frac{N \left( e^{\frac{\pi g^2}{\lambda}} - 1 \right)}{2e^{\frac{\pi g^2}{\lambda}} + N} \]

Works if \( \lambda \gtrsim \frac{\pi g^2}{\log(N)} \)

A. Altland, V. Gurarie, PRL (2008)
Semiclassical (adiabatic) regime

\[ \hat{H} = -\gamma \hat{b}^\dagger \hat{b} + \gamma \hat{S}^z + \frac{g}{\sqrt{N}} \left( \hat{b}^\dagger \hat{S}^- + \hat{b} \hat{S}^+ \right) \]

\[ \gamma = \lambda t \]

\[ \hat{b} \rightarrow \sqrt{N} \sqrt{n}, \ n \in [0, 1] \]

\[ \hat{S} \rightarrow \sqrt{\frac{N}{2}} \hat{n}, \ \hat{n} \rightarrow \varphi, \theta \text{ spherical angles} \]

The end result: classical problem

\[ H = -2\gamma n - 2gn\sqrt{1 - n \cos(\phi)} \]

with the equations of motion:

\[ \dot{n} = -\partial_\phi H \quad \dot{\phi} = \partial_n H \]

\[ \dot{n} = -2gn\sqrt{1 - n \sin(\phi)} \]

Little problem: the solution to these equations is \( n(t) = 0! \)
Truncated Wigner approximation

Need to match the initial quantum evolution with subsequent classical evolution.

Solution: initially $n$ is a random variable corresponding to the Wigner function of a harmonic oscillator in the ground state

$$W(n)dn = 2e^{-2Nn}Ndndn$$

Roughly speaking, initially $n \sim 1/N$

Now need to solve

$$H = -2\gamma n - 2gn\sqrt{1 - n}\cos(\phi)$$

with random initial conditions
Classical phase space

\[ H = -2\gamma n - 2gn\sqrt{1 - n\cos(\phi)} = \text{const} \]

\[
\begin{align*}
\gamma &= -1.5 \\
\gamma &= -0.65 \\
\gamma &= 0.5 \\
\gamma &= 2
\end{align*}
\]

Critical trajectory
Adiabatic Invariants

\[ I = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi n \]

Adiabatic evolution is best captured by adiabatic invariants

As a first approximation, they do not change during the evolution


\[ \gamma = -1.5 \quad \gamma = -0.65 \quad \gamma = 0.5 \quad \gamma = 2 \]

\[ t \to -\infty : \quad I_{ini} = n_{ini} \sim \frac{1}{N} \]

\[ t \to +\infty : \quad I_{final} = 1 - n_b/N \]

In the deep adiabatic regime, \( \lambda \) very small, \( I_{final} - I_{ini} = 0 \)

\[ I_{final} \approx 0 \quad n_b \approx N \]
Change in adiabatic invariants

Landau and Lifshits, Classical Mechanics, last sections:
For a smooth evolution, \( I_{\text{final}} - I_{\text{ini}} \sim e^{-\frac{1}{\lambda}} \)

Then \( n_b \sim N(1 - e^{-\frac{1}{\lambda}}) \) That’s Landau-Zener formula!

If an evolution crosses a “critical” trajectory (whose frequency vanishes), then \( I_{\text{final}} - I_{\text{ini}} \sim \lambda^\alpha \)
Then \( n_b \sim N(1 - \lambda^\alpha) \)

Our case is this critical case. Calculations following the adiabatic invariant theory give

\[
  n_b \approx N \left( 1 - \frac{\lambda}{\pi g^2 \log N} \right)
\]

A. Altland, VG, A. Polkovnikov, T. Kriecherbauer, PRA (2009)

\( \gamma = -0.65 \) Critical trajectory (it’s existence is related to the QPT)
Relationship with the Painlevé II

A. P. Itin, P. Törmä, arxiv:0901.4778

In the vicinity of a critical point \( n = 0, \phi = 0, \gamma = -1 \)
a substitution \( Y \sim n \sin(\phi/2) \)
leads approximately to \( s \sim \gamma + 1 \)

\[
\frac{d^2 Y}{ds^2} = sY - Y^3
\]

