7.1 An introduction to Dixit-Stiglitz CES preferences

D-S preferences are a special, symmetric case of CES preferences, elasticity of substitution $> 1$. A more general treatment of CES will be given later.

$Y$ will be a competitive, constant-returns industry while $X$ will consist of an endogenous number of differentiated varieties.

Utility of the representative consumer in each country is Cobb-
Douglas, and the symmetry of varieties within a group of goods allows us to write utility as follows (0 < \alpha < 1).

\[ U = X_c^\beta Y^{1-\beta}, \quad X_c = \left[ \sum_i (X_i)^\alpha \right]^{1/\alpha} \]

where the number of varieties N is endogenous.

This function permits the use of two-stage budgeting, in which the consumer first allocates total income (M) between Y and X_c.

Let e denote the minimum cost of buying one unit of X_c at price p for the individual varieties (i.e., e is the unit expenditure function for X_c). Y is numeraire. First-stage budgeting yields:

\[ Y = (1-\beta)M \quad X_c = \beta M/e \]
Let $M_x = \beta M$ be the expenditure on $X$ in aggregate. Solve for the demand for a given $X$ variety, and for the price index $e$.

The consumer’s sub-problem maximizing the utility from $X$ goods subject to an expenditure constraint (using $\lambda$ as a Lagrangean multiplier) and first-order conditions are:

$$e(p^k) = \min(X_i) \sum_i pX_i \quad st \quad X_c = 1$$

$$\max X_c = \left[ \sum X_i^\alpha \right]^{\frac{1}{\alpha}} + \lambda (M_x - \sum p_iX_i)$$

$$\Rightarrow \frac{1}{\alpha} \left[ \sum X_i^\alpha \right]^{\frac{1}{\alpha} - 1} \alpha X_i^{\alpha - 1} - \lambda p_i = 0$$
Let \( \sigma \) denote the elasticity of substitution among varieties. Dividing the first-order condition for variety \( i \) by the one for variety \( j \),

\[
\left[ \frac{X_i}{X_j} \right]^{\alpha-1} = \frac{p_i}{p_j} \quad \frac{X_i}{X_j} = \left[ \frac{p_i}{p_j} \right]^{\frac{1}{\alpha-1}} = \left[ \frac{p_i}{p_j} \right]^{-\sigma}
\]

since \( \sigma = \frac{1}{1 - \alpha} \)

\[
X_j = \left[ \frac{p_i}{p_j} \right]^\sigma X_i \quad p_jX_j = p_j p_j^{-\sigma} p_i^\sigma X_i
\]

\[
\sum p_jX_j = M_x = \left[ \sum p_j^{1-\sigma} \right] p_i^\sigma X_i
\]

Inverting this last equation, the demand for an individual variety \( i \):
Use $X_i$ to construct $X_c$ and then solve for $e$, noting the relationship between $\alpha$ and $\sigma$.

\[ X_i = p_i^{-\sigma} \left[ \sum p_j^{1-\sigma} \right]^{-1} M_x \] \quad \sigma = \frac{1}{1 - \alpha}, \quad \alpha = \frac{\sigma - 1}{\sigma} \]

\[ X_i^\alpha = X_i^\sigma p_i^{1-\sigma} \left[ \sum p_j^{1-\sigma} \right]^{1-\sigma} M_x^\alpha \]

\[ \sum X_i^\alpha = \left[ \sum p_i^{1-\sigma} \right] \left[ \sum p_j^{1-\sigma} \right]^{1-\sigma} M_x^\alpha = \left[ \sum p_j^{1-\sigma} \right]^{1} M_x^\alpha \]

\[ X_c = \left[ \sum X_i^\alpha \right]^{\frac{1}{\sigma}} = \left[ \sum X_i^\alpha \right]^{\frac{\sigma}{\sigma-1}} = \left[ \sum p_j^{1-\sigma} \right]^{\frac{1}{\sigma-1}} M_x \]
An increase in the range of goods lowers the cost of a unit of utility

Having derived $e$, we can then use equation (13) in (9) to get the demand for an individual variety.

$$X_i = p_i^{-\sigma}e^{\sigma-1}M_x$$

since

$$e^{\sigma-1} = \left[\sum p_j^{1-\sigma}\right]^{-1}$$

Now derive the price elasticity of demand for an individual good.

$$X_i = p_i^{-\sigma}\left(\sum p_j^{1-\sigma}\right)^{-1}M_x$$

$$s_i = \frac{p_iX_i}{M_x} = p_i^{1-\sigma}\left(\sum p_j^{1-\sigma}\right)^{-1}$$
\[(...) = \left( \sum p_j^{1-\sigma} \right) \]

\[
\frac{\partial X_i}{\partial p_i} = -\sigma p_i^{-\sigma-1} M_x - (1 - \sigma) p_i^{-\sigma} M_x - (\sigma - 1) p_i^{-2\sigma} M_x
\]

\[
= -\sigma p_i^{-\sigma-1} M_x + (\sigma - 1) p_i^{-2\sigma} M_x
\]

\[
p_i \frac{\partial X_i}{\partial p_i} = -\sigma p_i^{-\sigma} M_x + (\sigma - 1) p_i^{-2\sigma + 1} M_x
\]

\[
\frac{p_i}{X_i} \frac{\partial X_i}{\partial p_i} = -\sigma + (\sigma - 1) p_i^{-\sigma+1} (\sigma - 1) = -\sigma + s_i (\sigma - 1)
\]

A convention is to define the Marshallian price elasticity as positive
7.2 Monopoly with fixed costs

Now suppose that we have a two-sector economy, X and Y, where the representative consumer has CES preferences between the two goods.

X is produced with increasing returns to scale in the form of a fixed costs plus a constant marginal cost in terms of the single factor of production, labor (L).
We assume in this section that there is a single monopoly producer of X who must incur the fixed costs (FC). Agent ENTR receives markup revenues and demands fixed costs.

