Homework 7
ECON 4818
Professor Martins

1. From your textbook answer questions 1, 2, 5 and 8 from Appendix E in your textbook. For questions 5 and 8, note that what your textbook labels "The Gauss-Markov" assumptions, are precisely the assumptions we made in class. That is, for the multivariate regression model $Y = X\beta + U$ we have that

   (a) $X^T X$ is invertible

   (b) $E(U|X) = 0$

   (c) $V(U|X) = \sigma^2 I_n$

2. Use the MATLAB code `OLSwage2.m` and data set `WAGE1.mat` available from the class website to estimate the the following regression models:

   (a) $\log(wage) = \beta_0 + \beta_1\, educ + u$

   (b) $\log(wage) = \beta_0 + \beta_1\, educ + \beta_2\, exper + u$

   (c) $\log(wage) = \beta_0 + \beta_1\, educ + \beta_2\, exper + \beta_3\, exper^2 + u$

   For each of these models obtain the least squares estimators for β_k and construct a 95 percent confidence interval. For model (c), estimate the impact on wages of another year of experience.

3. Chapter 4: problems 2 and 8.

4. Chapter 6: problems 2 and 4.

5. Use the MATLAB code `MOLSwage2.m` and data set `WAGE1.mat` available from the class website to estimate the the following regression model:

$$\log(wage)_i = \beta_0 + \beta_1\, educ_i + \beta_2\, exper_i + \beta_3\, exper_i^2 + u_i,$$

   where $i = 1, \cdots, n$. Under the assumption that the assumption that $u_i \sim N(0, \sigma^2)$ for all i, that $\{u_i\}_{i=1}^n$ forms an independent sequence, test the following hypotheses at the 5 percent level:

(a) $H_0 : \beta_1 = \beta_2 = 0$

(b) $H_0 : \beta_3 = 0$

(c) $H_0 : \beta_1 = \beta_2 + \beta_3$.

6. Consider the following linear regression model

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + U_i$$

where $i = 1, \cdots, n$ and $U_i \sim N(0, \sigma^2)$ for all i and $\{U_i\}_{i=1,\cdots,n}$ forms an independent sequence of random variables. In class we have shown that if we want to test that $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ against the alternative hypothesis that some $\beta_j \neq 0$ for some $j = 1, 2, 3$ we can do so by using the statistic

$$f = \frac{m^{-1}(R\hat{\beta} - r)^T (R(X^T X)^{-1} R^T)^{-1} (R\hat{\beta} - r)}{\hat{\sigma}^2} \sim F_{m,n-4}$$

where $R = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, $m = 3$, $r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and $\hat{\sigma}^2 = \frac{1}{n-4} \hat{U}^T \hat{U}$ (refer to your notes for the definition of X, $\hat{U}$). Provide all assumptions needed to obtain that under the null hypothesis H_0 we have that $f \sim F_{m,n-4}$. For each assumption explain why and where it is needed in the proof of the distribution of f. Use mathematical arguments.