This is the Painlevé II equation describing a particle moving in a potential

\[
U(x) = \frac{Y^4}{4} - s\frac{Y^2}{2}
\]

The solutions to the Painlevé II equation are well known, leading to an improved result

\[
n_b = N \left( 1 - \frac{\lambda}{\pi g^2} \log \left[ \frac{N\lambda e^{\gamma_{\text{Euler}}}}{2\pi g^2} \right] + \ldots \right)
\]
Crossover to the super-adiabatic regime

\[ n_b = N \left( 1 - \frac{\lambda}{\pi g^2} \log \left( \frac{N \lambda e^{\gamma_{\text{Euler}}}}{2\pi g^2} \right) + \ldots \right) \]

This could work only if \( \lambda \geq \frac{1}{N} \)

Indeed, smaller \( \lambda \) should lead to such a slow evolution that individual levels are resolved, leading back to the Landau-Zener formula. This regime is unaccessible in the large \( N \) limit.
Summary of the analytic results

\[ \hat{H} = -\gamma \hat{b}^\dagger \hat{b} + \gamma \hat{S}^z + \frac{g}{\sqrt{N}} \left( \hat{b}^\dagger \hat{S}^- + \hat{b} \hat{S}^+ \right) \]
\[ \gamma = \lambda t \]

Regimes:

1. Holstein-Primakoff \( \lambda \gg \frac{\pi g^2}{\log(N)} \)
   \[ n_b = e^{\frac{\pi g^2}{\lambda}} - 1 \]

2. Intermediate \( \lambda \sim \frac{\pi g^2}{\log(N)} \)
   \[ n_b = \frac{N \left( e^{\frac{\pi g^2}{\lambda}} - 1 \right)}{2e^{\frac{\pi g^2}{\lambda}} + N} \]

3. Semiclassical (adiabatic) \( \lambda \ll \frac{\pi g^2}{\log(N)} \)
   \[ n_b \approx N \left( 1 - \frac{\lambda}{\pi g^2 \log N} \right) \]
Comparison with numerics

\[
\frac{1 - n_b / N}{\ln N}
\]

Holstein-Primakoff + rate at \( N = 10^3 \)

N = 10^3
N = 10^4
N = 10^6
N = 10^8

\( \lambda \)

Adiabatic limit
Distribution function

\[ \hat{H} = -\gamma \hat{b}^\dagger \hat{b} + \frac{\gamma}{2} \sum_{i=1}^{N} \sigma_i^z + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left( \hat{b}^\dagger \sigma_i^- + \hat{b} \sigma_i^+ \right) \]

\[ \gamma = \lambda t \]

Initially:

\[ \langle \hat{b}^\dagger \hat{b} \rangle = 0 \]
\[ \langle \sigma_i^z \rangle = 1 \]

We would now like to find, at \( t \to +\infty \)
the full probability distribution function of observing exactly \( n_b \) bosons

\[ P(n_b) \]

HP: \( \lambda \gg \frac{\pi g^2}{\log(N)} \)

Adiabatic: \( \lambda \ll \frac{\pi g^2}{\log(N)} \)

Exact solution:

\[ P(n_b) = e^{-n_b} e^{-\frac{\pi g^2}{\lambda}} - \frac{\pi g^2}{\lambda} \]

Gumbel distribution

\[ P(n_{\text{ini}}) \approx 2 e^{-2n_{\text{ini}}} \]
Comparison with numerics

$P(n_b) = e^{-n_b}e^{-\frac{\pi g^2}{\lambda}} - \frac{\pi g^2}{\lambda}$

HP: $\lambda \gg \frac{\pi g^2}{\log(N)}$

Adiabatic: $\lambda \ll \frac{\pi g^2}{\log(N)}$

$P(n_b) = e^{-\frac{\pi g^2}{\lambda}}(1 - \frac{n_b}{N}) - \frac{N\lambda}{\pi g^2}e^{-\frac{\pi g^2}{\lambda}}(1 - \frac{n_b}{N})$
Conclusions

\[
\hat{H} = -\gamma \hat{b}^\dagger \hat{b} + \gamma \hat{S}^z + \frac{g}{\sqrt{N}} \left( \hat{b}^\dagger \hat{S}^- + \hat{b} \hat{S}^+ \right)
\]
\[
\gamma = \lambda t
\]

