11.1 Comparative steady-state analysis

As in the case of labor supply, we would like to have models in which the stock of capital is endogenous. I will present a shortcut that is very valuable in many situations.

In models for which there exists a steady state, it is possible to represent this steady state as a complementarity problem.

We can then at least perform comparative steady-state experiments, in which a parameter change moves us from one steady state to another.
Model 11-1 incorporates optimal capital accumulation via the use of a rationing constraint and endogenous “taxes” to create a model for comparative steady state analysis when the capital stock is endogenously adjusted to its steady-state value.

Let \( r \) denote the rental price (for one period) of a unit of capital and let \( p_k \) denote the price of a new unit of capital.

\( \delta \) will denote the rate of depreciation of capital per period, and \( \rho \) will denote the discount rate between periods.

The steady-state optimal capital accumulation condition is a relationship between the price and rental rate on capital. The rental rate must be given by:
\[ r = \left[ 1 - \frac{1 - \delta}{1 + \rho} \right] p_k \]

The rental price \((r)\) is equal to the price of creating a new unit of capital \((p_k)\) minus the present value of what the capital could be sold for next period.

In the steady state, the price of a new unit next period is the same, so an old unit can be sold for its original value minus one period’s depreciation \((1 - \delta)\).

The present value of the undepreciated portion is thus \((1 - \delta)/(1 + \rho)\).

The dynamic steady-state problem is represented as a static problem using two tricks.
First, the use of capital is subsidized to create the desired wedge (denoted TAU) between the rental price and the price of new capital

$$\text{TAU} = - \frac{(1 - \text{DELTA})}{(1 + \text{RHO})};$$

This then appears as a subsidy to capital use, creating the wedge between the rental price and the price of producing a new unit of capital (PK).

PRF_X.. 100 * PL**0.4 * (PK*(1+TAU)/0.5)**0.6 =G= 100 * PX;

PRF_Y.. 100 * PL**0.6 * (PK*(1+TAU)/0.5)**0.4 =G= 100 * PY;

One unit of new capital is produced using one unit of labor.
Second, since in the steady state newly produced capital is equal to depreciation and depreciation is equal to a share DELTA of total capital, an auxiliary constraint is specified to "endow" consumers with the carryforward from the previous period.

Let $K_s$ denote the capital stock and $K_n$ the production of new capital. In the steady state, these are related by:

$$ K_n = \delta K_s \quad K_s = \frac{K_n}{\delta} $$
The carryforward from the previous period is the steady state stock minus new production. It is called \( K_{FORWD} \) and given by:

\[
\text{carry forward} = K_{FORWD} = K_s - K_n = \frac{K_n}{\delta} - K_n = \frac{(1 - \delta)K_n}{\delta}
\]

We will give the consumer an endowment, via the rationing multiplier \( K_{FORWD} \), the quantity of capital on the right-hand side of the above equation.

\[
A_{KFORWRD} = 140 \times K_{FORWD} = \frac{60 \times K \times (1 - \Delta)}{\Delta};
\]

Parameters: \( \rho = 0.4, \quad \delta = 0.3 \) imply \( \tau = 0.5, \quad r = 0.5 \times p_k \).
### Production Sectors | Consumers

<table>
<thead>
<tr>
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<th>X</th>
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<th>K</th>
<th>W</th>
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Carryforward: **140**

New capital: **60**

Total capital available for production: **200**

Carryforward = 140 = 60*(1 - δ) / δ  where  δ = 0.3
Price of capital chosen to be equal to one.

