Corruption and Socially Optimal Entry∗

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Abstract

The paper investigates the effects of corruption in the entry-certifying process on market structure and social welfare for a Cournot industry with linear demand and costs. To gain entry, a firm must pay a bribe-maximizing official an exogenous percentage of anticipated profit, in addition to the usual set-up cost. This would lead to a monopoly, but only in markets without pre-existing or shadow-economy firms. A benevolent social planner may turn bribery to the benefit of society by either manipulating the number of pre-existing firms in the market, or by setting up two independent (corrupt) licensing authorities. A socially optimal number of firms in the market may be reached by choosing the right number of pre-existing firms or by having exactly two licensing authorities. These mechanisms may be seen as restoring second-best efficiency in settings characterized by two major sources of distortion: Imperfect competition and corruption. We also show in an extension that the qualitative insights carry over to a model with quadratic costs and first best entry regulation.

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1 Introduction

Pervasive throughout human history, corruption is a complex phenomenon that raises a myriad of challenges for the social sciences and beyond. For economists, the importance of corruption has been noted in multiple areas of research, giving rise to various strands of literature (Tullock, 1967; Rose-Ackerman, 1978; Aidt, 2003; Jain, 2001). However, in its predominantly normative outlook, much of traditional economic analysis has tended to ignore the corruption dimension. Yet a positive perspective in economics often must integrate the distortive effects of corruption to be able to capture some essential features of economic activity, whenever actors other than pure market forces are present. In particular, corruption has the potential to emerge as a critical component whenever it can have a direct effect on important dimensions of market outcomes. One of these is undeniably market structure. This paper is an attempt to investigate the effects of corruption in the entry-certifying process on the endogenous market structure of a given industry. As such, the present paper is closest in spirit to the pioneering work of Bliss and Di Tella (1997), which considers firms that are asymmetric in production costs and engage in perfect competition upon entry. The paper joins a small but growing literature that restricts attention to the effects of corruption on market outcomes, and thus defines corruption as the sale by bureaucrats of public property for purely private gains. The prototypical papers in this strand also include Shleifer and Vishny (1993), Acemoglu and Verdier (2000), Choi and Thum (2003) and Emerson (2006).

In contrast to Bliss and Di Tella (1997), we adopt as an appropriate baseline setting the standard two stage entry game commonly used in industrial organization, to allow for strategic behavior on the part of firms (as in Emerson, 2006). In the first stage, a large number of identical firms simultaneously decide whether or not to enter a market, with entry requiring a fixed set-up cost. Upon entry, in the second stage, the firms compete in the quantity dimension, in an industry with a homogeneous good. For tractability, we consider the usual specification of linear demand and cost functions for the main part of the paper. Corruption is introduced into this standard setting by positing that a corrupt official charges a bribe to each firm in order to issue the requisite license for industry entry. The official is fully corrupt in that his goal is to maximize the expected total revenue collected from these bribes, taking into account that he may be detected and punished with some exogenous probability. In addition, as in Acemoglu and Verdier (2000), the bribes are taken to be an exogenously fixed percentage of each firm’s final equilibrium profit. This bribe-setting rule may be justified on multiple grounds. First, there is anecdotal evidence that corrupt officials tend to set their fees in this manner, often in line with historical practice in a given industry. Second, motivated by a natural desire to lay low so as to be able to operate over the long run,
a sophisticated corrupt official might well understand at some level that this simple pricing
rule actually minimizes the distortionary consequences of his actions on firms’ behavior, by
simple analogy to ad valorem taxes.

It is quite intuitive that, under this benchmark, the corrupt official would allow just one
firm into the industry, since his objective is to maximize (a fixed proportion of) industry
profit.\footnote{This observation was clearly hinted at by Bliss and Di Tella (1997) and Emerson (2006).} The starting point for this paper is the observation that this drastic curtailment of
competition in an industry is possible only in cases where the corrupt official begins his tenure
in the entry-certifying office prior to the launch time of the industry at hand. Otherwise,
in cases where the official inherits an ongoing industry, there will be incumbent firms that
typically no longer require a license, and are thus beyond the reach of the official in terms
of bribe extraction. The same holds for firms operating in the shadow economy (Choi and
Thum, 2005). In such cases, the official will still maximize personal revenue but only from
bribes extracted from new non-shadow entrants. It turns out that this can dramatically
curtail his ability to limit competition. Indeed, instead of letting in one firm only, the official
will actually find it in his personal interest to allow multiple entrants in (as many as the
number of existing firms in the industry plus one, when entry costs are zero and under the
present specification). Intuitively, this is because by allowing one more firm in, he captures a
percentage of a higher market share accruing to the new firms together, but at the same time
a lower bribe from each firm due to the increased competition. Balancing these conflicting
effects will generally give rise to more than one new firm entering the market.