Suppose demand for good X is just written in inverse form \( p(X) \) so the monopolist’s revenue is \( R = p(X)X \). Marginal revenue is then given by:
where \( \eta \) is the Marshallian elasticity of demand, defined as positive. The monopoly markup is just the inverse of this elasticity.

\[
\frac{\partial R}{\partial X} = p + X \frac{\partial p}{\partial X} = p + p \left[ \frac{X}{p} \frac{\partial p}{\partial X} \right] = p \left[ 1 - \frac{1}{\eta} \right] \equiv MR
\]

\[
MR = p (1 - mk) \quad mk = \frac{1}{\eta} \quad \eta \equiv -\left[ \frac{p}{X} \frac{\partial X}{\partial p} \right]
\]

where \( mk \) is the optimal markup.

\( \eta \) is the Marshallian elasticity of demand, defined as positive. The monopoly markup is just the inverse of this elasticity.

Let’s calibrate under the assumption that this is a “natural monopoly”: the profit-maximizing entrepreneur just breaks even.
<table>
<thead>
<tr>
<th>Markets</th>
<th>C</th>
<th>FC</th>
<th>Y</th>
<th>W</th>
<th>CONS ENTR</th>
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<td>PX</td>
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<td>MK</td>
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</table>

Choose units so that the price of X and the marginal cost of X = 1.

Then $1 \times (1 - mk) = 0.8$, so $mk = 0.2$. The observed expenditure share on X is $s = 0.5$ in the benchmark.

$$mk = 1/[(\sigma - s(\sigma-1)) = 0.8, \text{ so } [\sigma - s(\sigma-1)] = [\sigma - 0.5*(\sigma-1)] = 5$$

implies $\sigma = 9 \quad (9 - 0.5*8) = 5$
Two unknowns are added to a standard competitive model:

\[
\text{SHAREX} \quad \text{Share of X in consumption (value share)}
\]
\[
\text{MARKUP} \quad \text{Markup};
\]

And two equations (where sigma is a parameter = 9):

\[
\text{SHX..} \quad \text{SHAREX} = \frac{100 \times PX \times X}{100 \times PX \times X + 100 \times PY \times Y};
\]
\[
\text{MK..} \quad \text{MARKUP} = \frac{1}{\text{SIGMA} - (\text{SIGMA} - 1) \times \text{SHAREX}};
\]

We could break out the entrepreneur (whoever has the property rights to the income stream) as a separate consumer.

The problem with doing this is if profits are negative, then the model will not solve.
Suppose that the entrepreneur has the same preferences as everyone else.

Then we will just calculate aggregate income and break out monopoly profits after solving.

\[
\text{INCOME.. CONS} = PL \times \text{ENDOWL} + (100 \times PX \times X \times \text{MARKUP} - PL \times FC);
\]
$TITLE: M7-2.GMS:  Monopoly with fixed costs

$ONTEXT

Production Sectors          Consumers

<table>
<thead>
<tr>
<th>Markets</th>
<th>C</th>
<th>Y</th>
<th>W</th>
<th>CONS</th>
<th>ENTR</th>
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$OFFTEXT

PARAMETERS

SIGMA: elasticity of substitution among varieties
FC: parameter setting the level of fixed costs
ENDOWL: endowment of labor
INCOMEM: monopoly profit share (markup revenues - fixed costs)
INCOME: income share of the "the people"
MODELSTAT: statistic indicating model solved: 0 = solved;

SIGMA = 9;
FC = 20;
ENDOWL = 200;
POSITIVE VARIABLES

X     Activity level for X (output per firm)
Y     Activity level of Y output
W     Activity level for welfare
PX    Price of X
PY    Price of Y
PW    Price index for utility (consumer price index)
PL    Price of labor
CONS  Income of the representative consumer
SHAREX Share of X in consumption (value share)
MARKUP Markup;

EQUATIONS

PRICEX  MR = MC in X (associated with X output per firm)
PRICEY  Zero profit condition for Y (PY = MC)
PRICEW  Zero profit condition for W (PW = MC of utility)

MKT_X   Supply-demand balance for X (individual variety)
MKT_Y   Supply-demand balance for Y
MKT_W   Supply-demand balance for utility W (welfare)
MKT_L   Supply-demand balance for labor
INCOME   National income 
SHX      Share of X in expenditure 
MK       Markup equation; 

PRICEX.. 80*PL =G= 100*PX*(1-MARKUP); 

PRICEY.. 100*PL =G= 100*PY; 

PRICEW.. (0.5*PX**(1-SIGMA) + 0.5*PY**(1-SIGMA))**(1/(1-SIGMA)) 
          =G= PW; 

MKT_X..  X*100 =G= PX**(-SIGMA)*(PW**(SIGMA-1))*CONS/2; 

MKT_Y..  Y*100 =G= PY**(-SIGMA)*(PW**(SIGMA-1))*CONS/2; 

MKT_W..  200*W =G= CONS/PW; 

MKT_L..  ENDOWL =E= Y*100 + X*80 + FC; 

INCOME.. CONS =E= PL*ENDOWL + (100*PX*X*MARKUP - PL*FC); 

SHX.. SHAREX =E= 100*PX*X / (100*PX*X + 100*PY*Y); 

MK.. MARKUP =E= 1/(SIGMA - (SIGMA-1)*SHAREX);
MODEL MONOPOLY /PRICEX.X, PRICEY.Y, PRICEW.W,
    MKT_X.PX, MKT_Y.PY, MKT_W.PW, MKT_L.PL,
    INCOME.CONS, SHX.SHAREX, MK.MARKUP/;

OPTION MCP=PATH;

