$TITLE Model M6-1: 2x2 (two goods, two factors) benchmark taxes
* Positive tax in the X sector in the benchmark

$ONTEXT

Production Sectors       Consumers

<table>
<thead>
<tr>
<th>Markets</th>
<th>X</th>
<th>Y</th>
<th>W</th>
<th>CONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PX</td>
<td>100</td>
<td>0</td>
<td>-100</td>
<td></td>
</tr>
<tr>
<td>PY</td>
<td>0</td>
<td>100</td>
<td>-100</td>
<td></td>
</tr>
<tr>
<td>PW</td>
<td></td>
<td>0</td>
<td>200</td>
<td>-200</td>
</tr>
<tr>
<td>PL</td>
<td>-20</td>
<td>-60</td>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>PK</td>
<td>-60</td>
<td>-40</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>TAX</td>
<td>-20</td>
<td>0</td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

Assume that this is a 100% tax on labor in X: TLX = 1.
Let the CONSUMER price (wage) of labor equal 1: PL = 1.
The PRODUCER price (cost) of labor in X is equal to 2:
\[ PL \times (1 + TLX) = 2 \]

$OFFTEXT

SCALAR

TX     Proportional output tax on sector X,
TY     Proportional output tax on sector Y,
TLX    Ad-valorem tax on labor inputs to X,
TKX  Ad-valorem tax on capital inputs to X
TAXREV  Total tax revenue from all sources;

**POSITIVE VARIABLES**

X  Activity level for sector X
Y  Activity level for sector Y
W  Activity level for sector W
PX  Price index for commodity X
PY  Price index for commodity Y
PL  Price index for primary factor L
PK  Price index for primary factor K
PW  Price index for welfare (expenditure function)
CONS  Income definition for CONS
PPLX  Producer price for L in X
PPKX  Producer price for K in X
PPX  Producer price for X
PPY  Producer price for Y;

**EQUATIONS**

PRF_X  Zero profit for sector X
PRF_Y  Zero profit for sector Y
PRF_W  Zero profit for sector W
MKT_X  Supply-demand balance for commodity X
MKT_Y   Supply-demand balance for commodity Y
MKT_L   Supply-demand balance for primary factor L
MKT_K   Supply-demand balance for primary factor L
MKT_W   Supply-demand balance for aggregate demand

I_CONS  Income definition for CONS

RPPLX   Relation between consumer and producer price L in X
RPPKX   Relation between consumer and producer price K in X
RPPX    Relationship between producer and consumer price of X
RPPY    Relationship between producer and consumer price of Y;

*       Zero profit conditions:

PRF_X..  100*(PPLX/2)**0.4 * (PPKX)**0.6 =G= 100*PPX;
PRF_Y..  100*PL**0.6 * PK**0.4 =G= 100*PPY;
PRF_W..  200*PX**0.5 * PY**0.5 =G= 200*PW;

*       Market clearing conditions:

MKT_X..  100*X =G= 100*W*PW/PX;
MKT_Y..  100*Y =G= 100*W*PW/PY;
MKT_W.. 200*W =G= CONS/PW;

MKT_L.. 80 =G= 20*X*PPX/(PPLX/2) + 60*Y*PPY/PL;

MKT_K.. 100 =G= 60*X*PPX/PPKX + 40*Y*PPY/PK;

* Income constraints:

I_CONS.. CONS =E= 80*PL + 100*PK + 100*PX*X*TX + 100*PY*Y*TY +
TLX*PL*20*X* PPX /(PPLX/2) +
TKX*PK*60*X* PPX /(PPKX);

RPPLX.. PPLX =E= PL*(1+TLX);
RPPKX.. PPKX =E= PK*(1+TKX);
RPPX.. PPX =E= PX*(1-TX);
RPPY.. PPY =E= PY*(1-TY);

MODEL BENCHTAX /PRF_X.X, PRF_Y.Y, PRF_W.W,
MKT_X.PX, MKT_Y.PY, MKT_L.PL, MKT_K.PK,
MKT_W.PW, I_CONS.CONS,
RPPLX.PPLX, RPPKX.PPKX, RPPX.PPX, RPPY.PPY /;

X.L =1;
Y.L =1;
W.L =1;
PL.L = 1;
PX.L = 1;
PY.L = 1;
PK.L = 1;
PW.FX = 1;
PPLX.L = 2;
PPKX.L = 1;
PPX.L = 1;
PPY.L = 1;

CONS.L = 200;

TX = 0;
TY = 0;
TLX = 1;
TKX = 0;

BENCHTAX.ITERLIM = 0;
SOLVE BENCHTAX USING MCP;

BENCHTAX.ITERLIM = 1000;
SOLVE BENCHTAX USING MCP;

TAXREV = 100*PX.L*X.L*TX + 100*PY.L*Y.L*TY +
         TLX*PL.L*20*X.L* PPX.L / (PPLX.L/2) +
         TKX*PK.L*60*X.L* PPX.L / (PPKX.L);
DISPLAY TAXREV;

* In the first counterfactual, we replace the tax on
* labor inputs by a uniform tax on both factors:

TLX = 0.25;
TKX = 0.25;
TX = 0;
TY = 0;

SOLVE BENCHTAX USING MCP;

TAXREV = 100*PX.L*X.L*TX + 100*PY.L*Y.L*TY +
    TLX*PL.L*20*X.L* PPX.L /(PPLX.L/2) +
    TKX*PK.L*60*X.L* PPX.L /(PPKX.L);

DISPLAY TAXREV;

* Now demonstrate that a 25% tax on all inputs
* is equivalent to a
* 20% tax on the output (or all outputs if more than one)

TLX = 0;
TKX = 0;
TX = 0.2;
TY = 0;
SOLVE BENCHTAX USING MCP;

TAXREV = 100*PX.L*X.L*TX + 100*PY.L*Y.L*TY +
        TLX*PL.L*20*X.L* PPX.L /(PPLX.L/2) +
        TKX*PK.L*60*X.L* PPX.L /(PPKX.L);

DISPLAY TAXREV;

* Demonstrate that a 20% tax on the X sector output is
* equivalent to a 25% subsidy on Y sector output
* (assumes that the funds for the subsidy can be raised
  lump sum from the consumer!)

TKX = 0;
TLX = 0;
TX = 0;
TY = -0.25;

SOLVE BENCHTAX USING MCP;
TAXREV = 100*PX.L*X.L*TX + 100*PY.L*Y.L*TY +
        TLX*PL.L*20*X.L* PPX.L /(PPLX.L/2) +
        TKX*PK.L*60*X.L* PPX.L /(PPKX.L);

DISPLAY TAXREV;

* Show welfare under non-distortionary taxation

TX = 0.20;
TY = 0.20;

**SOLVE** BENCHTAX USING MCP;
TAXREV = 100*PX.L*X.L*TX + 100*PY.L*Y.L*TY +
   TLX*PL.L*20*X.L* PPX.L / (PPLX.L/2) +
   TKX*PK.L*60*X.L* PPX.L / (PPKX.L);
**DISPLAY** TAXREV;

TX = 0.0;
TY = 0.0;

**SOLVE** BENCHTAX USING MCP;
TAXREV = 100*PX.L*X.L*TX + 100*PY.L*Y.L*TY +
   TLX*PL.L*20*X.L* PPX.L / (PPLX.L/2) +
   TKX*PK.L*60*X.L* PPX.L / (PPKX.L);
**DISPLAY** TAXREV;