The second part of the paper proposes another policy response aimed at avoiding corruption-
induced monopoly in new industries. Instead of delegating the entry process to a single offi-
cial, the government could put in place multiple independent officials, each fully empowered
to certify entry. We assume that all officials would charge the same percentage of each firm’s
profit as bribe for entry, or some variant thereof, so that the underlying competition among
officials is not played out in terms of fees but rather in the numbers of firms allowed in. Under
this mechanism, the firms allowed in by one official might be seen as playing the role of the
existing firms in the previous discussion for the other officials. While it is intuitive then that
this set up will give rise to more competition in the industry, the surprising result is that,
with exactly two officials, it will actually lead to the second-best socially optimal number of
firms, as defined in Mankiw and Whinston (1986) or Suzumura and Kiyono (1987).

The intuition for this provocative result comes from the theory of Cournot mergers
(Salant, Switzer and Reynolds, 1983) and divisionalization (Baye, Crocker and Ju, 1996).\footnote{The mathematical structures of the models used in the present papers are formally equivalent to models of divisionalization, even though the economic settings are drastically different. This equivalence can be seen by identifying a corrupt official in our model with a parent firm in the divisionalization literature.}
Each official foresees that by letting one more potential firm into the market, he lowers the profits of the other firms he has already let in ("his firms"), but he increases the market share of his firms relative to other officials’ firms. It turns out that these two conflicting effects balance each other out for the case of two officials and induce socially optimal behavior on the part of the two officials.

The conclusions of this paper reveal an interesting new finding with no existing counterparts in this literature: That the number of existing firms along with the shadow firms in the same industry turns out to be an important determinant of the (endogenous) market structure in corrupt environments. One consequence is that with such firms, the socially optimal number of officials is either two or one, depending on the number of existing firms (including those in the shadow economy). This result sets welcome limits on the benefits of inter-official competition in curtailing the negative consequences of corruption, thus making the idea more realistically applicable. The fact that a "monopoly official" will often emerge as the socially optimal solution is rather counter-intuitive, and goes against what has emerged as the conventional view on the benefits of inter-official competition in corrupt environments (e.g., Rose-Ackerman, 1978 and Shleifer and Vishny, 1993).

The importance of the number of existing firms opens up some important policy options. To formulate these, we assume throughout this paper that, while the relevant officials are always corrupt, the government in place is benevolent, and aware of the presence and extent of corruption, though unable to implement any effective eradication strategies (despite setting up a random auditing scheme). This dichotomy is commonly postulated in the closely related literature although Bliss and Di Tella (1997), Shleifer and Vishny (1993) and Dhillon and Rigolini (2011) posit that corruption operates with full impunity. The first of the policy options is to always rely on two independent entry-certifying officials for new industry launches. Another policy option is to issue a suitable number of initial "special" licenses to enter a new industry through some other (special) authority before placing that industry under the authority of a single corrupt officer. This would clearly undermine the ability of the official(s) to curtail the level of competition in the industry, in light of the aforementioned results. These points will be discussed in more detail later on.

Since the conclusions described so far were obtained in the context of oligopoly with linear demand and costs, it is natural to raise the issue of robustness of the analysis. To address this issue, in a third part of the paper, we consider two inter-related extensions for the model without pre-existing firms, a quadratic cost function and a first best planning criterion (while keeping a linear demand function). This allows us to compare free entry, first best entry, second best entry and entry under multiple corrupt officials. The main conclusion of this part of the paper is that the basic insights of the paper essentially carry
over in a qualitative sense. More precisely, we show that (i) instituting competition in entry
certification will increase entry and welfare, and (ii) second best and first best numbers of
firms may respectively be reached by suitably tailoring the number of competing officials
to the strength of diseconomies of scale. However, the first best welfare level cannot be
replicated under corrupt entry, unless the diseconomies of scale are sufficiently strong. In
the latter case, an unexpected result of independent interest for the normative theory of entry
is that first best and second best entry regulation actually yield nearly identical outcomes.

