$TITLE: M6-6b.GMS: Pollution modelled as an MPEC to solve for optimal TX

$ONTEXT

Follows from M6-5a: two goods and one factor, one consumer
Pollution is generated by the production of X, pollution reduces utility
Pollution is modeled as a reduction in the endowment of CLEAN AIR
Initial endowment of clear air is 200, with 100 reduced by X pollution and 100 entering utility.
Solves for the welfare maximizing level of the pollution tax

<table>
<thead>
<tr>
<th>Markets</th>
<th>Production Sectors</th>
<th>Consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td>PX</td>
<td>100</td>
<td>-100</td>
</tr>
<tr>
<td>PY</td>
<td>100</td>
<td>-100</td>
</tr>
<tr>
<td>PW</td>
<td>300</td>
<td>-300</td>
</tr>
<tr>
<td>PL</td>
<td>-100 -100</td>
<td></td>
</tr>
<tr>
<td>PCA</td>
<td>-100</td>
<td>(200 - 100)</td>
</tr>
</tbody>
</table>

$OFFTEXT

$PARAMETERS

POLINT  polution intensity multiplier;
POLINT = 1;

VARIABLES
  WELFARE welfare
  TX pollution tax on X;

POSITIVE VARIABLES
  X activity level for X production
  Y activity level for Y production
  W activity level for the "production" of welfare from X Y
  PX price of good X
  PY price of good Y
  PCA price of clean air
  PW price of a unit of welfare (real consumer-price index)
  PL price of labor
  CONS income of the representative consumer
  POL pollution;

EQUATIONS
  OBJ Objective function: maximize welfare
  PRF_X zero profit for sector X
  PRF_Y zero profit for sector Y
PRF_W  zero profit for sector W (Hicksian welfare index)

MKT_X  supply-demand balance for commodity X
MKT_Y  supply-demand balance for commodity Y
MKT_CA market for clean air (determines shadow value PCA)
MKT_L  supply-demand balance for primary factor L
MKT_W  supply-demand balance for aggregate demand

I_CONS income definition for CONS

PPOL pollution caused by production - consumption of X;

* Zero profit inequalities

OBJ..   WELFARE =E= W;

PRF_X..  100*PL*(1+TX) =G= 100*PX;

PRF_Y..  100*PL =G= 100*PY;

PRF_W..  200*(PX**(1/3) * PY**(1/3) * PCA**(1/3)) =G= 200*PW;

* Market clearance inequalities

MKT_X..  100*X =G= 100 * W * PW / PX;

MKT_Y..  100*Y =G= 100 * W * PW / PY;
MKT_CA.. 200 - 100*POL =G= 100 * W * PW / PCA;

MKT_W.. 300*W =E= CONS / PW;

MKT_L.. 200 =G= 100*X + 100*Y;

* Income balance equations (don't forget tax revenue)

I_CONS.. CONS =E= 200*PL + (200-100*POL)*PCA + TX*100*X*PL;

PPOL.. 100*POL =G= POLINT*100*X;

MODEL POLLUTE / OBJ, PRF_X.X, PRF_Y.Y, PRF_W.W,
                    MKT_X.PX, MKT_Y.PY, MKT_CA.PCA, MKT_L.PL,
                    MKT_W.PW, I_CONS.CONS, PPOL.POL /;

* Chose a numeraire: real consumer price index

PW.FX = 1;

* Set initial values of variables:

X.L=1; Y.L=1; W.L=1;
PX.L=1; PY.L=1; PL.L=1; POL.L = 1; PCA.L = 1;
CONS.L=300; WELFARE.L = 1;

**OPTION** MPEC = nlpec;
POLLUTE.ITERLIM = 0;
**SOLVE** POLLUTE USING MPEC MAXIMIZING WELFARE;

TX.L = 0.3;
WELFARE.L = 1.2;

POLLUTE.ITERLIM = 1000;
**SOLVE** POLLUTE USING MPEC MAXIMIZING WELFARE;

* make pollution worse

POLINT = 1.5;

**SOLVE** POLLUTE USING MPEC MAXIMIZING WELFARE;