$TITLE: Model M3-4b: Closed TWOxTWO Economy - adds taxes
* adds taxes to model M3-1

$ONTEXT

<table>
<thead>
<tr>
<th>Markets</th>
<th>X</th>
<th>Y</th>
<th>W</th>
<th>CONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PX</td>
<td>100</td>
<td>-100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PY</td>
<td>100</td>
<td>-100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PW</td>
<td></td>
<td>200</td>
<td></td>
<td>-200</td>
</tr>
<tr>
<td>PL</td>
<td>-25</td>
<td>-75</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>PK</td>
<td>-75</td>
<td>-25</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

PX and PY will denote the consumer prices of X and Y
since PX(1+TX) = MC (marginal cost), the producer price of X (MC) is PX/(1+TX) and similarly for Y

$OFFTEXT

PARAMETERS

TX        ad-valorem tax rate for X sector inputs
TY        ad-valorem tax rate for Y sector inputs
LENDOW    labor endowment multiplier
KENDOW    capital endowment multiplier;
TX = 0; TY = 0;
LENDOW = 1;
KENDOW = 1;

**POSITIVE VARIABLES**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>activity level for X production</td>
</tr>
<tr>
<td>Y</td>
<td>activity level for Y production</td>
</tr>
<tr>
<td>W</td>
<td>activity level for the &quot;production&quot; of welfare from X Y</td>
</tr>
<tr>
<td>PX</td>
<td>price of good X</td>
</tr>
<tr>
<td>PY</td>
<td>price of good Y</td>
</tr>
<tr>
<td>PW</td>
<td>price of a unit of welfare (real consumer-price index)</td>
</tr>
<tr>
<td>PL</td>
<td>price of labor</td>
</tr>
<tr>
<td>PK</td>
<td>price of capital</td>
</tr>
</tbody>
</table>

**CONS** income of the representative consumer;

**EQUATIONS**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRF_X</td>
<td>zero profit for sector X</td>
</tr>
<tr>
<td>PRF_Y</td>
<td>zero profit for sector Y</td>
</tr>
<tr>
<td>PRF_W</td>
<td>zero profit for sector W (Hicksian welfare index)</td>
</tr>
<tr>
<td>MKT_X</td>
<td>supply-demand balance for commodity X</td>
</tr>
</tbody>
</table>
MKT_Y  supply-demand balance for commodity Y
MKT_L  supply-demand balance for primary factor L
MKT_K  supply-demand balance for primary factor L
MKT_W  supply-demand balance for aggregate demand

ICONS  income definition for CONS;

*        Zero profit inequalities

PRF_X..  100*(PL**0.25 * PK**0.75) * (1+TX) =G= 100*PX;
PRF_Y..  100*(PL**0.75 * PK**0.25) * (1+TY) =G= 100*PY;
PRF_W..  200*(PX**0.50 * PY**0.50) =G= 200*PW;

*        Market clearance inequalities

MKT_X..  100*X =G= 100 * W * PW / PX;
MKT_Y..  100*Y =G= 100 * W * PW / PY;
MKT_W..  200*W =E= CONS / PW;
MKT_L..  100*LENDOW =G= 25 * X * (PX/(1+TX)) / PL +
               75 * Y * (PY/(1+TY)) / PL;
MKT_K.. 100*KENDOW =G= 75 * X * (PX/(1+TX)) / PK + 
25 * Y * (PY/(1+TY)) / PK;

* Income balance equations (don't forget tax revenue)

I_CONS.. CONS =E= 100*LENDOW*PL + 100*KENDOW*PK + 
TX*100*X*(PX/(1+TX)) + 
TY*100*Y*(PY/(1+TY));

MODEL TWOxTWO /PRF_X.X, PRF_Y.Y, PRF_W.W,
MKT_X.PX, MKT_Y.PY, MKT_L.PL,MKT_K.PK,
MKT_W.PW,I_CONS.CONS /;

* Chose a numeraire: real consumer price index

PW.FX = 1;

* Set initial values of variables:

X.L=1; Y.L=1; W.L=1; PX.L=1; PY.L=1; PK.L=1; PL.L=1; 
CONS.L=200;

SOLVE TWOxTWO USING MCP;
* counterfactual 1: 50% tax on X

TX = 0.5;
*SOLVE TWOxTWO USING MCP;

* counterfactual 1: 50% tax on X and Y

TX = 0.5;
TY = 0.5;
*SOLVE TWOxTWO USING MCP;

* counterfactual 2: zero taxes, double the labor endowment

TX = 0;
TY = 0;
LENDOW = 2;
*SOLVE TWOxTWO USING MCP;

* counterfactual 3: double both endowments from the benchmark

LENDOW = 2;
KENDOW = 2;
*SOLVE TWOxTWO USING MCP;
$ontext
We emphasize that the above formulation uses a simplifying trick: the marginal costs of X, Y, and W can be replaced by the producer prices PX/(1+TX), PY/(1+TY), and PW. This should "always" work because when marginal cost and price are not equal in equilibrium, the quantity is zero: marginal cost times quantity = producer price times quantity (MC_X*X = PX*X regardless of X > 0 or X = 0). But below is the model done "properly" with Shepard's lemma.
$offtext

EQUATIONS
   MKT_X2
   MKT_Y2
   MKT_L2
   MKT_K2
   I_CONS2;

MKT_X2.. 100*X =G= 100 * W * (PX**0.5 * PY**0.5) / PX;
MKT_Y2.. 100*Y =G= 100 * W * (PX**0.5 * PY**0.5) / PY;
MKT_L2.. 100*LENDOW =G= 25 * X * PL**0.25 * PK**0.75 / PL +
             75 * Y * PL**0.75 * PK**0.25 / PL;
MKT_K2..  100*KENDOW =G=  75 * X * PL**0.25 * PK**0.75 / PK +
25 * Y * PL**0.75 * PK**0.25 / PK;

*   Income balance equations (don't forget tax revenue)

I_CONS2..  CONS =E= 100*LENDOW*PL + 100*KENDOW*PK +
 TX*100*X*(PL**0.25*PK**0.75) +
 TY*100*Y*(PL**0.75*PK**0.25);

MODEL TWOxTWOa /PRF_X.X, PRF_Y.Y, PRF_W.W,
 MKT_X2.PX, MKT_Y2.PY, MKT_L2.PL, MKT_K2.PK,
 MKT_W.PW, I_CONS2.CONS /;

TX = 0;  TY = 0;
LENDOW = 1;  KENDOW = 1;

SOLVE TWOxTWOa USING MCP;

TX = 0.5;
SOLVE TWOxTWOa USING MCP;

$ontext
Exercise: declare a parameter alpha, which is a productivity shift parameter producing X. Higher alpha, more output per input.
Code this up. Hint: alpha will appear more than in the program. Change alpha and interpret results. $offtext