While the institutional arrangement of introducing “competition in corruption” appears
a priori to be of a normative nature, it may also be construed as a policy measure that has
been previously applied in some real world settings. This issue figures prominently in Shleifer
and Vishny (1993), as one possible scenario of interest in the “industrial organization” of
corruption. These authors emphasize the importance of the latter in determining key mar-
et outcomes and the actual level of corruption. However, in contrast to the present paper,
the underlying mechanism in their perspective is simple Bertrand-style competition among
officials, which drives bribes down to zero, thus increasing consumer surplus in the relevant
markets.\textsuperscript{3} In their terminology, one might rephrase the motivation of the present paper as
being concerned with the social efficiency effects of conducting some (discrete-type) compar-
ative statics of varying the industrial organization of corruption in an exogenous manner.
As such, the approach taken in this paper could also be construed as fitting the general
theme of mechanism design as it is commonly invoked in such fields as public and regulation
economics. The idea is indeed that the hidden principal (or benevolent government leader)
embeds the self-interested behavior of corrupt officials within a suitably designed larger game
to induce them to collectively take actions that mimic those of an honest social planner in
terms of the relevant outcome of interest: The emerging market structure. There remains
one important caveat of course: Bribes are still collected in the process.

This brings us to the need to list important limitations of our analysis, which lie well be-
yond the scope of this paper and of the closely related literature. Our analysis is silent about
the broader implications of corruption, e.g., in breeding more corruption in other areas, in
destroying citizens’ faith in government and its civil servants, and in contributing to weak-
ening the rule of law, among other factors. More importantly, within the narrower present
context of industry entry, our model does not incorporate welfare losses related to allowing
entry by firms that would otherwise fail honest and often crucial health, environmental or
safety tests, thereby endangering consumers’ well-being and perhaps even lives.\textsuperscript{4} Indeed,

\textsuperscript{3}For instance, they conjecture that the lack of bribes in issuing passports in the US is probably linked to
the fact that citizens can choose between several offices for this service.

\textsuperscript{4}In an interesting survey on corruption, Svensson (2005) reports some important real-life examples of
these hidden and potentially very high social welfare costs.
our postulates of purely selfish behavior on the part of the entry officials and of absence of effective monitoring could hardly be consistent with the possibility that they might be conducting their certification tasks with much conscientiousness.

That competition among corrupt officials may indeed restore efficiency in the entry process is an interesting illustration of the view presented by Lipsey and Lancaster (1957) in their unified treatment of the theory of second best.\(^5\) In the presence of imperfect competition as a key distortion for the entry process, it is not all that surprising that corruption, treated as yet another distortion, might improve overall efficiency. Here, the right amount of corruption actually goes all the way to restoring (second-best) efficiency, in yet another instance of a common theme in the broader literature on corruption. What is much harder to anticipate is the main take-away message of the paper: That the socially optimal “industrial organization” of corruption is achieved with only one or two independent certifying offices. Yet, in contrast to the conventional wisdom, this optimal structure does not eradicate corruption in any sense; bribes are still part of the solution. Instead, corruption is co-opted to neutralize free entry, the other market imperfection in this economic environment.

The rest of the paper is organized as follows. The next section summarizes the three well known benchmark cases for endogenous entry: Free entry, second best socially optimal entry, and corruption-induced monopoly. Section 3 covers the novel case of entry under a corrupt official when the market has pre-existing firms. Section 4 introduces competition in corruption and covers the case of entry under multiple corrupt officials. In addition to a conclusion, Section 5 describes two possible extensions, a linear-quadratic cost function and first best entry regulation. An appendix contains the proofs of all the results of the paper.

### 2 The model and the benchmarks

To study the effects of corruption on entry and market structure, we begin our analysis with three benchmarks scenarios of interest: free entry, second-best socially optimal entry, and corruption-induced monopoly. While the first two correspond to standard concepts in the normative theory of industrial organization (e.g, Mankiw and Whinston, 1986), the third scenario is a useful benchmark for the positive theory of industry entry proposed in the present paper. This theory is laid out in the next two sections.

