$TITLE: M10-1.GMS: steady state capital stock, comparative steady-states

$ONTENT
"closure rule": instead of the capital stock being fixed (quantity closure), the stock adjusts to satisfy the steady-state relationship between the rental rate and the price of producing new capital (price closure):
  \[ \delta = \text{depreciation rate} \]
  \[ \rho = \text{rate of time preference} \]

rental rate = \( (1 - (1 - \delta)/(1 + \rho)) \) * (price of new capital)

this is done via a subsidy to capital use that creates the wedge
subsidy = \( (1 - \delta)/(1 + \rho) \)

<table>
<thead>
<tr>
<th>Production Sectors</th>
<th>Consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Markets</td>
<td>X</td>
</tr>
<tr>
<td>PX</td>
<td>100</td>
</tr>
<tr>
<td>PY</td>
<td></td>
</tr>
<tr>
<td>PW</td>
<td></td>
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<tr>
<td>PL</td>
<td>-40</td>
</tr>
<tr>
<td>PK</td>
<td>-120</td>
</tr>
<tr>
<td>SUB</td>
<td>60</td>
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</tbody>
</table>
PARAMETERS
RHO          Time preference parameter
DELTA        Depreciation rate
TAU          Effective capital use subsidy
KTAX         Tax on new capital production
NEWCAP       New capital stock after counterfactual (= 1 initially);

RHO     = 0.4;
DELTA   = 0.3;
TAU     = -(1 - DELTA)/(1 + RHO);
KTAX    = 0;

NONNEGATIVE VARIABLES
X            Activity level for sector X
Y            Activity level for sector Y
W            Activity level for sector W (Hicksian welfare index)
K            Capital stock index

PX           Price index for commodity X
PY           Price index for commodity Y
PL           Price index for primary factor L
PK           Price index for primary factor K
PW           Price index for welfare (expenditure function)
CONS    Income definition for CONS
KFORWRD Capital stock from previous period;

EQUATIONS

PRF_X   Zero profit for sector X
PRF_Y   Zero profit for sector Y
PRF_W   Zero profit for sector W (Hicksian welfare index)
PRF_K   Zero profit for capital index

MKT_X   Supply-demand balance for commodity X
MKT_Y   Supply-demand balance for commodity Y
MKT_L   Supply-demand balance for primary factor L
MKT_K   Supply-demand balance for factor K
MKT_W   Supply-demand balance for aggregate demand

I_CONS   Income definition for CONS
A_KFORWRD Auxiliary equation to determine the carry forward;

*       Zero profit conditions:

PRF_X..  100 * PL**0.4 * (PK*(1+TAU)/0.5)**0.6 =G= 100 * PX;
PRF_Y..  100 * PL**0.6 * (PK*(1+TAU)/0.5)**0.4 =G= 100 * PY;
PRF_W..  200 * PX**0.5 * PY**0.5 =G= 200 * PW;
PRF_K..  60*PL  =G= 60 * PK * (1-KTAX);

*       Market clearing conditions:

MKT_X..  100 * X  =G= 100 * W * PW/PX;
MKT_Y..  100 * Y  =G= 100 * W * PW/PY;
MKT_W..  200 * W  =G= CONS / PW;
MKT_L..  160  =G= 60*K + 40* X*PX/PL + 60*Y*PY/PL;
MKT_K..  140*KFORWRD + 60*K  =G= 120 * X * PX*0.5/(PK*(1+TAU)) + 
                               80 * Y * PY*0.5/(PK*(1+TAU));

*       Income constraints:

I_CONS..  CONS  =E= 160 * PL + 140*KFORWRD *PK + 
            PK * TAU * (120 * X * PX*0.5/(PK*(1+TAU)) + 
            80 * Y * PY*0.5/(PK*(1+TAU))) + 
            60* PK * KTAX * K;

*       Auxiliary constraints:

A_KFORWRD..  140*KFORWRD  =E= 60*K * (1-DELTA) / DELTA;

X.L =1;
Y.L =1;
W.L =1;
K.L =1;

PX.L =1;
PY.L =1;
PK.L =1;
PW.FX =1;
PL.L =1;

CONS.L =200;

KFORWRD.L = 1;

ALGEBRAIC.ITERLIM = 0;
SOLVE ALGEBRAIC USING MCP;
ALGEBRAIC.ITERLIM = 2000;
SOLVE ALGEBRAIC USING MCP;

* Raise the rate of time preference from 0.4 to 0.6:
RHO  = 0.6;
TAU  = - (1 - DELTA)/(1 + RHO);

SOLVE  ALGEBRAIC USING MCP;

NEWCAP = K.L/60;
DISPLAY  NEWCAP;

*  Set rho back to 0.4, tax new capital at 0.20

RHO  = 0.4;
TAU  = - (1 - DELTA)/(1 + RHO);

KTAUX = 0.20;

SOLVE  ALGEBRAIC USING MCP;

NEWCAP = K.L/60;
DISPLAY  NEWCAP;