*       set benchmark values:
X.L = 1;
Y.L = 1;
W.L = 1;
PX.L = 1;
PY.L = 1;
PL.L = 1;
PW.L = 1;
CONS.L = 200;
SHAREX.L = 0.5;
MARKUP.L = 0.20;

* choose the price of good Y as numeraire
PY.FX = 1;

* check for calibration and starting-value errors
MONOPOLY.ITERLIM = 0;
SOLVE MONOPOLY USING MCP;
MONOPOLY.ITERLIM = 1000;
SOLVE MONOPOLY USING MCP;

MODELSTAT = MONOPOLY.MODELSTAT - 1.;
DISPLAY MODELSTAT;

INCOMEM = (MARKUP.L*PX.L*X.L*100 - PL.L*FC)/CONS.L;
INCOME = (PL.L*ENDOWL)/CONS.L;

DISPLAY INCOMEM, INCOME;

* Counterfactual: contract the size of the economy

ENDOWL = 100;

SOLVE MONOPOLY USING MCP;

INCOMEM = (MARKUP.L*PX.L*X.L*100 - PL.L*FC)/CONS.L;
INCOME = (PL.L*ENDOWL)/CONS.L;

DISPLAY INCOMEM, INCOME;

* Counterfactual: expand the size of the economy

ENDOWL = 400;
SOLVE MONOPOLY USING MCP;

INCOMEM = (MARKUP.L*PX.L*X.L*100 - PL.L*FC)/CONS.L;
INCOME C = (PL.L*ENDOWL)/CONS.L;

DISPLAY INCOMEM, INCOME C;
7.3 Oligopoly: Cournot competition with identical products and free entry

Model will be characterized by variable markups and pro-competitive gains from trade.

A single factor of production $L$ (call it labor) divided between the $Y$ and $X$ sectors and among firms in the $X$ sector.

Marginal cost in units of labor is denoted by $mc$ and total cost ($tc$) and average cost ($ac$) for an $X$ firm are as follows:

\[
    tc = cX + f \\
    ac = \frac{tc}{X} = c + \frac{f}{X}
\]

General equilibrium production frontier shown in Figure 2.
Figure 11.1

\[ ac = mc + \frac{fc}{X} \]
The average cost of producing $X$ at point $A$ is given by the labor needed for $X$ divided by the output of $X$.

$$ac = \frac{\bar{L} - L^0}{X^0} = \frac{\bar{Y} - Y^0}{X^0}$$

Too little $X$ is produced at too high a price.

Cournot-Nash (or Cournot for short) competition in which firms pick a quantity as a best response to their rivals’ quantities.

Revenue for a Cournot firm $i$ and selling in country $j$ is given by the price in $j$ times quantity of the firm’s sales. Price is a function of all firms’ sales.
\[ R_{ij} = p_j(X_j)X_{ij} \]  

\( X_j \) is total sales in market \( j \):  
\[ X_j = \sum_i X_{ij} \]

Cournot conjectures imply that \( \partial X_j / \partial X_{ij} = 1 \); a one-unit increase in the firm’s own supply is a one-unit increase in market supply.

Marginal revenue is then

\[
\frac{\partial R_{ij}}{\partial X_{ij}} = p_j + X_{ij} \frac{\partial p_j}{\partial X_j} \frac{\partial X_j}{\partial X_{ij}} = p_j + X_{ij} \frac{\partial p_j}{\partial X_j} \text{ since } \frac{\partial X_j}{\partial X_{ij}} = 1
\]

Now multiple and divide the right-hand equation by total market supply and also by the price.
\[
\frac{\partial R_{ij}}{\partial X_{ij}} = p_j + X_{ij} \frac{\partial p_j}{\partial X_j} = p_j + p_j \frac{X_{ij}}{X_j} \left[ \frac{X_j \frac{\partial p_j}{\partial X_j}}{p_j \frac{\partial X_j}{\partial p}} \right]
\]

The term in square brackets in is just the inverse of the price elasticity of demand.

\[
\frac{\partial R_{ij}}{\partial X_{ij}} = p_j \left[ 1 - \frac{X_{ij}}{X_j} \frac{1}{\eta_j} \right] \quad \eta_j = -\left[ \frac{p_j \frac{\partial X_j}{\partial p_j}}{X_j \frac{\partial p_j}{\partial X_j}} \right]
\]

(top of next page)

The term \(X_{ij}/X_j\) in (11.6) is just firm i's market share in market j, which we can denote by \(s_{ij}\).

\[
mr_{ij} = p \left[ 1 - \frac{s_{ij}}{\eta_j} \right] = mc_i
\]
Let the price of $Y$ be numeraire, equal to one.

\[ U = (nX)^\alpha Y^{1-\alpha} \quad \text{with income} \quad I = \bar{L} + \Pi = pnx + Y \]

The elasticity of demand for $X$, we will find that $\eta = 1$. The markup is just the firm's market share, which in turn is just $1/n$

\[ p(1 - 1/n) = c \quad \Rightarrow \quad p = \frac{n}{n-1}c \]

Consider the free entry and exit version of the Cobb-Douglas case.

\[ p(1 - 1/n) = c \quad \text{pricing for } X \quad (1) \]
Free entry or zero-profits

\[ pX = cX + f \quad \Rightarrow \quad (p/n)X = f \]  

(pricing for n)

(markups revenues equal fixed costs)

\[ nX = \alpha \bar{L}/p \]  

market clearing

Use the first two equations:

\[ \frac{n}{n-1} = 1 + \frac{f}{cX} \quad \Rightarrow \quad \frac{n}{n-1} - \frac{n-1}{n-1} = \frac{f}{cX} \]

which gives us output per firm.

\[ X = (n - 1) \frac{f}{c} \quad (2) \]
Multiple both sides of pricing equation (1) by $X$, and substitute for $pX$ from market clearing.