All three benchmarks share the common features that entry is endogenous (though entry decisions are made by different entities in different benchmarks) and competition among firms in the market follows standard Cournot oligopoly. For simplicity, the industry is

\(^5\)In addition, this is also reminiscent of Tiebout’s (1956) well known result that multiple jurisdictions may restore social efficiency in public goods provision (Wooders, 1980).
characterized throughout the paper by a linear market demand for a homogeneous good, given by \( P(Q) = \max\{a - Q, 0\} \), where \( Q \) is the total output produced in the market, and a constant marginal cost of production \( c > 0 \) (common to all firms).

Given the linear nature of demand, it is well known that at the unique and symmetric equilibrium, the per firm profit in the Cournot stage of the game is given by

\[
\pi_n = \left( \frac{a - c}{n + 1} \right)^2, \text{ for all } n = 1, 2, \ldots
\]  

All the scenarios under consideration, with or without corruption, are modeled as multi-stage games with Cournot competition among entrants, and attention is restricted to sub-game perfect equilibrium. The scenarios differ in (key aspects of) their entry processes only.

For those scenarios where corruption is an integral feature, bribery is introduced as an additional cost of entry for firms in the first stage. In order to gain entry into the market, firms must pay a bribe to a corrupt official who certifies their compliance with existing (safety or other) regulation. It is assumed throughout that corrupt officials operate under an exogenous probability of being audited and caught, in which case their gains are forfeited as punishment. Although we posit a benevolent government leader, in view of our interest in the effects of corruption, we shall focus on cases where the monitoring of corruption is ineffective so that the entry official is deterred only by market forces (details below).

We first review as useful benchmarks the three well known cases of free entry, second best social optimum and single corrupt official. The latter benchmark introduces corruption in the entry process and was first suggested by Bliss and Di Tella (1997) in a related model with perfect competition in the market stage. This benchmark may be viewed as a first step in the study of corruption, paving the way for the two novel models of the next two sections.

In all scenarios involving a corrupt official in charge of certifying entry, he is allowed full control of the number of firms that enter the market by issuing a limited number of licenses. In this capacity, he can then extract bribes from firms as a fixed proportion (or commission) \( \lambda \in (0, 1) \) of their anticipated profits. As in the related literature, we have in mind markets where entry requires an inspection procedure to determine compliance with environmental, safety, hygiene or other requirements.

### 2.1 Free entry

This is the standard, pure market solution to the problem of endogenous entry into an industry. There is no interference from the government or any corrupt official. Formally,
consider a two-stage game, where in the first (entry) stage a large (infinite) number of potential entrants simultaneously decide whether to enter the market or not. To enter each firm must incur a one time sunk cost of entry \( K \geq 0 \). In the second (production) stage of the game, upon observing the number of entrants, the firms that have entered compete among each other in Cournot fashion. Firms that have decided not to enter the market in the first stage are assumed to get a payoff of zero (as outside option).

Ignoring the integer constraint for simplicity, it is well known that at the unique symmetric (subgame perfect) equilibrium, each firm is indifferent between the choices of entering or not. Hence, the per firm profit in the second stage of the game is equal to the cost of entry. Hence the uniquely defined free entry number of firms in the market, \( \hat{n} \), is characterized by

\[
\pi_{\hat{n}} = \left( \frac{a - c}{\hat{n} + 1} \right)^2 = K \quad \text{or} \quad \hat{n} = \frac{a - c}{\sqrt{K} - 1} \quad (2)
\]

We now define the second-best solution to this market as our realistic ideal, as in Mankiw and Whinston (1986) and Suzumura and Kiyono (1987).

2.2 Second best socially optimal entry

This is the standard regulated solution to the problem of endogenous entry into an industry. Consider a benevolent social planner who is empowered to control the number of firms that can operate in an industry (e.g., by issuing licenses), but not to directly influence firms’ behavior once in the market.\(^7\) The entry cost is still the same \( K > 0 \). Formally, this scenario is also modeled as a two-stage game. In the first stage, the social planner decides on the number of firms (out of a large number of potential entrants) to be allowed in the market, with a view to maximize social welfare. In the second stage, the firms that have entered observe the number of firms chosen by the social planner and compete in Cournot fashion.

Along a subgame perfect equilibrium, anticipating the firms’ equilibrium outputs in the second stage, the social planner solves

\[
\max_n W(n) = \int_0^{Q_n} P(z)dz - cQ_n - nK,
\]

where \( Q_n = n\frac{a-c}{n+1} \) is the usual Cournot equilibrium industry output in an \( n \)-firm market.

