$TITLE M8-6b.GMS: 2x2x2 of M8-6a in set notation

TABLE BENCH(*,*)
    XHH YHH XHF YHF XFF YFF XFH YFH UH UF CONSH CONSF
PXH  150   -50                                               -100
PYH   50  50                                             -100
PXF    50   50                                               -100
PYF   150   -50  -100
PUH                                                    200       -200
PUF                                                      200       -200
PLH    -120 -10                                                130
PKH    -30  -40                                                 70
PLF                                                -40   -30  70
PKF   -10  -120                                              130;

DISPLAY BENCH;

SETS I goods /1, 2/
   J factors /1, 2/
   C countries /H, F/;

ALIAS (J, JJ), (I, II);

PARAMETERS

VS(J, C) endowment of factor j in country c
TC(I) trade cost of importing good i for country c
BETA(I,J) share of factor j in the production of good i
GAMMA(I) share of good i in the utility function
WELW(C) welfare weight of country c in world welfare
RESULTS(*,*), assemble the results;

VS("1", "H") = BENCH("PLH", "CONSH");
VS("2", "H") = BENCH("PKH", "CONSH");
VS("1", "F") = BENCH("PLF", "CONSF");
VS("2", "F") = BENCH("PKF", "CONSF");

BETA("1","1") = -BENCH("PLH", "XHH")/ BENCH("PXH", "XHH");
BETA("1","2") = -BENCH("PKH", "XHH")/ BENCH("PXH", "XHH");
BETA("2","1") = -BENCH("PLH", "YHH")/ BENCH("PYH", "YHH");
BETA("2","2") = -BENCH("PKH", "YHH")/ BENCH("PYH", "YHH");

GAMMA("1") = -BENCH("PXH", "UH")/ BENCH("PUH", "UH");
GAMMA("2") = -BENCH("PYH", "UH")/ BENCH("PUH", "UH");

WELW("H") = BENCH("PUH", "UH")(BENCH("PUH", "UH") + BENCH("PUF", "UF"));
WELW("F") = BENCH("PUF", "UF")(BENCH("PUH", "UH") + BENCH("PUF", "UF"));

DISPLAY VS, BETA, GAMMA, WELW;

TC(I) = 1.0001;
VARIABLES

JWELMAX  joint welfare maximization
WELMAXH  welfare of country H
WELMAXF  welfare of country F
TR(I,C)   tariff of country C on good I;

POSITIVE VARIABLES

X(I,C)   Activity level for production of I by C
M(I,C)   Activity level for imports of I by C
E(I,C)   Activity level for exports of I by C
U(C)     Activity level for sector U in country C

PX(I,C)  Price of commodity I in country C
PW(I)    World price of commodity I
PV(J,C)  Price of factor J in country C
PU(C)    Price of welfare (expenditure function) in country C

CONS(C)  Aggregate income;

EQUATIONS

OBJJ
OBJH, OBJF
PRX(I,C) Zero profit for sector I in country J
PRM(I,C) Zero profit for imports of I by country j
PRE(I,C) Zero profit for exports of I by country j
PRU(C)   Zero profit for sector U
MKX(I,C)  Supply-demand balance for commodity I
MKW(I)    Supply-demand balance for imports and exports of I
MKV(J,C)  Supply-demand balance for primary factor V1
MKU(C)    Supply-demand balance for welfare (aggregate demand)

ICONS(C)  Income definition for CONS;

OBJJ..    JWELMAX =E= PROD(C, U(C)**WELW(C));
OBJH..    WELMAXH =E= U("H");
OBJF..    WELMAXF =E= U("F");

* Zero profit inequalities

PRX(I,C).. PROD(J, PV(J,C)**BETA(I,J)) =G= PX(I,C);
PRM(I,C).. PW(I)*TC(I)*TR(I,C) =G= PX(I,C);
PRE(I,C).. PX(I,C) =G= PW(I);
PRU(C)..   PROD(I, PX(I,C)**GAMMA(I)) =G= PU(C);

