Suppose $\dfrac{x}{h} < 1 \iff x < h \iff x \in [0, h)$. Then,

$$\int_{-1}^{1} K(u) f(x - hu)\, du = \int_{-1}^{x/h} K(u) f(x - hu)\, du + \int_{x/h}^{1} K(u) f(x - hu)\, du.$$

Note that for the second integral, $\dfrac{x}{h} < u \leq 1 \implies x < uh \leq h \implies -x > -uh \geq -h \implies 0 > x - uh \geq x - h$. Since the support of the density is $[0, \infty)$, $f(x - uh) = 0$ and

$$\int_{-1}^{1} K(u) f(x - hu)\, du = \int_{-1}^{x/h} K(u) f(x - hu)\, du.$$

Also,

$$\int_{-1}^{1} K(u)\, du = \int_{-1}^{x/h} K(u)\, du + \int_{x/h}^{1} K(u)\, du.$$

Hence,

$$\begin{aligned}
\int K(u) f(x - hu)\, du - f(x) \int K(u)\, du &= \int_{-1}^{x/h} K(u) f(x - hu)\, du - f(x) \int_{-1}^{x/h} K(u)\, du \\
&\quad - f(x) \int_{x/h}^{1} K(u)\, du \\
&= \int_{-1}^{x/h} K(u) \big[f(x - hu) - f(x) \big] du - f(x) \int_{x/h}^{1} K(u)\, du.
\end{aligned}$$

Thus, if $f \in \Sigma(\beta, C)$ and for some $\lambda \in (0, 1)$,

$$\begin{aligned}
\mathrm{E}\, \hat{f}(x) - f(x) &= \int_{-1}^{x/h} K(u) \Bigg\{ f'(x)(-hu) + \frac{1}{2!} f^{(2)}(x)(-hu)^2 + \cdots \\
&\qquad + \frac{1}{(\lfloor \beta \rfloor - 1)!} f^{(\lfloor \beta \rfloor - 1)}(x)(-hu)^{\lfloor \beta \rfloor - 1} + \frac{1}{\lfloor \beta \rfloor !} f^{(\lfloor \beta \rfloor)}(x - hu\lambda)\, (-hu)^{\lfloor \beta \rfloor} \Bigg\} du \\
&\qquad - f(x) \int_{x/h}^{1} K(u)\, du \\
&= -f'(x)\, h \int_{-1}^{x/h} K(u)\, u\, du + \frac{h^2}{2!} f^{(2)}(x) \int_{-1}^{x/h} u^2 K(u)\, du + \cdots \\
&\quad + \frac{(-h)^{\lfloor \beta \rfloor}}{\lfloor \beta \rfloor !} \int_{-1}^{x/h} f^{(\lfloor \beta \rfloor)}(x - hu\lambda)\, u^{\lfloor \beta \rfloor} K(u)\, du \\
&\quad - f(x) \int_{x/h}^{1} K(u)\, du.
\end{aligned}$$

Suppose $x = ch$ for $c \in [0, 1)$. Then $0 \leq x < h$ and $\int_{x/h}^{1} K(u)\, du = \int_{c}^{1} K(u)\, du$, with $0 < \displaystyle\int_{c}^{1} K(u)\, du \leq \frac{1}{2}$. Hence, the last term in the bias expansion stays constant as $n \to \infty$ and $h \to 0$. All other terms vanish as $h \to 0$.

Now consider the variance of $\hat{f}(x)$.

$$\begin{aligned}
\mathrm{V}\big(\hat{f}(x)\big) &= \frac{1}{n^2h^2}\, n\, \mathrm{V}\bigg(K\bigg(\frac{x-X}{h}\bigg)\bigg) \\
&\le \frac{1}{nh}\int_{-1}^{1} K^2(u) f(x-hu)\, du = \frac{1}{nh}\int_{-1}^{x/h} K^2(u) f(x-hu)\, du,
\end{aligned}$$

where the last equality follows since $x < h \implies f(x-hu) = 0$ when $u > \dfrac{x}{h}$. Then, under the conditions of Theorem 7, $[0,\infty)$,

$$\mathrm{V}\big(\hat{f}(x)\big) \le f(x)\,\frac{1}{nh}\int_{-1}^{c} K^2(u)\, du \longrightarrow 0 \quad \text{if } nh \to \infty,\ h \to 0 \text{ as } n \to \infty.$$