1. (20 points) For each of the following unrelated questions, answer either ALWAYS TRUE or NOT ALWAYS TRUE. No justification is necessary.

   (a) $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = (\mathbf{A} \cdot \mathbf{C}) \times (\mathbf{B} \cdot \mathbf{C})$.

   (b) $(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{D} \times \mathbf{C}) \cdot (\mathbf{B} \times \mathbf{A})$

   (c) $(\mathbf{A} \times \mathbf{B}) \times (\mathbf{B} \times \mathbf{A}) = \mathbf{0}$.

   (d) If an object moves along a smooth path in the x-y plane, then its acceleration, $\mathbf{a}$, is orthogonal to its velocity, $\mathbf{v}$.

2. (20 points) The door to a secret tunnel in a hillside is flat and triangular in shape. It is hinged along the side that extends from the point P_1 at $(1, 0, 0)$ to the point P_2 at $(0, 1, 0)$. When closed, the third, and upper, corner of the door is at point P_3 at $(0, 0, 2)$. One end of a thin cord is attached to the upper corner of the door so it can only swing open 90 degrees. When open, the third corner of the door is at location P_4 which should still be in the first octant. The other end of the cord is attached to the hillside at location P_3.

   (a) Determine the standard equation of the plane for the door when it is closed, and its unit normal vector, $\mathbf{n}_1$.

   (b) Determine the standard equation of the plane for the door when it is open, and its unit normal vector, $\mathbf{n}_2$.

   (c) How long should the cord be?

   (d) If you wanted to cover the door with camouflage material, what area of material would you need?

3. (20 points) Consider a particle moving along the parabolic curve $y = ax^2$ in the x-y plane, where $a > 0$.

   (a) Determine an equation for the curvature, κ, at any point (x, y) on the parabola.

   (b) What is the curvature at the point $(1, a)$?

   (c) Is there a point on the parabola where κ has a maximum value? If so, what is κ at this point?

   (d) Is there a point on the parabola where κ has a minimum value? If so, what is κ at this point?

   (e) Suppose the particle travels along the curve with constant speed $|\mathbf{v}| = 5$ m/s. Write down the acceleration vector, $\mathbf{a}$, of the particle when it is at the point $(1, a)$.

   (f) What torsion, τ, is the particle experiencing at the point $(1, a)$?

4. (20 points) Joey the cat loves to jump off the top of his Kitty-condo. At time $t = 0$, Joey jumps from the top to the ground, and takes the path $\mathbf{r}(t) = \frac{t}{\sqrt{2}}\,\mathbf{i} + \frac{t}{\sqrt{2}}\,\mathbf{j} + \left(1 - \frac{t^2}{2}\right)\mathbf{k}$, where the plane $z = 0$ represents the ground.

   (a) At what time does Joey hit the ground?

   (b) Set up the calculation to determine how far Joey travels through the air. (Note: we are not asking for Joey's displacement.)

   (c) Evaluate your expression from part (b).

5. (20 points) For each of the surfaces described below (a through d), list the possible equations (1 through 8) that would fit the description. Note that there might be more than one equation that fits the description. Also, there might not be any equations that fit the description, in which case you should state NONE.

(a) Paraboloid parallel to the y-axis.

(b) A set of cones parallel to the x-axis.

(c) Hyperboloid of two sheets parallel to the x-axis.

(d) Hyperboloid of one sheet parallel to the y-axis.

(1) $\frac{z^2}{4} + \frac{y^2}{16} = \frac{x^2}{25} - 1$

(2) $\frac{y^2}{16} = \frac{x}{25} + \frac{z^2}{4}$

(3) $\frac{y^2}{16} + \frac{z^2}{4} = \frac{x^2}{9}$

(4) $\frac{x^2}{4} + \frac{z^2}{16} = \frac{y}{9}$

(5) $\frac{x^2}{16} - \frac{y^2}{9} + \frac{z^2}{4} = 1$

(6) $\frac{x^2}{16} + \frac{y^2}{9} = \frac{z^2}{4} - 1$

(7) $\frac{x^2}{9} + \frac{z^2}{4} + 9 = \frac{y}{9}$

(8) $\frac{z^2}{4} = \frac{x}{5} + \frac{y^2}{9}$

Projections, and distances from a point to a line and a plane

$$\text{proj}_{\mathbf{A}}\mathbf{B} = \left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \qquad d = \frac{|\overrightarrow{PS}\times\mathbf{v}|}{|\mathbf{v}|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

Arc length, Frenet formulas, and tangential and normal acceleration components

$$ds = |\mathbf{v}|\,dt \qquad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N} \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N} \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3} = \frac{|f''(x)|}{[\,1+(f'(x))^2\,]^{3/2}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}} \qquad \tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}$$

$$\mathbf{a} = a_N\mathbf{N} + a_T\mathbf{T} \qquad a_T = \frac{d|\mathbf{v}|}{dt} \qquad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$

Fond memories from Calculus II

$$\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C \qquad \int\sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln(x+\sqrt{a^2+x^2}) + C$$

$$\int x^2\sqrt{a^2+x^2}\,dx = \frac{x}{8}(a^2+2x^2)\sqrt{a^2+x^2} - \frac{a^4}{8}\ln(x+\sqrt{a^2+x^2}) + C$$