\(^7\)We shall consider a first best social planner’s solution as an extension in Section 5. This postulates an omnipotent and benevolent social planner that can control both the number of entrants and the outputs they must produce in the market. For interested readers, a characterization of this scenario is given in von Weizsacker (1980) and Suzumura and Kiyono (1987).
Solving yields the unique second best socially optimal number of firms as

\[ n^* = \left( \frac{(a - c)^2}{K} \right)^{\frac{1}{3}} - 1. \]

This number of firms is taken as the second best ideal for much of the paper. It maximizes total welfare under the constraint of no government interference in firms’ market conduct.

In the next benchmark and in the new models of later sections, we investigate how corrupt officials may distort the number of firms in the market for their own selfish gains.

### 2.3 Corruption-induced monopoly

As a final benchmark and a first pass at corruption, we consider a modification of the previous scenario wherein a corrupt official is fully in charge of the entry process to the industry. Each firm needs a license to enter the market, and the license can be acquired only upon approval from the official. It is assumed that there is no intrinsic cost for the license itself or alternatively that the cost of the license is already included in the entry cost \( K > 0 \).

In terms of formal structure, the model is a two-stage game analogous to the preceding model of regulated entry but with the critical modification that the benevolent social planner is replaced by a corrupt official.\(^8\) The latter’s goal, when choosing the number of firms to enter, is to maximize his expected private revenue from the bribes extracted from the entering firms, knowing that he is subject to a random audit with probability \( 1 - p \) (i.e. \( p \) is the probability of no audit). If audited, he forfeits all his gains (and receives a zero payoff). He is constrained by historical practice, which forces him to set the bribe he extracts from each firm at an exogenously fixed percentage, \( \lambda \in [0, 1] \), of the firm’s profit.\(^9\)

As described, the subgame perfect equilibrium of this simple two-stage game captures precisely one of the possible effects of corruption on market structure that we take as benchmark. However, some tacit assumptions therein warrant further discussion and justification in terms of the plausibility and level of realism of this model. These justifications apply equally well to the other two models with corrupt officials of the next two sections.

First, as implied by the very structure of the two-stage game, the official credibly commits

\(^{8}\) In practice, the socially optimal solution is of course implemented by a public official who issues licenses to entering firms. So effectively, the two models may be regarded as being equivalent except for the (key) fact that in the socially optimal solution, the official is honest and executes his true mandate, whereas in the present model the official is corrupt and cares only about his personal gain.

\(^{9}\) There is anecdotal evidence suggesting that the value of \( \lambda \) is often of the order of 10% in some countries that score high on the corruption index. For instance, as a former top-level official who at various times held top posts including president and federal investment minister in Pakistan, Asif Zardari is widely known as Mr. 10% for exactly this reason (The Economist, 2012).
to a set number of firms in the market in the first stage of the game. Thus he cannot engage in one or more rounds of further entry and bribe collection once the game as described above ends. The presence of this commitment technology may be due to institutional details of the entry process (e.g., the respective lists of the firms allowed and denied entry may be made public), and/or reputational concerns on the part of the corrupt official.

As will become clear, the specific value of $\lambda \in [0, 1]$ is immaterial. While $\lambda = 1$ is allowed, it is unrealistic in that the corrupt official is then able to extract all the surplus from firms. In such a case, though indifferent, firms are assumed to enter as a tie-breaking rule.

The two-stage game at hand might include an additional middle period to allow for the firms selected to enter to either accept the official’s offer, pay the bribe and enter or reject the offer and stay out. While this feature would make the game more realistic, it would not change the equilibrium outcomes in any way.

We now characterize the subgame perfect equilibria of the game. The official has direct control over how many firms he allows to operate in the equilibrium and can foresee the profits of the firms in the final stage of the game. His objective is to select $n$ such that his graft income is maximized. His expected payoff is

$$B(n) = \lambda pn(\pi_n - K) + (1 - p)0 = \lambda pn[(P(nq_n) - c)q_n - K],$$

(3)

where $q_n$ is the per-firm output in a market with $n$ firms and $p$ is the probability of no audit.

