Optimal Trigger Strategy under Uncertainty

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1 Basic Model: Oligopoly Collusion (Porter1983)

1.1 Setting up the model

• There are \(N\) risk-neutral oil-producing states. In each period of the repeated play, state \(i\) chooses a production level \(q_{it}\), so that the total production is \(Q_t = \sum q_{it}\). The vector \((q_1, q_2, \ldots, q_N)\) is denoted by \(q_t\).

• The inverse demand function is:

\[
P(Q) = a - bQ
\]  

(1.1)

• The market price \(\hat{p}\) is determined by the demand and a multiplicative stochastic component \(\theta\), so that \(\hat{p} = P(Q)\theta\). \(\theta\) has distribution \(F(\cdot)\) with \(E[\theta] = \mu\).

• The states face identical cost functions:

\[
C(q) = c_0 + c_1q
\]  

(1.2)

• The marginal cost of producing an item is less than the price at zero quantity: \(0 < c_1 < \mu a\)

1.2 No Collusion: the Cournot Outcome

The expected profit function for state \(i\) is given by combining (1.1) and (1.2):

\[
E[\pi_i(q)] = (a - bQ)q_i \mu - c_0 - c_1q_i
\]  

(1.3)

\[
= [\mu a - c_1 - \mu bQ]q_i - c_0
\]  

(1.4)

Letting \(A = \mu a - c_1\) and \(B = \mu b\),

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\[ E[\pi_i(q)] = (A - BQ)q_i - c_0 \quad (1.5) \]

The optimal choice of production for state \( i \) given other the production by other state is provided by the first order condition

\[ \frac{\partial E\pi_i(q)}{\partial q_i} = A - BQ - Bq_i = 0 \quad (1.6) \]
\[ \Leftrightarrow q_i = \frac{A - BQ_i}{2B} \quad (1.7) \]

where \( Q_i = Q - q_i \). Since all the states are homogeneous, the equilibrium \( q_i = s \) is identical to all states, which provides \( Q_i = s(N-1) \). Taking this into (1.6), we have

\[ s = \frac{A - Bs(N-1)}{2B} \quad (1.8) \]
\[ \Leftrightarrow s = \frac{A}{B(N+1)} \quad (1.9) \]

which provides the expected profit of

\[ E\pi_i(s) = \frac{A^2}{B(N+1)^2} - c_0 \]

1.3 Perfect Collusion: the Monopoly Outcome

If the collusion among the states are perfectly enforceable, the outcome is more favorable. The total revenue is

\[ \Pi = AQ - BQ^2 - Nc_0 \]

which is maximized when \( A - 2BQ = 0 \) or \( Q = A/2B \). Dividing \( Q \) among the states leads to production vector \( r \) with identical components

\[ r = \frac{A}{2BN} \]

The profit per state given this monopoly level of production is

\[ \pi_M = \frac{A^2}{4BN} - c_0 \]

It is obvious that \( r < s \) and \( \pi_M > E\pi_i(s) \) for \( N > 1 \).
1.4 Restriction Production with Trigger Strategy

1.4.1 Detering Optimal Deviation

Now suppose the states are able to reach an agreement in which all states agree to produce an amount \( \tilde{q} \) that is smaller than that of the Cournot outcome \( s \) and greater than that of the monopoly outcome \( r \). Assume all other states stick to the agreement, the profit function of state \( i \) is given by

\[
\pi_i(q) = [A - B(N-1)\tilde{q} - Bq_i]q_i - c_0
\]

whose first-order condition yields

\[
q_i = \frac{A - B(N-1)\tilde{q}}{2B}.
\]

It is obvious that \( q_i > \tilde{q} \). Thus the state \( i \) could gain for that period from deviating to \( q_i \). This is balanced by a \( T-1 \) period loss of the difference of the cooperative profit and the Cournot profit. To deter such deviation, the other states can use trigger punishment strategy. The strategy will deter deviation to \( q_i \) as long as the punishment period is long enough.

1.4.2 Detering Incremental Deviation

There is the other form of deviation where the state increases the production quantity marginally instead of optimally in one period. This type of deviation is hard to detect given the uncertainty in the price determination mechanism. Trigger punishment strategy is still able to deter deviation to some extent. Given that the quantity of production by each state is not observable, the trigger strategy could only target the price on the market. \( \hat{p} \) is set such that \( T \) periods of Cournot game is triggered if the price \( \hat{p} = P(Q)\theta \) is ever greater than \( \tilde{p} \).

The state tempted by incremental deviations faces the following recursive equation of payoff if the deviation is carried out:

\[
V_i(q) = \pi_i(q) + \Pr(P(Q)\theta < \hat{p}) \cdot \delta V_i(q)
\]

\[
+ \Pr(P(Q)\theta \geq \hat{p}) \cdot \left[ \sum_{t=1}^{T-1} \delta^t \pi_i(s) + \delta^T V_i(q) \right]
\]

(1.10)

Note

\[
\Pr(P(Q)\theta < \hat{p}) = \Pr \left( \theta < \frac{\hat{p}}{P(Q)} \right)
\]

(1.12)

\[
= F(\hat{p}/P(Q))
\]

(1.13)

Using recursion to solve for \( V_i \) in (1.10), it is obtained that

\[
V_i(q) = \frac{\pi_i(s)}{1-\delta} + \frac{\pi_i(q) - \pi_i(s)}{1-\delta^T - F(\hat{p}/P(Q))}(\delta - \delta^T)
\]

(1.14)
In contemplating incremental deviation, state \( i \) chooses the quantity \( q_i \) that maximize the value function \( V_i(q) \), which is thus

\[
\max_{q_i} V_i(q) \quad (1.15)
\]

The first-order condition provides:

\[
\frac{\partial V_i(q)}{\partial q_i} = 0
\]

or

\[
[1 - \delta + (\delta - \delta^T)F]\pi_i^*(q^*) + (\delta - \delta^T)[\hat{p}p_i(Q^*)/p(Q^*)^2][f[\pi_i(q^*) - \pi_i(s)] = 0 \quad (1.16)
\]

The argument of \( F \) and \( f \), \( \hat{p}/P(Q^*) \), is suppressed. \( \pi_i^*(q^*) = \frac{d\pi_i(q^*)}{dq_i} \) and \( p_i(Q^*) = \frac{dp_i(Q^*)}{dq_i} \).