User cost of capital in X: 120 units of K at a price of 0.5
User cost of capital in Y: 80 units of K at a price of 0.5

Counterfactuals:

(1) raise the rate of time preference to $\rho = 0.6$

(3) restore $\rho$ to 0.4 and set a tax on new capital production to 0.20
$TITLE: M11-1.GMS: steady state capital stock, comparative steady-states

$ONTEXT
"closure rule": instead of the capital stock being fixed (quantity closure), the stock adjusts to satisfy the steady-state relationship between the rental rate and the price of producing new capital (price closure):

\[ \text{delta} = \text{depreciation rate} \]
\[ \text{rho} = \text{rate of time preference} \]

\[ \text{rental rate} = (1 - (1 - \text{delta})/(1 + \text{rho}))\times(\text{price of new capital}) \]

this is done via a subsidy to capital use that creates the wedge

\[ \text{subsidy} = (1 - \text{delta})/(1 + \text{rho}) \]

KFORWARD is (undepreciated) capital taken forward from previous period and K is new capital production, which can be used instantly.

The steady-state condition is depreciation equal new capital formation

\[ (\text{KFORWARD} + K)\times\text{DELTA} = K \quad \text{or} \quad \text{KFORWARD} = K\times(1-\text{DELTA})/\text{DELTA} \]
Production Sectors          Consumers

<table>
<thead>
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$OFFTEXT

PARAMETERS

RHO          Time preference parameter
DELTA        Depreciation rate
TAU          Effective capital use subsidy
KTAX         Tax on new capital production
NEWCAP       New capital stock after counterfactual (= 1 initially);

RHO     = 0.4;
DELTA   = 0.3;
TAU     = - (1 - DELTA)/(1 + RHO);
KTAX    = 0;

NONNEGATIVE VARIABLES
X       Activity level for sector X
Y       Activity level for sector Y
W       Activity level for sector W (Hicksian welfare index)
K       Capital stock index

PX      Price index for commodity X
PY      Price index for commodity Y
PL      Price index for primary factor L
PK      Price index for primary factor K
PW      Price index for welfare (expenditure function)

CONS   Income definition for CONS
KFORWRD Capital stock from previous period;

EQUATIONS
PRF_X   Zero profit for sector X
PRF_Y   Zero profit for sector Y
PRF_W   Zero profit for sector W (Hicksian welfare index)
PRF_K   Zero profit for capital index

MKT_X   Supply-demand balance for commodity X
MKT_Y   Supply-demand balance for commodity Y
MKT_L   Supply-demand balance for primary factor L
MKT_K   Supply-demand balance for factor K
MKT_W   Supply-demand balance for aggregate demand
I_CONS  Income definition for CONS
A_KFORWRD  Auxiliary equation to determine the carry forward;

*  Zero profit conditions:

PRF_X..  100 * PL**0.4 * (PK*(1+TAU)/0.5)**0.6 =G= 100 * PX;

PRF_Y..  100 * PL**0.6 * (PK*(1+TAU)/0.5)**0.4 =G= 100 * PY;

PRF_W..  200 * PX**0.5 * PY**0.5 =G= 200 * PW;

PRF_K..  60*PL =G= 60 * PK * (1-KTAX);

*  Market clearing conditions:

MKT_X..  100 * X =G= 100 * W * PW/PX;

MKT_Y..  100 * Y =G= 100 * W * PW/PY;

MKT_W..  200 * W =G= CONS / PW;

MKT_L..  160 =G= 60*K + 40* X*PX/PL + 60*Y*PY/PL;

MKT_K..  140*KFORWRD + 60*K =G= 120 * X * PX*0.5/(PK*(1+TAU)) + 80 * Y * PY*0.5/(PK*(1+TAU));
* Income constraints:

\[ I_{\text{CONS}}.. \quad \text{CONS} = 160 \times PL + 140 \times K_{\text{FORWRD}} \times PK + \]
\[ \quad PK \times TAU \times (120 \times X \times PX \times 0.5/(PK \times (1 + TAU)) + \]
\[ \quad 80 \times Y \times PY \times 0.5/(PK \times (1 + TAU))) + \]
\[ \quad 60 \times PK \times KTAX \times K; \]