Alternatively, we may assume as in Acemoglu and Verdier (2000) that the official decides whether to act honestly and receive a fixed wage $w$, or engage in corruption and receive a payoff of $\lambda n(\pi_n - K) + w$ with probability $p$ (i.e., if not caught) and a payoff of $0$ with probability $1 - p$ (i.e., if caught). Since our focus is on corruption, we posit that the parameters of the model are then such that he rationally decides to engage in corruption. It is easy to see that this amounts to assuming that the wage $w$ is small enough, just as in Acemoglu and Verdier (2000). In this case, this set up would not change the results of the paper.

Since the official’s objective (3) amounts to maximizing industry profits, the unique solution is to pick $\bar{n} = 1$, as observed next.

Remark 1. Corruption in a new industry with a single licensing authority leads to a monopoly.

Thus, perhaps not surprisingly, corruption too causes social inefficiency in the form of extreme under-entry into the market. Combining this with the main result in Mankiw and

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$^{10}$It is trivial to see that the solution $\bar{n} < \hat{n}$, since overall per firm profit is zero at the free entry equilibrium. In other words, the official will deny entry to some firms wishing to enter.

$^{11}$The idea that officials or economic agents may be deterred from “malfeasance” via the payment of a high enough wage goes back to Becker and Stigler (1974).

$^{12}$A version of this result is discussed in Bliss and Di Tella (1997) and in Emerson (2006).
Whinston (1986) yields (when ignoring the integer constraint)

\[ \tilde{n} = 1 \leq n^* \leq \hat{n}. \]

In other words, when the public official in charge of the entry process is fully corrupt, the resulting market structure is always monopoly, or under-entry at its extreme. The outcome is thus what one may aptly term corruption-induced monopoly.\(^{13}\)

In the next two sections, we present two new models based on the interplay between corruption and endogenous entry.

3 Single official in a pre-existing market

By their very definition, the three benchmark scenarios presented above all tacitly assume that the industry is *new*, in the sense that no existing firms were present in the market prior to the play of the two-stage game. For the third benchmark, this amounts to positing that the tenure of the corrupt official (in the entry-certifying office) began before the industry itself, or at least before its certifying process was instituted. One natural avenue of extension is to consider the realistic case where there are pre-existing firms active in the market. Another extension of interest considers the situation where entry may be certified by any one of several officials, who compete by choosing the number of licenses they would each issue, independently of others (see Section 4).

We shall demonstrate that a benevolent government that has no means for eradicating corruption may nevertheless design optimal institutions (choosing the correct number of officials or allocating a number of special licenses) that yields the optimal number of firms without direct interference. This is a common theme in the corruption literature, where optimal designs are suggested that integrate corruption as a fact-of-life feature of the model, with a view to reach or move towards social optimality (Shleifer and Vishny, 1993).

To study a market with pre-existing firms, we modify the model by positing that a number \( \ell \) of pre-existing firms in the market are currently in operation. Now, in the first stage the official must choose the number of new firms, \( m \), to issue licenses to. To do so, as before, the official extracts a fixed proportion \( \lambda \) of firm profits as a personal bribe. However only the new entrants pay the bribe, as pre-existing firms are beyond the control of the official and thus not under any obligation to pay any proportion of their profits to the official.\(^{14}\)

\(^{13}\)This result is clearly general in nature, since it holds whenever industry profit is maximized by monopoly (see Amir and Lambsom, 2000).

\(^{14}\)While there is anecdotal evidence of officials harassing pre-existing firms for bribes to allow them to continue operating in some industries, we tacitly assume here that firms’ licenses are granted with an indefinite
An alternative interpretation of the presence of the pre-existing firms is to think of them as forming the shadow or informal economy, the relevance and estimated size of which is well documented in the public economics literature; for instance, Choi and Thum (2005) provide a detailed account of this sector with a wealth of supporting data for many countries. Such an interpretation would presume that the corrupt official is sophisticated enough to foresee that the presence of these shadow firms side-by-side with registered firms would affect the profitability of the latter, and hence his own bribe revenue. Here, we are tacitly positing that the shadow firms are outside the reach of the official for bribing purposes, a realistic assumption in most settings.\footnote{The shadow firms are not explicitly modeled as strategic agents. It is realistic to assume that their number is relatively small so as to leave room for legal firms. As to the timing of their entry decision, it is most convenient to think of them as first movers, and of the legal firms as second movers. (All firms are then simultaneous movers in the Cournot market). All firms may alternatively be seen as simultaneous movers in entry (in the usual sense that each firm has no direct observability of others’ actual entry times), but then one needs to assume rational expectations of the entry of shadow firms on the part of the legal firms.} Although we think of this alternative explanation as an important remark for the overall relevance of our model here as well as in the next sections, we henceforth return for future discussion to the “pre-existing firms” version for definiteness.