\[ p \left(1 - \frac{1}{n}\right)X = p \left(\frac{n-1}{n}\right)X = \left(\frac{n-1}{n}\right) \frac{\alpha L}{n} = cX \quad (3) \]

Now substitute the expression for $X$ in (2) to give us a solution for $n$, the endogenous number of firms.

\[ \left(\frac{n-1}{n}\right) \frac{\alpha L}{n} = c(n-1) \frac{f}{c} \quad n^2 = \frac{\alpha L}{f} \quad (4) \]

Take the square root of the right-hand equation to get $n$ and then substitute this into to get $X$. 
\[ n = \sqrt{\frac{\alpha L}{f}} \quad X = \left[ \sqrt{\frac{\alpha L}{f}} - 1 \right] \frac{f}{c} \]

\[ \text{markup} = \frac{f}{\alpha L} \]

Pro-competitive, pro-efficiency gains from trade: a larger market supports more competition and more output per firm.

In the model, we use another feature which is not needed, but allows us to keep track of markup revenues relative to total income.

We introduce a (dummy) agent called “ENTRE” for entrepreneur, who received the markup revenues and demands fixed costs.
Figure 11.4
This is a way of capturing free entry: in equilibrium, markup revenues are exactly exhausted in paying for fixed costs.

In the data for the model, we assume that the fixed costs of 20 are the combined fixed costs of 5 firms initially in the market.
If preferences are Cobb-Douglas, then the initial markup is $1/N = 1/5 = 0.20$ which is consistent with the initial data.

Then in the calibration, the initial value of $N$ is $N.L = 5$. The marginal cost of $X$ is 1, so the price of $X$ must be $P_X.L = 1.25$.

\begin{align*}
\text{DN..} & \quad N \times 4 \quad =G= \quad \text{ENTRE/PN;} \\
\text{IENTRE..} & \quad \text{ENTRE} \quad =E= \quad \text{MARKUP} \times P_X \times X \times 80; \\
\text{MK..} & \quad \text{MARKUP} \times N \quad =E= \quad 1;
\end{align*}

where MARKUP is an endogenous auxiliary variable.
$TITLE M7-3.GMS: Oligopoly with Free Entry, homogeneous good, Cournot competition. Uses Cobb-Douglas demand

$ONTEXT

<table>
<thead>
<tr>
<th>Production Sectors</th>
<th>Consumers</th>
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</tbody>
</table>

$OFFTEXT

PARAMETERS

SIGMA = "Elasticity of substitution"
ENDOW = "Endowment scale multiplier"
MODELSTAT = "statistic indicating model solved: 0 = solved"
XPF = "X output per firm;"

SIGMA = 1;
ENDOW = 200;
**POSITIVE VARIABLES**

X  Aggregate X production by all firms
N  Number of X sector firms
Y  Activity level of Y output
W  Activity level for welfare

PX  Price of an individual X variety
PN  Price of fixed costs (price of entering)
PY  Price of Y
PW  Price index for utility (consumer price index)

PL  Price of labor

CONS  Income of the representative consumer
ENTRE  Income of the agent ENTRE = markup revenue
MARKUP  Endogenous markup rate = 1 over N;

**EQUATIONS**

PRICEX  MR = MC in X
PRICEN  Zero profit condition for fixed costs
PRICEY  Zero profit condition for Y (PY = MC)
PRICEW  Zero profit condition for W

DX       Supply-Demand for X
DN       Supply-Demand for fixed costs
DY       Supply-Demand for Y
DW       Supply-Demand for W

LAB      Supply-demand balance for labor

ICONS    Consumer (factor owners') income
IENTRE   Entrepreneur's profits
MK       Markup equation;

PRICEX.. PL  =G=  PX*(1 - MARKUP);
PRICEN.. PL  =G=  PN;
PRICEY.. PL  =G=  PY;
PRICEW.. ((PX/1.25)**0.5)*(PY**0.5) =G=  PW;
DX..     X*80  =E=  0.5*CONS/PX;
DN..     N*4   =G=  ENTRE/PN;
DY..       Y*100 =E= 0.5*CONS/PY;

DW..       W*200 =E= CONS/PW;

LAB..      ENDOW =E= Y*100 + X*80 + N*4;

ICONS..    CONS =E= PL*ENDOW;

IENTRE..   ENTRE =E= MARKUP*PX*X*80;

MK..       MARKUP*N =E= 1;

MODEL M52 /DX.PX, DY.PY, DW.PW, DN.PN, PRICEX.X, PRICEY.Y, PRICEW.W, PRICEN.N,LAB.PL, ICONS.CONS, IENTRE.ENTRE, MK.MARKUP/;

OPTION MCP=MILES;
OPTION LIMROW=0;
OPTION LIMCOL=0;
$OFFSYMLIST OFFSYMXREF OFFUELLIST OFFFUELXREF

CONS.L = 200;
X.L = 1;
Y.L = 1;
W.L = 1;
N.L = 5;
PX.L = 1.25;
PY.L = 1;
PL.L = 1;
PW.L = 1;
PN.L = 1;
ENTRE.L = 20;
MARKUP.L = 0.20;
PY.FX = 1;

M52.ITERLIM = 0;
SOLVE M52 USING MCP;
MODELSTAT = M52.MODELSTAT - 1.;

M52.ITERLIM = 1000;
SOLVE M52 USING MCP;
MODELSTAT = M52.MODELSTAT - 1.;

XPF = 80*X.L/N.L;
DISPLAY XPF;

* counterfactual: double the size of the economy

ENDOW = 400;
SOLVE M52 USING MCP;

XPF = 80*X.L/N.L;
DISPLAY XPF;

* show welfare as a function of the economy's size

SETS I indexes 25 different size levels /I1*I25/;

