$TITLE M6-6c.GMS: Pollution tax set optimally via a
 * "first-order condition"
 * TX is set by an equation equation the price of X to it's full cost:
 * \( PX = PL + PCA \)

$ONTEXT
This model is a closed economy: two goods and one factor, one consumer
Pollution is generated by the production of X, pollution reduces utility
Pollution is modeled as a reduction in the endowment of CLEAN AIR
Initial endowment of clear air is 200, with 100 reduced by X pollution
and 100 entering utility.

\[
\begin{array}{l|rl|l}
\text{Production Sectors} & \text{Consumers} \\
\hline
\text{Markets} & X & Y & W & \text{CONS} \\
\hline
PX & 100 & & -100 & \\
PY & & 100 & -100 & \\
PW & & 300 & & -300 \\
PL & -100 & -100 & & 200 \\
\hline
PCA & -100 & & (200 - 100) \\
\end{array}
\]

$OFFTEXT
PARAMETERS

POLINT  pollution intensity multiplier
WELOPT  welfare under the optimal tax
TAUOPT  value of the optimal tax;

POLINT = 1;

NONNEGATIVE VARIABLES

X  activity level for X production
Y  activity level for Y production
W  activity level for the "production" of welfare from X Y

PX  price of good X
PY  price of good Y
PCA price of clean air
PW  price of a unit of welfare (real consumer-price index)
PL  price of labor

CONS income of the representative consumer
POL pollution
TX  pollution tax;
EQUATIONS

PRF_X  zero profit for sector X
PRF_Y  zero profit for sector Y
PRF_W  zero profit for sector W (Hicksian welfare index)

MKT_X  supply-demand balance for commodity X
MKT_Y  supply-demand balance for commodity Y
MKT_CA market for clean air (determines shadow value PCA)
MKT_L  supply-demand balance for primary factor L
MKT_W  supply-demand balance for aggregate demand

I_CONS income definition for CONS
PPOL pollution caused by production - consumption of X
ATX sets pollution tax optimally;

* Zero profit inequalities

PRF_X..  100*PL*(1+TX) =G= 100*PX;
PRF_Y..  100*PL =G= 100*PY;
PRF_W..  200*(PX**(1/3) * PY**(1/3) * PCA**(1/3)) =G= 200*PW;

* Market clearance inequalities
MKT_X.. 100*X =G= 100 * W * PW / PX;
MKT_Y.. 100*Y =G= 100 * W * PW / PY;
MKT_CA.. 200-100*POL =G= 100 * W * PW / PCA;
MKT_W.. 300*W =E= CONS / PW;
MKT_L.. 200 =G= 100*X + 100*Y;

* Income balance equations (don't forget tax revenue)

I_CONS.. CONS =E= 200*PL + (200-100*POL)*PCA + TX*100*X*PL;
PPOL.. 100*POL =G= POLINT*100*X;
ATX.. PX =E= PL + PCA*POLINT;

* or since PX = PL*(1 + TX), equivalently

*ATX.. TX =E= PCA*POLINT / PL;

MODEL ALGEBRAIC /PRF_X.X, PRF_Y.Y, PRF_W.W, MKT_X.PX, MKT_Y.PY, MKT_CA.PCA, MKT_L.PL, MKT_W.PW, I_CONS.CONS, PPOL.POL, ATX.TX /;
* Chose a numeraire: real consumer price index

PW.FX = 1;

* Set initial values of variables:

X.L=1; Y.L=1; W.L=1; PX.L=1; PY.L=1; PL.L=1; POL.L = 1; PCA.L = 1; CONS.L=300;

ALGEBRAIC.ITERLIM = 0;
SOLVE ALGEBRAIC USING MCP;
ALGEBRAIC.ITERLIM = 1000;
SOLVE ALGEBRAIC USING MCP;

WELOPT = 100*W.L;
TAXOPT = TX.L;
DISPLAY WELOPT, TAXOPT;

POLINT = 1.5;

SOLVE ALGEBRAIC USING MCP;

WELOPT = 100*W.L;
TAXOPT = TX.L;
DISPLAY WELOPT, TAXOPT;