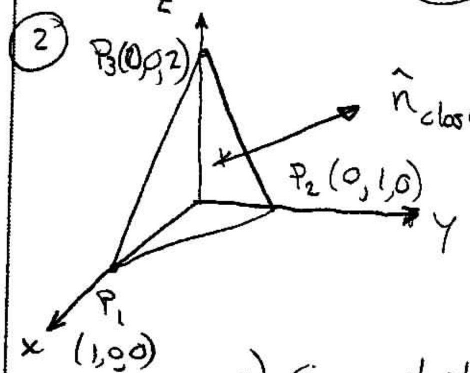
(1) a) Not always true
b) always true
c) always true
d) not always true

(5) a) 4, 7
b) 3
c) 1
d) 5

(2)



Define vectors $\underline{P}_{12} = -\hat{i} + \hat{j}$

$\underline{P}_{13} = -\hat{i} + 2\hat{k}$

Then $\underline{n}_{closed} = \underline{P}_{12} \times \underline{P}_{13} = 2\hat{i} + 2\hat{j} + \hat{k}$

a) Given that (non-unit) normal to the closed plane is $\underline{n}_{closed} = 2\hat{i} + 2\hat{j} + \hat{k}$, then the std. eqn. of the plane must be $2x + 2y + z = D$.

Evaluating this at P_1 gives $D = 2$.

So $\hat{n}_{closed} = \dfrac{2\hat{i} + 2\hat{j} + \hat{k}}{3}$ and $2x + 2y + z = 2$

b) Note that a normal to the closed position would be

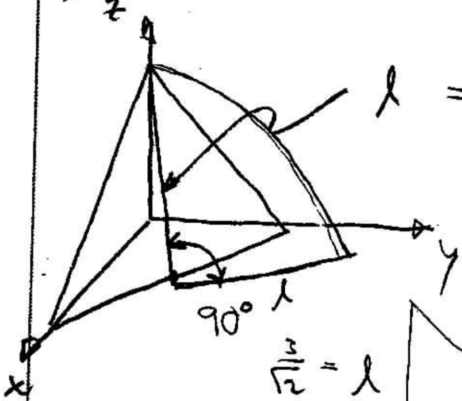
$$\underline{n}_{open} = \underline{n}_{clos.} \times \underline{P}_{12} = \ldots = \hat{i} + \hat{j} - 4\hat{k}$$

Thus the open plane must be of the form

$x + y - 4z = d$. Eval. at P_1 to get $d = 1$.

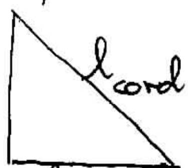
Thus $\hat{n}_{open} = \dfrac{\hat{i} + \hat{j} - 4\hat{k}}{\sqrt{18}}$ $\quad x + y - 4z = 1$

c) Calc. dist from P_3 to line through P_1 + P_2



$$l = \frac{|\underline{P}_{13} \times \underline{P}_{12}|}{|\underline{P}_{12}|} = \frac{|-\underline{n}_c|}{|\underline{P}_{12}|} = 3/\sqrt{2}$$

$\frac{3}{\sqrt{2}} = l$

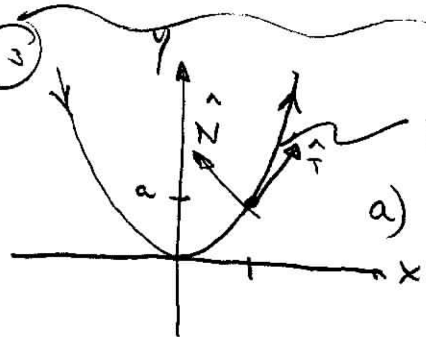


$l_{cord} = \sqrt{l^2 + l^2} = 3$

$l = 3/\sqrt{2}$

2 d) $\text{area}_{door} = \dfrac{|\underline{P}_{12} \times \underline{P}_{13}|}{2} = \dfrac{|\underline{n}_c|}{2} = 3/2$

3



path is on curve $y = f(x) = ax^2$

a) $$k = \frac{|f''|}{\left[1+(f')^2\right]^{3/2}} = \frac{2a}{\left(1+(2ax)^2\right)^{3/2}}$$

b) $$k\Big|_{(1,a)} = \frac{2a}{\left(1+(2a)^2\right)^{3/2}}$$

c) k is max. when the denom. of k is minimized, ie when $(1+4a^2x^2)$ is minimized at $x=0$, $k = 2a$

d) k decreases as $x \to \infty$. In fact $\lim\limits_{x \to \pm\infty} k = 0$

but k actually has no minimum.

e) $\underline{a} = a_T \hat{T} + a_N \hat{N}$ where $a_T = \dfrac{d|\underline{v}|}{dt} = 0$

and $a_N = k|\underline{v}|^2 = \dfrac{2a|\underline{v}|^2}{(1+4a^2x^2)^{3/2}}$

so $$\underline{a}\Big|_{(1,a)} = \frac{2a|\underline{v}|^2}{(1+4a^2)^{3/2}}\hat{N}$$ but we still need $\hat{N}\Big|_{(1,a)}$

Since $f'(x) = 2ax$, then $\underline{T}\Big|_{(1,a)} = \hat{i} + 2a\hat{j}$ and $\underline{N} = -2a\hat{i} + \hat{j}$ (non-unit versions)

so $$\hat{N}\Big|_{(1,a)} = \frac{-2a\hat{i}+\hat{j}}{\sqrt{1+4a^2}}$$ Hence $$\underline{a}\Big|_{(1,a)} = \frac{2a|\underline{v}|^2}{(1+4a^2)^{3/2}}\frac{(-2a\hat{i}+\hat{j})}{(1+4a^2)^{1/2}} = \frac{2a|\underline{v}|^2}{(1+4a^2)^2}\left(-2a\hat{i}+\hat{j}\right)$$ + $|\underline{v}| = 5\ m/s$

f) Torsion of a path in a plane is $\tau = 0$.

(4)

$\underline{r}(t) = \frac{t}{\sqrt{2}}\hat{i} + \frac{t}{\sqrt{2}}\hat{j} + \left(1 - \frac{t^2}{2}\right)\hat{k}$

$\underline{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + (-t)\hat{k} \quad + \quad |\underline{v}| = \sqrt{1+t^2}$

a) Joey hits ground when $\hat{k}$ component of $\underline{r}$ is 0.

so $1 - \frac{t^2}{2} = 0$ gives $t = \sqrt{2}$ ⟵

b) Arc length $S = \int_{t=0}^{\sqrt{2}} |\underline{v}|\,dt = \int_{t=0}^{\sqrt{2}} \sqrt{1+t^2}\,dt$

c)
$$= \left[\frac{t}{2}\sqrt{1+t^2} + \frac{1}{2}\ln\left(t + \sqrt{1+t^2}\right)\right]_{t=0}^{\sqrt{2}}$$

$$= \frac{\sqrt{6}}{2} + \frac{\ln(\sqrt{2}+\sqrt{3})}{2}$$ ⟵

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