PARAMETERS
  SIZE(I)
  WELFARE(I)
  WELFCAP(I)
  FIRMSIZE(I)
  FIRMNUMB(I)
  MARKUPO(I)
  RESULTS(I,*);

LOOP (I,

  SIZE(I) = 5.2 - 0.2*ORD(I);
  ENDOW = 200*SIZE(I);

SOLVE M52 USING MCP;
WELFARE(I) = W.L;
WELFCAP(I) = WELFARE(I)/SIZE(I);
FIRMSIZE(I) = X.L/N.L*5;
FIRMNUMB(I) = N.L/5;
MARKUPO(I) = MARKUP.L;

);  

RESULTS(I, "SIZE") = SIZE(I);
RESULTS(I, "WELFARE") = WELFARE(I);
RESULTS(I, "WELFCAP") = WELFCAP(I);
RESULTS(I, "FIRMSIZE") = FIRMSIZE(I);
RESULTS(I, "FIRMNUMB") = FIRMNUMB(I);
RESULTS(I, "MARKUP") = MARKUPO(I);

DISPLAY RESULTS;

* Write parameter RESULTS to an Excel file M7.XLS,
* starting in Sheet1,

$LIBINCLUDE XLDUMP RESULTS M7.XLS SHEET1!A3

Execute_Unload 'M7.gdx' RESULTS
execute 'gdxxrw.exe M7.gdx par=RESULTS rng=SHEET2!A3'
7.4 Monopolistic-competition I: large group with D-S CES

The assumption in “large-group” monopolistic competition is that there are many firms: individual firms view e, M as *constants*.

Thus the elasticity of demand for an individual variety is just $\sigma$.

Equilibrium in the X sector involves two equations in two unknowns. The unknowns are X, output per variety and N, the numbers of varieties or firms.

The two equations are the firm’s optimization condition, marginal revenue equals marginal cost, and the free-entry or zero profit condition, prices equals average cost.
Gains from increased final and intermediate goods variety.

Total income is given by $L$ when the wage is chosen as numeraire.

Symmetry I: all X goods are imperfect but symmetric substitutes

Symmetry I: all X goods have the same cost function

Symmetry III: fixed and marginal costs have the same functional form: $f/c$ is a constant.

$X$ and $p_x$ will denote the price of a representative good which are the same for all goods actually produced

$$U = \left[ \sum_i X_i^\alpha \right]^\frac{\beta}{\alpha} Y^{1-\beta} \quad \sigma = \frac{1}{1 - \alpha} \quad L = np_x X + p_y Y$$ (1)
the consumer’s demands for $X$ varieties and $Y$ are

\begin{align*}
Y &= (1 - \beta) \frac{L}{p_y} \\
X_i &= p_{xi}^{-\sigma} \left[ \sum_i p_{xi}^{1-\sigma} \right]^{-1} \beta L \\
nX &= \beta \frac{L}{p_x} \tag{2}
\end{align*}

The variety’s own price appears both as the first term on the right-hand side of the second equation of (2) but also appears in the summation term inside the square brackets.

The effect of a change in a firm’s price on the summation term in square brackets become extremely small as the number of varieties (firms) $n$ becomes large.

Assumes that an individual firm is too small to affect the summation term in (2), an assumption known as “large-group monopolistic competition.”
The price elasticity of demand for an individual good is given simply by $\sigma$, the elasticity of substitution among the $X$ goods

$$- \frac{p_x \, \partial X}{X \, \partial p_x} = \sigma \quad mr_x = p_x \left(1 - 1/\sigma\right) = mc_x$$ (3)

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Definition</th>
<th>Complement Var</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_x(1 - 1/\sigma) \leq mc_x$</td>
<td>pricing for $X$</td>
<td>$X$</td>
</tr>
<tr>
<td>$(p_x/\sigma)X \leq fc_x$</td>
<td>pricing for $n$ (free entry)</td>
<td>$n$</td>
</tr>
<tr>
<td>$p_y \leq mc_y$</td>
<td>pricing for $Y$</td>
<td>$Y$</td>
</tr>
</tbody>
</table>
Then there are three market-clearing conditions, which require that supply equal demand (strictly speaking supply is greater than or equal to demand)

\[(1 - \beta)L/p_y \leq Y \quad \text{demand/supply } Y \quad p_y \quad (7)\]

\[\beta L/p_x \leq nX \quad \text{demand/supply } X \text{ varieties } \quad p_x \quad (8)\]

\[(mc_y)Y + n(mc_x)X + n(fc_x) = L \quad \text{demand/supply } L \quad w \quad (9)\]

Equations (4) and (5) can be solved for both \(X\) and \(p_x\). Then these solution values can be used in (8) to get \(n\).

\[X = (\sigma - 1)\frac{fc_x}{mc_x} \quad n = \frac{\beta L}{\sigma fc_x} \quad nX = \frac{(\sigma - 1) \beta L}{\sigma mc_x} \quad (10)\]
The output of any good that is produced is a constant and that any expansion in the economy creates a proportional increases in variety $n$.

Let $X/L$, the consumption of a representative variety per capita, be given by $C$. Then note from the last equation of (10) that $nC$ is a constant:

$$C = \frac{X}{L} = \frac{(\sigma - 1)}{\sigma} \frac{\beta}{m_{c_x} n} \equiv \frac{\gamma}{n} \quad \text{(11)}$$

$$U_x = \left[ nC^\alpha \right]^\frac{1}{\alpha} = n^{\frac{1}{\alpha}} C = n^{\frac{1-\alpha}{\alpha}} \gamma = n^{\frac{1}{\sigma - 1}} \gamma = \left[ \frac{\beta L}{\sigma f_{c_x}} \right]^{\frac{1}{\sigma - 1}} \gamma$$
<table>
<thead>
<tr>
<th>Markets</th>
<th>C</th>
<th>FC</th>
<th>Y</th>
<th>W</th>
<th>Cons Entr</th>
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<tbody>
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<td></td>
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<td>MK</td>
<td>-20</td>
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<td></td>
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</tr>
</tbody>
</table>

There are a number of ways to organize the benchmark data, this is one of them.