• In a large \( N \) many-body system, it is hard to reach the adiabatic regime (possible consequences for the adiabatic quantum computing)

• To be adiabatic, \( \lambda \ll \frac{g^2}{\text{Log}(N)} \)

• Quasiclassical evolution must be supplemented by the quantum initial conditions to study the adiabatic regime

• Very broad distribution of boson numbers despite the applicability of the quasiclassical approximation
Molecule creation in a Feshbach resonance experiment
The model

\[ H = \sum_{p, \sigma = \uparrow, \downarrow} \frac{p^2}{2m} \hat{a}_{p\sigma}^{\dagger} \hat{a}_{p\sigma} + \sum_{p} \left( \frac{q^2}{4m} - 2\gamma \right) \hat{b}_{q}^{\dagger} \hat{b}_{q} + \frac{g}{\sqrt{V}} \sum_{p,q} \left( \hat{b}_{q} \hat{a}_{q/2+p}^{\dagger} \hat{a}_{q/2-p}^{\dagger} + h.c. \right) \]

\[ \gamma = \lambda t \]

fermionic atoms \hspace{2cm} \text{bosonic molecules} \hspace{2cm} \text{interconversion}

Initially, at \( t \rightarrow -\infty \), no bosons while fermions fill the Fermi sea up to \( \varepsilon_F \), having density \( n_F \).

Finally, at \( t \rightarrow +\infty \), how many bosons we create, as a function of the rate \( \lambda \)?

Dimensionless parameters

\[ \gamma_W = \frac{g^2 m^2}{n_F^{1/3}} \quad \text{Resonance width} \]

\[ \Gamma = \frac{\pi g^2 n_F}{\lambda} \quad \text{Landau-Zener parameter} \]

\[ n_b = n_F f (\Gamma, \gamma_W) \]
Fast rate

\[ H = \sum_{p, \sigma=\uparrow, \downarrow} \frac{p^2}{2m} \hat{a}_{p\sigma}^\dagger \hat{a}_{p\sigma} + \sum_p \left( \frac{q^2}{4m} - 2\gamma \right) \hat{b}_q^\dagger \hat{b}_q + \frac{g}{\sqrt{\pi}} \sum_{p,q} \left( \hat{b}_q \hat{a}_{q/2+p\uparrow}^\dagger \hat{a}_{q/2+p\downarrow} + h.c. \right) \]

\[ \gamma = \lambda t \]

conjecture: \[ n_b = n_F \left( 1 - e^{-\Gamma} \right) \]

Wrong!

\[ \Gamma = \pi g^2 n_F / \lambda \]


Fast rate \( \lambda \)

compare with

\[ n_b = n_F \left( \Gamma - \frac{88}{105} \Gamma^2 + \ldots \right) \]

\[ n_F \left( 1 - e^{-\Gamma} \right) = n_F \left( \Gamma - \frac{\Gamma^2}{2} + \ldots \right) \]
Simplified model (valid for narrow resonance only)

Broad resonance: hopeless (even the time-independent problem can be solved only numerically). Concentrate on narrow resonances.

$$\hat{H} = -2\gamma \hat{b}^{\dagger} \hat{b} + \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}_{p\sigma}^{\dagger} \hat{a}_{p\sigma} + \frac{g}{\sqrt{N}} \sum_p \left( \hat{b}^{\dagger} \hat{a}_{-p\downarrow} \hat{a}_{p\uparrow} + \hat{b} \hat{a}_{p\uparrow}^{\dagger} \hat{a}_{-p\downarrow}^{\dagger} \right)$$

$$\gamma = \lambda t$$

This model applies only to the narrow resonance case, since for broad resonance the bosonic momentum dependence becomes important

$$\gamma_W \ll 1$$

$$\gamma_W = g^2 m^2 / n_F^{1/3}$$

J. Levinsen, VG, PRA (2006)
Narrow resonance: close to a QPT

\[ \hat{H} = -2\gamma \hat{b}^{\dagger} \hat{b} + \sum_{p, \sigma = \uparrow, \downarrow} \frac{p^2}{2m} \hat{a}^{\dagger}_{p\sigma} \hat{a}_{p\sigma} + \frac{g}{\sqrt{N}} \sum_{p} \left( \hat{b}^{\dagger} \hat{a}_{-p\downarrow} \hat{a}_{p\uparrow} + \hat{b} \hat{a}^{\dagger}_{p\uparrow} \hat{a}^{\dagger}_{-p\downarrow} \right) \]