* Market clearance inequalities

MKX(I,C).. X(I,C) - E(I,C) + M(I,C)/TC(I) =G=
\[ \text{PROD}(\text{II}, \text{PX}(\text{II}, \text{C}))^{*} \text{GAMMA}(\text{II})) \times (\text{GAMMA}(\text{I})/\text{PX}(\text{I}, \text{C})) \times \text{U}(\text{C}); \]

\[ \text{MKW}(\text{I}).. \quad \text{SUM}(\text{C}, \text{E}(\text{I}, \text{C}) - \text{M}(\text{I}, \text{C})) = G = 0; \]

\[ \text{MKV}(\text{J}, \text{C}).. \quad \text{VS}(\text{J}, \text{C}) = G = \text{SUM}(\text{I}, \text{PROD}(\text{JJ}, \text{PV}(\text{JJ}, \text{C})^{*} \text{BETA}(\text{I}, \text{JJ})) \times (\text{BETA}(\text{I}, \text{J})/\text{PV}(\text{J}, \text{C})) \times \text{X}(\text{I}, \text{C}); \]

\[ \text{MKU}(\text{C}).. \quad \text{U}(\text{C}) = E = \text{CONS}(\text{C}) / \text{PU}(\text{C}); \]

* Income balance equations

\[ \text{ICONS}(\text{C}).. \quad \text{CONS}(\text{C}) = E = \text{SUM}(\text{J}, \text{VS}(\text{J}, \text{C}) \times \text{PV}(\text{J}, \text{C}) \times \text{SUM}(\text{I}, \text{PW}(\text{I}) \times (\text{TR}(\text{I}, \text{C}) - 1) \times \text{M}(\text{I}, \text{C});) \]

\textbf{MODEL} MCP /PRX.X, PRM.M, PRE.E, PRU.U,
MKX.PX, MKW.PW, MKV.PV, MKU.PU,
ICONS.CONS/;

\textbf{MODEL} MPEC /OBJJ, OBJH, OBJF, PRX.X, PRM.M, PRE.E, PRU.U,
MKX.PX, MKW.PW, MKV.PV, MKU.PU,
ICONS.CONS/;

\text{PU.L}(\text{C}) = 1;
\text{PU.FX}("H") = 1;

* Set initial values of variables:
X.L(I,C) = 100;
M.L(I,C) = 100;
E.L(I,C) = 100;
U.L(C) = 200;
PX.L(I,C) = 1;
PW.L(I) = 1;
PV.L(J,C) = 1;
PV.L(J,C) = 1;
CONS.L(C) = 200;
WELMAXH.L = U.L("H");
WELMAXF.L = U.L("F");
JWELMAX.L = 1;

* fix tariffs at zero (meaning TR = 1) to solve for free trade

TR.FX(I,C) = 1;

OPTION MPEC = nlpec;
SOLVE MPEC USING MPEC MAXIMIZING JWELMAX;

RESULTS("WELJ", "FREETR") = JWELMAX.L/200;
RESULTS("WELH", "FREETR") = U.L("H")/200;
RESULTS("WELF", "FREETR") = U.L("F")/200;
RESULTS("TARIFFH", "FREETR") = TR.L("2","H")-1;
RESULTS("TARIFFF", "FREETR") = TR.L("1","F")-1;
DISPLAY RESULTS;

* show that this can be done with the mcp version when TR variables
* are fixed

SOLVE MCP USING MCP;
TR.FX(I,"H") = 1.2;
SOLVE MCP USING MCP;

* solve for the optimal tariff for country h when TRF is still fixed
* at zero.

TR.UP("2", "H") = +INF;
TR.LO("2", "H") = -INF;

SOLVE MPEC USING MPEC MAXIMIZING WELMAXH;

RESULTS("WELJ", "UNIH") = JWELMAX.L/200;
RESULTS("WELH", "UNIH") = U.L("H")/200;
RESULTS("WELF", "UNIH") = U.L("F")/200;
RESULTS("TARIFFF", "UNIH") = TR.L("1","F")-1;
RESULTS("TARIFFH", "UNIH") = TR.L("2","H")-1;
DISPLAY RESULTS;

*$EXIT
SETS K iterative procedure to determine Nash tariffs /K1*K10/;

TR.FX(I,C) = 1;

LOOP(K,

TR.UP("2", "H") = +INF;
TR.LO("2", "H") = -INF;
TR.FX("1", "F") = TR.L("1", "F");

SOLVE MPEC USING MPEC MAXIMIZING WELMAXH;

TR.UP("1", "F") = +INF;
TR.LO("1", "F") = -INF;
TR.FX("2", "H") = TR.L("2", "H");

SOLVE MPEC USING MPEC MAXIMIZING WELMAXF;

RESULTS("WELJ", "NASH") = JWELMAX.L/200;
RESULTS("WELH", "NASH") = U.L("H")/200;
RESULTS("WELF", "NASH") = U.L("F")/200;
RESULTS("TARIFFF", "NASH") = TR.L("1","F")-1;
RESULTS("TARIFFF", "NASH") = TR.L("1","F")-1;
DISPLAY RESULTS;
$LIBINCLUDE XLDUMP RESULTS M8.XLS SHEET2!A3