Markup revenues (MK) are not directly observed by IO economists have techniques for estimating these.
I introduce an artificial or “dummy” agent ENTR (entrepreneur). ENTR receives the markup revenues and “demands” fixed costs.

In equilibrium, the total value of fixed costs produced equals markup revenues, which is a way of modeling the free-entry zero-profit condition.

The activity level for N (production of fixed costs) corresponds to the number of varieties produced in equilibrium, and so affects the price index and welfare.

\[
\text{marginal revenue} = \text{mc} \quad \text{price} = \text{average cost}
\]

\[
p(1 - 1/\sigma) = p(1 - mk) = mc \quad p = mc + fc/X
\]
Subtracting the second equation from the first:

\[ p(1 - mk) - p = mc - mc - fc/X \]

\[ p(mk)X = fc \quad \text{markup revenues = fixed costs.} \]

The counter-factual experiment doubles the size of the economy.

The \( X \) sector’s output is homogeneous of degree 1.25 in factor inputs with \( \sigma = 5 \), if by \( X \) sector’s output here we mean \( X_c \).

The \( X \) sector expands only through the entry of new firms, the output of a representative firm, \( X \), is constant. \( X_c \) is given by

\[ X_c = \left[NX^\alpha\right]^{1/\alpha} = \frac{1}{N}X = \frac{\sigma}{\sigma - 1}X = N^{1.25}X \]
$TITLE: M7-4.GMS: Large-Group Monopolistic Competition
* calibrated to an elasticity of substitution of 5

$ONTEXT

<table>
<thead>
<tr>
<th>Markets</th>
<th>XC</th>
<th>N</th>
<th>Y</th>
<th>W</th>
<th>CONS</th>
<th>ENTR</th>
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$OFFTEXT

PARAMETERS
SI SIGMA: elasticity of substitution among varieties
FC parameter setting the level of fixed costs
ENDOWL endowment of labor
MODELSTAT statistic indicating model solved: 0 = solved;

SI = 5;
FC = 20;
ENDOWL = 200;

**POSITIVE VARIABLES**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
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<tbody>
<tr>
<td>X</td>
<td>Activity level for X (output per firm)</td>
</tr>
<tr>
<td>XC</td>
<td>Composite X (utility value of agg X sector output)</td>
</tr>
<tr>
<td>N</td>
<td>Number of X sector firms (variety measure)</td>
</tr>
<tr>
<td>Y</td>
<td>Activity level of Y output</td>
</tr>
<tr>
<td>W</td>
<td>Activity level for welfare</td>
</tr>
<tr>
<td>PX</td>
<td>Price of an individual X variety</td>
</tr>
<tr>
<td>PE</td>
<td>Price index (unit expenditure function): cost of XC = 1</td>
</tr>
<tr>
<td>PN</td>
<td>Price of fixed costs (price of entering)</td>
</tr>
<tr>
<td>PY</td>
<td>Price of Y</td>
</tr>
<tr>
<td>PW</td>
<td>Price index for utility (consumer price index)</td>
</tr>
<tr>
<td>PL</td>
<td>Price of labor</td>
</tr>
<tr>
<td>CONS</td>
<td>Income of the representative consumer;</td>
</tr>
</tbody>
</table>

**EQUATIONS**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRICEX</td>
<td>MR = MC in X (associated with X output per firm)</td>
</tr>
<tr>
<td>PINDEX</td>
<td>Price index for X sector goods</td>
</tr>
<tr>
<td>PRICEN</td>
<td>Zero profits - free entry in X (associated with N)</td>
</tr>
<tr>
<td>PRICEY</td>
<td>Zero profit condition for Y (PY = MC)</td>
</tr>
</tbody>
</table>
PRICEW  Zero profit condition for W (PW = MC of utility)

DX        Supply-demand balance for X (individual variety)
DXC       Supply-demand balance for XC
DN        Supply-demand for firms N: markup rev = fixed cost
DY        Supply-demand balance for Y
DW        Supply-demand balance for utility W (welfare)

LAB       Supply-demand balance for labor

INCOME    National income;

PRICEX..  PL =G= PX*(1-1/SI);
PINDEX..  (N*PX**(1-SI))**(1/(1-SI)) =G= PE;
PRICEN..  PL =G= PN;
PRICEY..  PL =G= PY;
PRICEW..  (PE**0.5)*(PY**0.5) =G= PW;

DX..      X*80 =G= PX**(-SI)*(PE**(SI-1))*CONS/2;
DXC..  \[ \text{XC} = G = N^{*\left(\frac{\text{SI}}{(\text{SI}-1)}\right)}*X; \]

DN..  \[ \text{N*FC} = G = (\text{PX}/\text{SI})*X*80*N/\text{PN}; \]

DY..  \[ \text{Y*100} = G = \text{CONS}/(2*\text{PY}); \]

DW..  \[ 200*\text{W} = G = (1.25^{*0.5})*\text{CONS}/\text{PW}; \]

LAB..  \[ \text{ENDOWL} = E = \text{Y*100} + \text{N*X*80} + \text{N*FC}; \]

INCOME..  \[ \text{CONS} = E = \text{PL*ENDOWL}; \]

MODEL M62 /PRICEX.X, PRICEY.Y, PRICEW.W, PRICEN.N, PINDEX.XC,
\hspace{1cm} \text{DX.PX, DXC.PE, DN.PN, DY.PY, DW.PW,}
\hspace{1cm} \text{LAB.PL, INCOME.CONS/};

