$TITLE M6-2b.GMS: 2x2 Economy with income tax, endogenous tax rate
*       adds equal yield tax reform to model M6-2a

$ONTEXT
Illustrates equal yield tax reform to introduce auxiliary variable and constraint equation
Distortionary labor tax is lowered and capital tax raised endogenously (TXK is now a VARIABLE) to hold revenue constant

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$OFFTEXT

SETS S /1*6/;
PARAMETERS

T XL         Labor income tax rate
WELFARE(S)  Welfare
LABSUP(S)   Labor supply
INCOME(S)   Money income = consumption of X and Y
CAPTAX(S)   Endogenous capital tax for equal yield
TAXREV(S)   Tax revenue in terms of purchasing power;

POSITIVE VARIABLES

X       Activity level for sector X
Y       Activity level for sector Y
TL      Supply activity for L
TK      Supply activity for K
W       Activity level for sector W
PX      Price index for commodity X
PY      Price index for commodity Y
PL      Price index for primary factor L net of tax
PK      Price index for primary factor K net of tax
PLS     Price index for primary factor L gross of tax
PKS     Price index for primary factor K gross of tax
PW      Price index for welfare (expenditure function)
CONS    Income definition for CONS
TXK     Endogenous capital tax from equal yield constraint;
EQUATIONS

PRF_X   Zero profit for sector X
PRF_Y   Zero profit for sector Y
PRF_TL  Zero profit for sector TL
PRF_TK  Zero profit for sector TK
PRF_W   Zero profit for sector W

MKT_X   Supply-demand balance for commodity X
MKT_TK  Supply-demand balance for commodity TK
MKT_TL  Supply-demand balance for commodity TL
MKT_Y   Supply-demand balance for commodity Y
MKT_L   Supply-demand balance for primary factor L
MKT_K   Supply-demand balance for primary factor K
MKT_W   Supply-demand balance for aggregate demand

I_CONS  Income definition for CONS
A_TXK   Auxiliary eq associated with equal yield constraint;

*  Zero profit conditions:

PRF_X..  80*PLS**0.4 * PKS**0.6 =G= 120*PX;

PRF_Y..  80*PLS**0.6 * PKS**0.4 =G= 120*PY;

PRF_TL.. 80*PL*(1+TXL) =G= 80*PLS;
PRF_TK.. 80*PK*(1+TXK) =G= 80*PKS;


* Market clearing conditions:

MKT_X.. 120*X =G= 340*W*PW * (12/34)/PX;

MKT_Y.. 120*Y =G= 340*W*PW * (12/34)/PY;

MKT_W.. 340*W =G= CONS / PW;

MKT_L.. 180 =G= 80*TL + 340*W*(10/34)*(PW/PL);

MKT_K.. 80 =G= 80*TK;

MKT_TL.. 80*TL =G= 48*X*PX/PLS + 72*Y*PY/PLS;

MKT_TK.. 80*TK =G= 72*Y*PY/PKS + 48*X*PX/PKS;

* Income constraints:

I_CONS.. CONS =E= 180*PL + 80*PK + 80*TL*TXL*PL + 80*TK*TXK*PK;
\[ A_{TXK} = TXL \times PL \times TL \times 80 + TXK \times PK \times TK \times 80 = \frac{80}{\sqrt{P_X} \times \sqrt{P_Y}}; \]


\[ \begin{align*} X.L & = 1; \\
Y.L & = 1; \\
TK.L & = 1; \\
TL.L & = 1; \\
W.L & = 1; \\
PL.L & = 1; \\
PX.L & = 1; \\
PY.L & = 1; \\
PLS.L & = 1.5; \\
PKS.L & = 1.5; \\
PK.L & = 1; \\
PW.FX & = 1; \\
CONS.L & = 340; \\
TXL & = 0.5; \\
TXK.L & = 0.5; \end{align*} \]
ALGEBRAIC.ITERLIM = 0;
SOLVE ALGEBRAIC USING MCP;

*     Lets do some counter-factual with taxes shifted to the
*     factor which is in fixed supply:

ALGEBRAIC.ITERLIM = 1000;
SOLVE ALGEBRAIC USING MCP;

LOOP (S,

TXL = 0.60 - 0.10*ORD(S);

SOLVE ALGEBRAIC USING MCP;

WELFARE(S) = W.L;
LABSUP(S) = TL.L;
INCOME(S) = (PX.L*X.L + PY.L*Y.L)
            /(PX.L**0.5*PY.L**0.5)/2;
CAPTAX(S) = TXK.L;

TAXREV(S) = (TXL*PL.L*TL.L*80 + TXK.L*PK.L*TK.L*80)
            /(PX.L**0.5*PY.L**0.5));

DISPLAY WELFARE, LABSUP, INCOME, CAPTAX, TAXREV;
TXL = 0;
TXK.FX = 0;

**SOLVE** ALGEBRAIC USING MCP;