\[ \gamma = \lambda t \]

\[ n_b = n_F \left[ 1 - \left( \frac{\gamma}{\epsilon_F} \right)^{\frac{3}{2}} \right] \]

\[ \gamma_W = 0 \quad (g = 0) \]
The conversion, one pair at a time

\[ \hat{H} = -2\gamma \hat{b}^\dagger \hat{b} + \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}_{p\sigma}^\dagger \hat{a}_{p\sigma} + \frac{g}{\sqrt{N}} \sum_p \left( \hat{b}^\dagger \hat{a}_{-p\downarrow} \hat{a}_{p\uparrow} + \hat{b} \hat{a}_{p\uparrow}^\dagger \hat{a}_{-p\downarrow}^\dagger \right) \]

\[ \gamma = \lambda t \]

When \(-\epsilon_F \leq \gamma \leq 0\)

only those fermions are in resonance whose momentum

\[ \frac{p^2}{2m} = -\gamma \]
The transition happens one pair at a time

\[ \hat{H} = -2\gamma \hat{b}^\dagger \hat{b} + \sum_{p, \sigma = \uparrow, \downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{p\sigma} \hat{a}_{p\sigma} + \frac{g}{\sqrt{N}} \sum_p \left( \hat{b}^\dagger \hat{a}_{-p\downarrow} \hat{a}_{p\uparrow} + \hat{b} \hat{a}^\dagger_{p\uparrow} \hat{a}^\dagger_{-p\downarrow} \right) \]

\[ \gamma = \lambda t \]

only those fermions are in resonance whose momentum

\[ \frac{p^2}{2m} = -\gamma \]

When \(-\epsilon_F \leq \gamma \leq 0\)

\[ n_p^f = e^{-\frac{\pi g^2 n_b(p)}{\chi}} \]

\[ \frac{dn_b(x)}{dx} = e^{-\frac{g^2 \pi n_b(x)}{\chi}} - 1 \]

As before, a trivial solution

\[ n_b(x) = 0! \]

\[ x = \frac{p^3}{(6\pi^2)} \]

\[ x \in [0, n_F] \]
Solution

\[ n_b = n_F \left( 1 - \frac{1}{\Gamma} \log \frac{1}{\Gamma \gamma_W} + \ldots \right) = n_F \left( 1 - \frac{\lambda}{\pi g^2 n_F} \log \left[ \frac{\lambda}{\pi g^2 n_F \gamma_W} \right] + \ldots \right) \]

Compare with \[ n_b \approx N \left( 1 - \frac{\lambda}{\pi g^2 \log N} \right) \]

VG, arxiv:0812.4474
Conclusions II

\[ H = \sum_{\mathbf{p}, \sigma = \uparrow, \downarrow} \frac{p^2}{2m} \hat{a}_{\mathbf{p}\sigma} \hat{a}_{\mathbf{p}\sigma} + \sum_{\mathbf{p}} \left( \frac{q^2}{4m} - 2\gamma \right) \hat{b}_{\mathbf{q}} \hat{b}_{\mathbf{q}} + \frac{g}{\sqrt{V}} \sum_{\mathbf{p}, \mathbf{q}} \left( \hat{b}_{\mathbf{q}} \hat{a}_{\mathbf{q}/2 + \mathbf{p} \uparrow} \hat{a}_{\mathbf{q}/2 - \mathbf{p} \downarrow} + h.c. \right) \]

\[ \gamma = \lambda t \]

\[ n_b = n_F \left( 1 - \frac{\lambda}{\pi g^2 n_F} \log \left[ \frac{\lambda}{\pi g^4 n_F^2 m^2} \right] + \ldots \right) \]

- It is as hard to create molecules in this system as it is in the many-body time-dependent Dicke model.

- The adiabatic limit is approached linearly in driving rate, not exponentially as in the usual LZ problem.

- Although there is no QPT, the system is in the vicinity of a QPT, thus similar physics to the Dicke model.
The end