* \hspace{1cm} \text{set benchmark values:}

\[ \text{PE.L = 1.25;} \]
\[ \text{CONS.L = 200;} \]
\[ \text{X.L = 1;} \]
\[ \text{XC.L = 1;} \]
\[ \text{Y.L = 1;} \]
\[ \text{N.L = 1;} \]
W.L = 1;
PX.L = 1.25;
PN.L = 1;
PY.L = 1;

PL.L = 1;
PW.L = 1.25**0.5;

* choose the price of good Y as numeraire

PY.FX = 1;

* check for calibration and starting-value errors

M62.ITERLIM = 0;
SOLVE M62 USING MCP;

M62.ITERLIM = 1000;
SOLVE M62 USING MCP;

MODELSTAT = M62.MODELSTAT - 1.;

DISPLAY MODELSTAT;

* Counterfactual: expand the size of the economy
ENDOWL = 400;

SOLVE M62 USING MCP;

* show welfare as a function of the economy's size

SETS I indexes 25 different size levels /I1*I25/;

PARAMETERS
  SIZE(I)
  WELFARE(I)
  WELFCAP(I)
  FIRMSIZE(I)
  FIRMNUMB(I)
  MARKUPM(I)
  RESULTS(I,*)

LOOP (I,)

  SIZE(I) = 5.2 - 0.2*ORD(I);
  ENDOWL = 200*SIZE(I);

SOLVE M62 USING MCP;
WELFARE(I) = W.L;
WELFCAP(I) = WELFARE(I)/SIZE(I);
FIRMSIZE(I) = X.L;
FIRMNUMB(I) = N.L;
MARKUPM(I) = 1/SI;

RESULTS(I, "SIZE") = SIZE(I);
RESULTS(I, "WELFARE") = WELFARE(I);
RESULTS(I, "WELFCAP") = WELFCAP(I);
RESULTS(I, "FIRMSIZE") = FIRMSIZE(I);
RESULTS(I, "FIRMNUMB") = FIRMNUMB(I);
RESULTS(I, "MARKUP") = MARKUPM(I);

DISPLAY RESULTS;

* Write parameter RESULTS to an Excel file TRCOST.XLS,
  * starting in Sheet1,

$LIBINCLUDE XLDUMP RESULTS M7.XLS SHEET1!A31

**Execute_Unload** 'M7.gdx' RESULTS
execute 'gdxxrw.exe M7.gdx par=RESULTS rng=SHEET2!A31'
Market size effects on welfare per capita: large-group monopolistic competition versus free-entry oligopoly

Monopolistic competition

Oligopoly

Initial calibration point
7.5 Monopolistic-competition II: small group

Small-group monopolistic competition. Firms are Bertrand competitors, choosing their price holding the prices of the other firms constant.

The demand for an individual variety i:

\[ X_i = p_i^{-\sigma} \left[ \sum p_j^{1-\sigma} \right]^{-1} M_x \] \[ \sigma = \frac{1}{1 - \alpha}, \quad \alpha = \frac{\sigma - 1}{\sigma} \]

Large-group monopolistic competition assumes that the number of firms is large so that each firm views the term in brackets as a constant. But this is an approximation.
\[ \eta_i = -\frac{p_i}{X_i} \frac{\partial X_i}{\partial p_i} = \sigma - s_i (\sigma - 1) \]

\[ s_i = \frac{p_i X_i}{M_x} = \frac{p_i^{1-\sigma}}{\sum p_j^{1-\sigma}} \]

where \( s_i \) is the share of \( X \)-sector expenditure on good \( i \).

If all firms are identical (e.g., domestic firms that have the same price).

\[ \eta_i = \sigma - \frac{1}{N} (\sigma - 1) \quad \text{markup} = \frac{1}{\sigma - \frac{1}{N} (\sigma - 1)} \]

However, we will have some problem comparing this to our large-group monopolistic-competition model.
This new one would require re-calibrating the whole model, but then we would have a different elasticity of substitution.

An alternative is to use a simple parameter, a “cluge” or “fudge factor” that makes the two consistent.

In our case with $N = 1$ initially, we choose a constant $0.20 = 1/\sigma$ so that the same benchmark data fits the small-group model.

$$markup = \frac{0.20}{\sigma - \frac{1}{N}(\sigma - 1)} = \frac{N/\sigma}{N\sigma - (\sigma - 1)}$$

Now the large-group and small-group versions will both reproduce the benchmark data with $N = 1$. 
There is a problem with this formulation, which occurs when $N < 1$.

This arises due to the modeling of $N$ as a continuous variable when in reality it is discrete and bound from below by 1.

A quick fix is to replace $N$ in the markup formula with $(N+1)$, and then adjust the “fudge” or “cluge” to offset this.

$$\text{markup} = \frac{0.60}{\sigma - \frac{1}{(N + 1)}(\sigma - 1)}$$

Then $\eta \rightarrow 1$ as $N \rightarrow 0$.

