$TITLE M6-4.GMS: Economy with two consumers, public good,  
* optimal provision with an endogenous tax rate, Samuelson rule

$ONTEXT
Samuelson rule for optimal provision, PG = PG1 + PG2 
introduces an auxiliary variable and constraint equation
Here is the tax rate is a VARIABLE, set optimally
Generalizes M6-3.gms: two consumers with different preferences

<table>
<thead>
<tr>
<th>Production Sectors</th>
<th>Consumers</th>
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<tbody>
<tr>
<td>Markets</td>
<td>X   Y   G</td>
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<td>---------------------</td>
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</tr>
<tr>
<td>PX</td>
<td>100</td>
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<tr>
<td>PY</td>
<td>100</td>
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<tr>
<td>PG</td>
<td>50</td>
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<tr>
<td>PL</td>
<td>-80</td>
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<tr>
<td>TAX</td>
<td>-20</td>
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<tr>
<td>PW1</td>
<td></td>
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<tr>
<td>PW2</td>
<td></td>
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<tr>
<td>PG1</td>
<td></td>
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<td>PG2</td>
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</tbody>
</table>

$OFFTEXT
PARAMETERS
SHX1, SHY1, SHG1 shares of X Y and G in consumer 1's utility
SHX2, SHY2, SHG2 shares of X Y and G in consumer 2's utility;

SHG1 = 0.2;
SHX1 = 0.5 - SHG1/2;
SHY1 = 0.5 - SHG1/2;
SHG2 = 0.2;
SHX2 = 0.5 - SHG2/2;
SHY2 = 0.5 - SHG2/2;

POSITIVE VARIABLES
X Activity level for sector X,
Y Activity level for sector Y,
W1 Activity level for sector W1,
W2 Activity level for sector W2,
G Activity level for government sector,

PX Price index for commodity X,
PY Price index for commodity Y,
PL Price index for primary factor L,
PW1 Price index for welfare 1(expenditure function),
PW2 Price index for welfare 2(expenditure function),
PG1 Private valuation of the public good (consumer 1),
PG2 Private valuation of the public good (consumer 2),
PG      Price (marginal cost) of the public good

GOVT    Budget restriction for government,
CONS1   Income definition for CONS1,
CONS2   Income definition for CONS2,

LGP     Level of government provision
TAX     Uniform value-added tax rate;

EQUATIONS

PRF_X   Zero profit for sector X
PRF_Y   Zero profit for sector Y
PRF_W1  Zero profit for sector W1
PRF_W2  Zero profit for sector W2
PRF_G   Zero profit in government sector

MKT_X   Supply-demand balance for commodity X
MKT_Y   Supply-demand balance for commodity Y
MKT_L   Supply-demand balance for primary factor L
MKT_W1  Supply-demand balance for consumer 1
MKT_W2  Supply-demand balance for consumer 2
MKT_G1  Private valuation of the public good (consumer 1)
MKT_G2  Private valuation of the public good (consumer 2)
MKT_G   Supply-demand balance for commodity G

I_G     Budget restriction for government
I.Cons1 Income definition for CONS1
I.Cons2 Income definition for CONS2

A.LGP Auxiliary for government provision
A.TAX Auxiliary for government provision;

* Zero profit conditions:

PRF_X.. 80*PL *(1+TAX) =G= 100*PX;

PRF_Y.. 80*PL *(1+TAX) =G= 100*PY;

PRF_W1.. 125*PX**(SHX1) * PY**(SHY1) * (PG1/0.5)**(SHG1) =E= 125*PW1;

PRF_W2.. 125*PX**(SHX2) * PY**(SHY2) * (PG2/0.5)**(SHG2) =E= 125*PW2;

PRF_G.. 40*PL *(1+TAX) =G= 50*PG;

* Market clearing conditions:

MKT_X.. 100*X =G= 125*SHX1*W1*PW1/PX + 125*SHX2*W2*PW2/PX ;

MKT_Y.. 100*Y =G= 125*SHY1*W1*PW1/PY + 125*SHY2*W2*PW2/PY;
MKT_W1.. 125*W1 =G= CONS1/PW1;

MKT_W2.. 125*W2 =G= CONS2/PW2;

MKT_L.. 200 =G= (80*X + 80*Y + 40*G);

MKT_G1.. 50 * LGP =G= 125*SHG1 * W1 * PW1/PG1;

MKT_G2.. 50 * LGP =G= 125*SHG2 * W2 * PW2/PG2;

MKT_G.. 50*G =G= GOVT/ PG;

* Income constraints:

I_G.. GOVT =G= PL*(80*X + 80*Y + 40*G)*TAX;

I_CONS1.. CONS1 =E= 100*PL + 50*LGP*PG1;

I_CONS2.. CONS2 =E= 100*PL + 50*LGP*PG2;

* Auxiliary constraints:

A_LGP.. LGP =E= G;

A_TAX.. PG =E= PG1 + PG2;
MODEL PUBGOOD2 /PRF_X.X, PRF_Y.Y, PRF_W1.W1, PRF_W2.W2,
  PRF_G.G,
  MKT_X.PX, MKT_Y.PY, MKT_L.PL,
  MKT_W1.PW1, MKT_W2.PW2,
  MKT_G.PG, MKT_G1.PG1, MKT_G2.PG2,
  I_G.GOVT, I_CONS1.CONS1, I_CONS2.CONS2,
  A_LGP.LGP, A_TAX.TAX /;

  X.L = 1;
  Y.L = 1;
  W1.L = 1;
  W2.L = 1;
  G.L = 1;

  PL.FX = 1;
  PX.L = 1;
  PY.L = 1;

  PG.L = 1;
  PW1.L = 1;
  PW2.L = 1;
  PG1.L = 0.5;
  PG2.L = 0.5;

  CONS1.L = 125;
CONS2.L =125;
GOVT.L   =50;

LGP.L   =1;
TAX.L   =0.25;

PUBGOOD2.ITERLIM = 0;
SOLVE PUBGOOD2 USING MCP;
PUBGOOD2.ITERLIM = 2000;
SOLVE PUBGOOD2 USING MCP;

*   Change consumer 1's preferences, higher preference for the
*   public good, which now has a Cobb-Douglas share of 0.3

SHG1 = 0.3;
SHX1 = 0.5 - SHG1/2;
SHY1 = 0.5 - SHG1/2;

*PUBGOOD2.ITERLIM = 0;
SOLVE PUBGOOD2 USING MCP;

TAX.FX = 0.25;

SOLVE PUBGOOD2 USING MCP;