* Auxiliary constraints:

\[ A_{\text{KFORWRD}}.. \quad 140 \times K_{\text{FORWRD}} = 60 \times K \times (1 - \text{DELTA}) / \text{DELTA}; \]


X.L = 1;
Y.L = 1;
W.L = 1;
K.L = 1;
PX.L = 1;
PY.L = 1;
PK.L = 1;
PW.FX = 1;
PL.L = 1;
CONS.L = 200;

KFORWRD.L = 1;

ALGEBRAIC.ITERLIM = 0;
\textbf{SOLVE} ALGEBRAIC USING MCP;
ALGEBRAIC.ITERLIM = 2000;
\textbf{SOLVE} ALGEBRAIC USING MCP;

* Raise the rate of time preference from 0.4 to 0.6:

RHO = 0.6;
TAU = -(1 - DELTA)/(1 + RHO);

\textbf{SOLVE} ALGEBRAIC USING MCP;

NEWCAP = K.L;
\textbf{DISPLAY} NEWCAP;

* Set rho back to 0.4, tax new capital at 0.20

RHO = 0.4;
TAU = -(1 - DELTA)/(1 + RHO);

KTAX = 0.20;
SOLVE ALGEBRAIC USING MCP;

NEWCAP = K*L;
DISPLAY NEWCAP;
11.2 Converting an Infinite-Horizon Problem to an MCP

Comparative dynamics in a model with a steady-state

Consider a straight-forward dynamic model with an endogenous capital stock.

Sectors (Activities)

\( X_t \) production of composite good in period \( t \)
\( I_t \) production of new capital (investment) in period \( t \)
\( K_t \) transforms capital into capital services and future capital
Commodities (Markets)

\[ p_X (CX_t) \] price of X in period \( t \)
\[ p_r (CR_t) \] rental price of capital in period \( t \)
\[ p_k (CK_t) \] asset price of capital (price of a new capital good) in period \( t \)
\[ p_l (CL_t) \] price of labor in period \( t \)

Consumers

Infinitely lived representative consumer

\( \delta \) = rate of capital depreciation
\( \rho \) = rate of time preference (discounting utility)
\( KE_t \) = capital endowment at the beginning of a period
\( K_t \) = capital stock for production at time \( t \) \( (KE_t + I_t) \)
Conditions for Steady-State Equilibrium:

(1) \[ KE_{t+1} = KE_t \Rightarrow I_t = \delta KE_t \]

(2) rate of interest = \( \rho \):
\[ pj_{t+1} = \frac{pj_t}{1 + \rho} \quad \text{for all goods } j = X, I, K \]

(3) \[ pr_t = \left[ 1 - \frac{1 - \delta}{1 + \rho} \right] pk_t \quad \text{relationship between asset and rental prices} \]

(4) \[ KE_{t+1} = (1 - \delta)K_t = (1 - \delta)(KE_t + I_t) \quad \text{(KE: capital endowment)} \]
\[ pk_{t+1}KE_{t+1} = (1 - \delta) \frac{pk_t}{1 + \rho} K_t = (1 - \delta) \frac{(pk_tKE_t + pk_tI_t)}{1 + \rho} \]
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Parameters: $\rho = 0.2$, $\delta = 0.1$

Prices: $C_0 = C_1 = C_0 = 1$: $C_0 = 4$  
$C_1 = C_0 / (1 + \rho) = 3.3333$

$C_0 = (1 - (1 - \delta) / (1 + \rho)) \times C_0 = (1/4) \times C_0$
1. $360 = 90$ units at $CK_0 = 4$
2. $400 = 100$ units at $CK_0 = 4$
3. $100 = 100$ units at rental price = 1
4. $300 = \text{undepreciated capital } (1-\text{delta})*100 = 90$
   at a price of $CK_1 = 1/(1+RHO) = 4/1.2 = 3.3333$
   $300 = (1-\text{DELTA})*4*100/(1+RHO) = 300$

The amount $360 - 300 = 60$ can be thought of as net rental income:

rental income (90) minus the cost of replacing depreciated capital: $9*CK_0/(1+RHO) = 30$.