This is the second version of the model shown below, and calibrates to the same initial markup, 0.20, as before.
$TITLE: M7-5.GMS: Small-Group Monopolistic Competition
* markup formula is 1/(\sigma - (1/(1+N))(\sigma - 1)
* to calibrate to the same data, \sigma = 5, N = 1, a fudge-factor
* of 0.6 is used in the markup formula to reproduce the benchmark
* markup = 0.6/(\sigma - (1/(1+N))(\sigma - 1)) = 0.20

$ONTEXT

<table>
<thead>
<tr>
<th>Production Sectors</th>
<th>Consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Markets</td>
<td>Xc</td>
</tr>
<tr>
<td>PX</td>
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<td>PY</td>
<td></td>
</tr>
<tr>
<td>PN</td>
<td></td>
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<tr>
<td>PW</td>
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<tr>
<td>PL</td>
<td>-80</td>
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<tr>
<td>MK</td>
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$OFFTEXT

PARAMETERS
SI       SIGMA: elasticity of substitution among varieties
FC       parameter setting the level of fixed costs
ENDOWL   endowment of labor
MODELSTAT statistic indicating model solved: 0 = solved;
SI = 5;
FC = 20;
ENDOWL = 200;

**NONNEGATIVE VARIABLES**

- X: Activity level for X (output per firm)
- XC: Composite X (utility value of agg X sector output)
- N: Number of X sector firms (variety measure)
- Y: Activity level of Y output
- W: Activity level for welfare

- PX: Price of an individual X variety
- PE: Price index (unit expenditure function): cost of XC = 1
- PN: Price of fixed costs (price of entering)
- PY: Price of Y
- PW: Price index for utility (consumer price index)

- PL: Price of labor
- MK: Markup

CONS: Income of the representative consumer;
EQUATIONS

PRICEX  MR = MC in X (associated with X output per firm)
PINDEX  Price index for X sector goods
PRICEN  Zero profits - free entry in X (associated with N)
PRICEY  Zero profit condition for Y (PY = MC)
PRICEW  Zero profit condition for W (PW = MC of utility)

DX  Supply-demand balance for X (individual variety)
DXC  Supply-demand balance for XC
DN  Supply-demand for firms N: markup rev = fixed cost
DY  Supply-demand balance for Y
DW  Supply-demand balance for utility W (welfare)

LAB  Supply-demand balance for unskilled labor

MARKUP  Markup equation

INCOME  National income;

PRICEX..  PL =G= PX*(1 - MK);
PINDEX..  (N*PX**(1-SI))**(1/(1-SI)) =G= PE;
PRICEN..  PL =G= PN;
PRICEY..   PL =G= PY;
PRICEW..   (PE**0.5)*(PY**0.5) =G= PW;
DX..       X*80 =G= PX**(-SI)*(PE**(SI-1))*CONS/2;
DXC..      XC =G= N**((SI/(SI-1)))*X;
DN..       N*FC =G= (PX*MK)*X*80*N/PN;
DY..       Y*100 =G= CONS/(2*PY);
DW..       200*W =G= (1.25**0.5)*CONS/PW;
LAB..      ENDOWL =E= Y*100 + N*X*80 + N*FC;
MARKUP..   MK =E= 0.6/(SI - 1/(N+1)*(SI - 1));
INCOME..   CONS =E= PL*ENDOWL;

MODEL M62 /PRICEX.X, PRICEY.Y, PRICEW.W, PRICEN.N, PINDEX.XC,
        DX.PX, DXC.PE, DN.PN, DY.PY, DW.PW,
        LAB.PL, MARKUP.MK, INCOME.CONS/;
* set benchmark values:

PE.L = 1.25;
CONS.L = 200;
X.L = 1;
XC.L = 1;
Y.L = 1;
N.L = 1;
W.L = 1;
PX.L = 1.25;
PN.L = 1;
PY.L = 1;

PL.L = 1;
PW.L = 1.25**0.5;
MK.L = 0.20;
* choose the price of good Y as numeraire
PY.FX = 1;

* check for calibration and starting-value errors

M62.ITERLIM = 0;
SOLVE M62 USING MCP;

M62.ITERLIM = 1000;
SOLVE M62 USING MCP;

MODELSTAT = M62.MODELSTAT - 1.;

DISPLAY MODELSTAT;

* Counterfactual: expand the size of the economy

*ENDOWL = 400;

*SOLVE M62 USING MCP;

* show welfare as a function of the economy's size

SETS J scenario 1 = small-group mc 2 = large-group /J1*J2/;
SETS I indexes 25 different size levels /I1*I25/;

PARAMETERS
  SIZE(I)
  WELFARE(I,J)
  WELFCAP(I,J)
  MARKUPS(I,J)
  NUMBERF(I,J)
  RESULTS(I,*);
MK.L = 0.2;

LOOP(I,
   LOOP(J,

      SIZE(I) = 5.2 - 0.2*ORD(I);
      ENDOWL = 200*SIZE(I);

      MK.UP = +INF;
      MK.LO = 0;
      MK.FX$(ORD(J) EQ 2) = 0.20;

   SOLVE M62 USING MCP;

   WELFARE(I,J) = W.L;
   WELFCAP(I,J) = WELFARE(I,J)/SIZE(I);
   MARKUPS(I,J) = MK.L;
   NUMBERF(I,J) = N.L;

   );
   );

   RESULTS(I, "SIZE") = SIZE(I);
   RESULTS(I, "WELFCAP-L") = WELFCAP(I, "J2");
   RESULTS(I, "WELFCAP-S") = WELFCAP(I, "J1");
RESULTS(I, "NUMBERF-L") = NUMBERF(I, "J2");
RESULTS(I, "NUMBERF-S") = NUMBERF(I, "J1");
RESULTS(I, "MARKUP-S") = MARKUPS(I,"J1");

**DISPLAY**  RESULTS;

* Write parameter RESULTS to an Excel file MCOMP2.XLS,  
* starting in Sheet1, cell A3

**Execute_Unload**  'M7.gdx'  RESULTS  
**execute**  'gdxxrw.exe M7.gdx par=RESULTS rng=Sheet4!A3'
<table>
<thead>
<tr>
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<th>WELFCAP-S</th>
<th>NUMBERF-L</th>
<th>NUMBERF-S</th>
<th>MARKUP-S</th>
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<td>3.13</td>
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<td>4.00</td>
<td>1.19</td>
<td>1.18</td>
<td>4.00</td>
<td>3.00</td>
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<td>I7</td>
<td>3.80</td>
<td>1.18</td>
<td>1.18</td>
<td>3.80</td>
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