Homework 8
ECON 4818
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1. Let $\{W_i\}_{i=1}^n$ be a sequence of independent and identically distributed random variables such that $E(W_i) = \mu$ and $V(W_i) = \sigma^2$. Prove that
$$\frac{1}{n}\sum_{i=1}^{n} W_i \xrightarrow{p} \mu.$$
In this question you cannot use Kolmogorov's law of large numbers. Hint: For any random variable X and $\epsilon > 0$, $P(|X| > \epsilon) < E(X^2)/\epsilon^2$.

2. Consider the regression model
$$Y_i = X_i\beta + u_i \text{ where } i = 1, \cdots, n,$$
$u_i|X_i \sim \text{IID}(0, \sigma^2)$, X_i is a $1 \times (K+1)$ vector of regressors, Y_i is the regressand and β is a parameter to be estimated. Let Y be an $n \times 1$ vector that collects all observations on the regressand, X be an $n \times (K+1)$ matrix that collects all observations on the regressors and u be an $n \times 1$ vector that collects all unobserved regression errors.
   (a) Give conditions (that are sufficient for) and establish that $X^T X/n \xrightarrow{p} Q$, where $Q = E(X_i^T X_i)$.
   (b) Give conditions (that are sufficient for) and establish that $X^T u/n \xrightarrow{p} 0$.
   (c) Use (a) and (b) to show that $\hat{\beta} \xrightarrow{p} \beta$.
   (d) Establish that $\sqrt{n}(\hat{\beta} - \beta) \xrightarrow{d} Z \sim N(0, \sigma^2 Q^{-1})$.

3. Consider a random sample $\{X_i\}_{i=1,2,\cdots,n}$, i.e., a sequence of IID random variables, and suppose $E(X_i) = \mu$ and $V(X_i) = \sigma^2$. Suppose s^2 is an estimator for σ^2, such that $s^2 \xrightarrow{p} \sigma^2$. How would you test the hypothesis that $\mu = c$, c a known constant? You must obtain the distribution of the test statistics you propose under the null hypothesis.