$TITLE: M4-2.GMS: Find all pure-strategy Nash equilibrium
* with discrete strategy sets

$ONText
two firms, one in country h and one in country f
each firm chooses one of three strategies:
  don't enter: strategy 0
  enter with a single plant and export to the other country: strategy 1
  enter with plants in both countries (horizontal multinational)
    strategy 2
$OFFText

SETS  R strategies for firm h /SH0, SH1, SH2/ 
      C strategies for firm f /SF0, SF1, SF2/;

ALIAS (R,RR)
ALIAS (C,CC);

PARAMETERS
  ROWMAX(R,C)  maximum value over the rows for a given column C
  COLMAX(R,C)  maximum value over the columns for a given row R
  NASHEQ(R,C)  matrix of 0-1 where 1 is a Nash equilibrium
  PROFHNE(R,C) profit of firm h in Nash equilibrium
  PROFFNE(R,C) profit of firm f in Nash equilibrium;

* small maintenance costs -0.1 when not entering, not needed
TABLE  PAYOFFH(*,*)
    SF0  SF1  SF2
SH0  -.1  -.1  -.1
SH1   10    6    3
SH2   12    5    2 ;

TABLE  PAYOFFF(*,*)
    SF0  SF1  SF2
SH0  -.1   10   12
SH1  -.1    6    5
SH2  -.1    3    2 ;

DISPLAY  PAYOFFH, PAYOFFF;

ROWMAX(R,C) = 1$(PAYOFFH(R,C) EQ SMAX(RR, PAYOFFH(RR,C))) ;
COLMAX(R,C) = 1$(PAYOFFF(R,C) EQ SMAX(CC, PAYOFFF(R,CC))) ;

DISPLAY  ROWMAX, COLMAX;

NASHEQ(R,C) = ROWMAX(R,C) * COLMAX(R,C) ;

DISPLAY  NASHEQ;

PROFHNE(R,C) = PAYOFFH(R,C) $NASHEQ(R,C) ;
PROFFNE(R,C) = PAYOFFF(R,C) \$NASHEQ(R,C);

DISPLAY PROFHNE, PROFFNE;

*CASE 2: MARKETS TOO SMALL FOR A FIRM TO ENTER AGAINST A TWO-PLANT RIVAL
*subtract 4 from each payoff strategies 1 and 2

TABLE PAYOFFH2(*,*)
    SF0   SF1   SF2
SH0  -1   -1   -1
SH1   6    2   -1
SH2   8    1   -2 ;

TABLE PAYOFFF2(*,*)
    SF0   SF1   SF2
SH0  -1    6    8
SH1  -1    2    1
SH2  -1   -1   -2 ;

ROWMAX(R,C) = 1$(PAYOFFH2(R,C) EQ SMAX(RR, PAYOFFH2(RR,C)))
COLMAX(R,C) = 1$(PAYOFFF2(R,C) EQ SMAX(CC, PAYOFFF2(R,CC)))

NASHEQ(R,C) = ROWMAX(R,C) * COLMAX(R,C);

DISPLAY NASHEQ;
PROFHNE(R,C) = PAYOFFH2(R,C)$NASHEQ(R,C);  
PROFFNE(R,C) = PAYOFFF2(R,C)$NASHEQ(R,C);

DISPLAY PROFHNE, PROFFNE;

*CASE 3: LOWER FIRM FIXED COSTS, RAISE PLANT FIXED COSTS
*makes two-plant production more profitable
*add 2 when playing strategy 2

TABLE PAYOFFH3(*,*)

<table>
<thead>
<tr>
<th></th>
<th>SF0</th>
<th>SF1</th>
<th>SF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>SH0</td>
<td>-.1</td>
<td>-.1</td>
<td>-.1</td>
</tr>
<tr>
<td>SH1</td>
<td>10</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>SH2</td>
<td>14</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

TABLE PAYOFFF3(*,*)

<table>
<thead>
<tr>
<th></th>
<th>SF0</th>
<th>SF1</th>
<th>SF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>SH0</td>
<td>-.1</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>SH1</td>
<td>-.1</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>SH2</td>
<td>-.1</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

ROWMAX(R,C) = 1$(PAYOFFH3(R,C) EQ SMAX(RR, PAYOFFH3(RR,C)));  
COLMAX(R,C) = 1$(PAYOFFF3(R,C) EQ SMAX(CC, PAYOFFF3(R,CC)));
NASHEQ(R, C) = ROWMAX(R, C) * COLMAX(R, C);

DISPLAY NASHEQ;

PROFHNE(R, C) = PAYOFFH3(R, C)$NASHEQ(R, C);
PROFFNE(R, C) = PAYOFFF3(R, C)$NASHEQ(R, C);

DISPLAY PROFHNE, PROFFNE;