Problem: Suppose we want to represent this infinite-horizon problem as a finite dimension complementarity problem

Approaching the last period the consumer would have no incentive to accumulate capital and would want to run down the capital stock.
(1) Assume a finite number of periods plus a terminal period.

(2) Assume an extra dummy agent: DEITY

(3) Assume that the dummy agent is endowed with an extra good HEAVEN

(4) Assume that the dummy agent will only sell Heaven in exchange for terminal period capital (does not demand any other good)

(5) Assume that the representative agent has a demand for heaven

(6) Use a tax/subsidy on heaven to ensure that the asset/rental price relationship holds on terminal capital (so that the economy is forced onto the steady-state path at terminal time)
## Terminal period

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$TITLE: M11-2.GMS: Infinite horizon dynamic model, MPS/GE formulation

$ONTEXT
Converts infinite horizon model into a fixed time horizon MCP
Trick is that there is an outside agent "DEITY" who demands terminal period capital in exchange for the good "HEAVEN"
Price of HEAVEN relative to terminal capital is set by a tax/subsidy so that the steady-state condition is satisfied in the terminal period
$OFFTEXT

SETS T time periods /1*25/;

PARAMETERS
   DELTA rate of depreciation
   RHO rate of time preference
   PV present value from terminal period to first period
   TERM number of the terminal period (25)
   RTEM present value of terminal values at t = 1
   INITK initial capital stock
   R(T) present value of period t values at t = 1
   D(T) remaining undepreciated portion of initial K at t
   PVUTIL present value of the utility path at t = 1
   TLAST(T) switch to indicate the terminal (last) period
   TFIRST(T) switch to indicate the first time period
   SOLUTION(T,*) stores values of the solution path
   CONSUME(T) consumption at time t
INVEST(T)  investment at time t
KSTOCK(T)  capital stock at time t;

RHO = 0.2;
DELTA = 0.1;
INITK = 90;
TERM = CARD(T);
RTERM = (1/(1+RHO))**(CARD(T) - 1);
R(T) = (1/(1+RHO))**(ORD(T)-1);
D(T) = (1-DELTA)**(ORD(T) - 1);
PV = 200*SUM(T, R(T)) + 90*4*RTERM/(1+RHO));
TLAST(T) = 0;
TLAST('25') = 1;
TFIRST('1') = 1;

$ONTEXT
$MODEL: BASIC

$SECTORS:

X(T)  ! production at time t
I(T)  ! production of new capital (investment) at time t
K(T)  ! capital stock at time t
U     ! present value of utility
$COMMODITIES:
CX(T)    ! present value price of output at t
CR(T)    ! present value rental rate for capital at time t
CK(T)    ! present value price of capital (cost of production) at t
CL(T)    ! present value price of labor at time t
CKT      ! terminal period present value price of capital
CU       ! price of utility (intertemporal consumer price index)
HEAVEN   ! price of heaven

$CONSUMERS:
CONS     ! representative consumer
DEITY    ! deity who demand terminal capital stock and sells heaven

$AUXILIARY:
TRANS    ! endogenous tax or subsidy achieves steady-state at TLAST

$PROD:K(T)
O:CK(T+1) Q:(100*(1-DELTA)) P:(4*R(T+1))
O:CKT$TLAST(T) Q:(100*(1-DELTA)) P:(4*R(T)/(1+RHO))
O:CR(T) Q:100 P:(R(T))
I:CK(T) Q:100 P:(4*R(T))

$PROD:I(T)
O:CK(T) Q:10
I:CL(T) Q:40
$\text{PROD: } X(T) \ s: 1
   O: \text{CX}(T) \quad Q: 200
   I: \text{CL}(T) \quad Q: 100
   I: \text{CR}(T) \quad Q: 100

