$TITLE M6-2a.GMS: 2x2 Economy with labor supply and income tax

$ONTEXT

Production Sectors | Consumers

<table>
<thead>
<tr>
<th>Markets</th>
<th>X</th>
<th>Y</th>
<th>W</th>
<th>TL</th>
<th>TK</th>
<th>CONS</th>
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<tbody>
<tr>
<td>PX</td>
<td>120</td>
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<td>-120</td>
<td></td>
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<tr>
<td>PY</td>
<td></td>
<td>120</td>
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<td>-120</td>
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<td>-72</td>
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<td>-40</td>
<td>80</td>
</tr>
</tbody>
</table>

$OFFTEXT

SETS S /1*6/;

PARAMETERS

TXL Labor income tax rate,
TXK Capital income tax rate,
WELFARE(S) Welfare,
LABSUP(S) Labor supply
INCOME(S) Money income = consumption of X and Y
CAPTAX(S)  The level of the capital tax
TAXREV(S)  Tax revenue generated;

POSITIVE VARIABLES

X       Activity level for sector X
Y       Activity level for sector Y
TL      Supply activity for L
TK      Supply activity for K
W       Activity level for sector W
PX      Price index for commodity X
PY      Price index for commodity Y
PL      Price index for primary factor L net of tax
PK      Price index for primary factor K net of tax
PLS     Price index for primary factor L gross of tax
PKS     Price index for primary factor K gross of tax
PW      Price index for welfare (expenditure function)
CONS    Income definition for CONS;

EQUATIONS

PRF_X   Zero profit for sector X
PRF_Y   Zero profit for sector Y
PRF_TL  Zero profit for sector TL
PRF_TK  Zero profit for sector TK
PRF_W   Zero profit for sector W

MKT_X   Supply-demand balance for commodity X
MKT_TK  Supply-demand balance for commodity TK
MKT_TL  Supply-demand balance for commodity TL
MKT_Y   Supply-demand balance for commodity Y
MKT_L   Supply-demand balance for primary factor L
MKT_K   Supply-demand balance for primary factor K
MKT_W   Supply-demand balance for aggregate demand

I_CONS  Income definition for CONS;

* Zero profit conditions:

PRF_X.. 80*PLS**0.4 * PKS**0.6 =G= 120*PX;
PRF_Y.. 80*PLS**0.6 * PKS**0.4 =G= 120*PY;
PRF_TL.. 80*PL*(1+TXL)   =G= 80*PLS;
PRF_TK.. 80*PK*(1+TXK)   =G= 80*PKS;
        =G= 340 * PW;
* Market clearing conditions:

MKT_X.. 120*X =G= 340*W*PW * (12/34)/PX;

MKT_Y.. 120*Y =G= 340*W*PW * (12/34)/PY;

MKT_W.. 340*W =G= CONS / PW;

MKT_L.. 180 =G= 80*TL + 340*W*(10/34)*(PW/PL);

MKT_K.. 80 =G= 80*TK;

MKT_TL.. 80*TL =G= 48*X*PX/PLS + 72*Y*PY/PLS;

MKT_TK.. 80*TK =G= 72*Y*PY/PKS + 48*X*PX/PKS;

* Income constraints:

I_CONS.. CONS =E= 180*PL + 80*PK + 80*TL*TXL*PL + 80*TK*TXK*PK;

MODEL INCOMETAX /PRF_X.X, PRF_Y.Y, PRF_TK.TK, PRF_TL.TL,
       PRF_W.W, MKT_X.PX, MKT_Y.PY, MKT_L.PL,
       MKT_TK.PKS, MKT_TL.PLS,
       MKT_K.PK, MKT_W.PW, I_CONS.CONS /;

X.L =1;
Y.L  =1;
TK.L =1;
TL.L =1;
W.L  =1;
PL.L =1;
PX.L =1;
PY.L =1;
PLS.L =1.5;
PKS.L =1.5;
PK.L =1;
PW.FX =1;
CONS.L =340;

TXL =0.5;
TXK =0.5;

INCOMETAX.ITERLIM = 0;
SOLVE INCOMETAX USING MCP;

*       Lets do some counter-factual with taxes shifted to the
*       factor which is in fixed supply:

INCOMETAX.ITERLIM = 1000;
SOLVE INCOMETAX USING MCP;
LOOP(S, \\

TXL = 0.60 - 0.10*ORD(S);  
TXK = 0.40 + 0.10*ORD(S);  

SOLVE INCOMETAX USING MCP;  

WELFARE(S) = W.L;  
LABSUP(S) = TL.L;  
INCOME(S) = ((PX.L/1.5)*X.L + (PY.L/1.5)*Y.L) 
/((PX.L/1.5)**0.5*(PY.L/1.5)**0.5)/2;  
CAPTAX(S) = TXK;  
TAXREV(S) = (TXL*PL.L*TL.L*80 + TXK*PK.L*TK.L*80) 
/((PX.L/1.5)**0.5*(PY.L/1.5)**0.5);  

);  

DISPLAY WELFARE, LABSUP, INCOME, CAPTAX, TAXREV;  

PARAMETER  
  RESULTS(S, *);  

RESULTS(S, "WELFARE") = WELFARE(S);  
RESULTS(S, "LABSUP") = LABSUP(S);  
RESULTS(S, "TAXREV") = TAXREV(S);
DISPLAY RESULTS;

TXL = 0;
TXK = 0;

SOLVE INCOMETAX USING MCP;