$\text{PROD: } U \ s: 1 \ a: 2
   O: \text{CU} \quad Q: \text{PV}
   I: \text{CX}(T) \quad Q: 200 \quad P: \text{R}(T) \quad a:
   I: \text{HEAVEN} \quad Q: 90 \quad P: (4 \times \text{RTERM}/(1+\text{RHO})) \quad A: \text{CONS} \quad N: \text{TRANS}

$\text{DEMAND: } \text{CONS}
   D: \text{CU} \quad Q: \text{PV}
   E: \text{CL}(T) \quad Q: 140
   E: \text{CK}(T) \ $\text{TFIRST}(T) \quad Q: \text{INITK}

$\text{DEMAND: } \text{DEITY}
   D: \text{CKT} \quad Q: 90
   E: \text{HEAVEN} \quad Q: 90

$\text{CONSTRAINT: TRANS}
   \text{CR('25')} \ =E= (1 - (1-\text{DELTA})/(1+\text{RHO})) \times \text{CL('25')} \times 4;

$\text{OFFTEXT}
$\text{SYSINCLUDE MPSGESET BASIC}
TRANS.UP = +INF;
TRANS.LO = -INF;

CX.L(T) = R(T);
CL.L(T) = R(T);
CR.L(T) = R(T);
CK.L(T) = 4*R(T);
CKT.L = 4*R('25')/(1+RHO);
HEAVEN.L = 4*R('25')/(1+RHO);
TRANS.L = 0;

*BASIC.ITERLIM = 0;

$INCLUDE BASIC.GEN
SOLVE BASIC USING MCP;

PVUTIL = SUM(T, X.L(T)*R(T)) + (X.L('25')*R('25'))/RHO;

DISPLAY PVUTIL;

CONSUME(T) = X.L(T);
INVEST(T) = I.L(T);
KSTOCK(T) = K.L(T);
SOLUTION(T,"X") = X.L(T);
SOLUTION(T,"I") = I.L(T);
SOLUTION(T,"K") = K.L(T);

$LIBINCLUDE XLDUMP SOLUTION M11.xls SHEET1!A2

* counterfactual: lower the capital stock below is ss value

INITK = 30;

$INCLUDE BASIC.GEN
SOLVE BASIC USING MCP;

PVUTIL = SUM(T, X.L(T)*R(T)) + (X.L('25')*R('25'))/RHO;

DISPLAY PVUTIL;

CONSUME(T) = X.L(T);
INVEST(T) = I.L(T);
KSTOCK(T) = K.L(T);
SOLUTION(T,"X") = X.L(T);
SOLUTION(T,"I") = I.L(T);
SOLUTION(T,"K") = K.L(T);

$LIBINCLUDE XLDUMP SOLUTION M11.xls SHEET1!F2

* counterfactual: lower the rate of time preference
INITK = 90;

RHO = 0.1;
RTERM = (1/(1+RHO))**((CARD(T) - 1));
R(T) = (1/(1+RHO))**((ORD(T)-1));
D(T) = (1-DELTA)**((ORD(T) - 1));
PV = 200*SUM(T, R(T)) + 90*(4*RTERM/(1+RHO));

$INCLUDE BASIC.GEN
SOLVE BASIC USING MCP;

PVUTIL = SUM(T, X.L(T)*R(T)) + (X.L('25')*R('25'))/RHO;

DISPLAY PVUTIL;

CONSUME(T) = X.L(T);
INVEST(T) = I.L(T);
KSTOCK(T) = K.L(T);
SOLUTION(T,"X") = X.L(T);
SOLUTION(T,"I") = I.L(T);
SOLUTION(T,"K") = K.L(T);

$LIBINCLUDE XLDUMP SOLUTION M11.xls SHEET1!K2
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Destroy 2/3 of initial (steady state) capital stock
Lower rate of time preference from 0.2 to 0.1