The fact that the bribe has the structure of an ad valorem tax is a critical feature of this model. Such bribes tend to minimize distortions in economic activity and are also observed in practice. This assumption follows Acemoglu and Verdier (2000) but departs from Bliss and Di Tella (1997), where entry bribes are modeled in the form of a fixed cost that ends up driving inefficient firms out of the market.

The official’s maximization problem now adjusts for the change in market conditions; new entrants face higher competition (than in the previous model) and generate lower profits and thus bribe revenue. Given a small enough initial number of firms and suitable market conditions that allow for further entry, the official’s maximization problem is given by

\[
\max_m B(m; \ell) = \lambda pm (\pi_{\ell+m} - K) = \lambda pm \left[ \left( \frac{a-c}{1+\ell+m} \right)^2 - K \right],
\]

where \( \pi_{\ell+m} = \frac{(a-c)^2}{(\ell+m+1)^2} \) is the per firm profit in equilibrium when the official allows \( m \) firms to enter the market, so that a total of \( (\ell + m) \) firms compete in the second stage.

The following lemma yields that the optimal number of firms that maximize the official’s bribe income is unique and independent of the bribe rate \( \lambda \).\footnote{All the proofs of the paper are relegated to the Appendix.} Here, the assumption \( \ell < \sqrt{\frac{(a-c)^2}{K}} - 1 \) is there to rule out the uninteresting cases where there are more existing firms in the industry than under free entry. For such cases, we always have the trivial solution \( m = 0 \).

(or sufficiently long) horizon and need no renewal.
Lemma 1. In a market with $\ell$ pre-existing firms and entry cost $K$ such that $\ell < \sqrt{\frac{(a-c)^2}{K}} - 1$, the unique bribe maximizing number of licenses $m(\ell)$ is implicitly defined by the equation

$$(a-c)^2(1+\ell-m(\ell)) = K(1+\ell+m(\ell))^3. \quad (5)$$

For the special case $K = 0$, this reduces to the closed-form solution $m(\ell) = \ell + 1$. When $K > 0$, the solution is such that $m(\ell) < \ell + 1$.

The model is a generalization of the third benchmark (corruption-induced monopoly) and so it is to be expected that $m(0) = 1$, that is when the market is new the corrupt official allows the entry of only one new firm. However the presence of pre-existing firms changes the total number of licenses issued by the official in a critical way. The following example illustrates Lemma 1 more precisely.

Example 1. Let $a - c = 20$ and $K = 1$, the number of firms that the official allows entry, $m(\ell)$, corresponding to the number of pre-existing firms ($\ell$) is

<table>
<thead>
<tr>
<th>$\ell$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m(\ell)$</td>
<td>0.98</td>
<td>1.86</td>
<td>2.57</td>
<td>3.10</td>
<td>3.48</td>
<td>3.71</td>
<td>3.83</td>
<td>3.85</td>
<td>3.78</td>
<td>3.65</td>
<td>3.45</td>
</tr>
</tbody>
</table>

While the result might come as a surprise at first glance, it is actually quite intuitive on second thought, upon invoking basic insights from the Cournot theory of mergers (Salant et al., 1983) and divisionalization (Baye et al., 1996). We first observe that since his objective amounts to maximizing the total profit generated by the entrants ($m\pi_{\ell+m}$), instead of industry profit ($(\ell+m)\pi_{\ell+m}$), he will select $m > 1$ firms. For otherwise, the choice of one firm would mean that a merger of $m$ firms is profitable in an industry with $(\ell + m)$ firms for any $m > 1$, a clear violation of the well known 80% rule (Salant et al., 1983). More generally, at an intuitive level, when considering adding one more firm in, the official faces the following trade off: This action will lower (his cut of) the profit of each of the new firms he has already allowed in, but will also increase the overall market share of the new firms in the industry. In terms of overall bribe revenue, the first effect is negative while the second is positive. For the simple benchmark of zero entry cost ($K = 0$), he reaches the optimal trade off between these two conflicting effects by letting in $m(\ell) = \ell + 1$ firms, as is easily seen from (5).

The interplay between the level of corruption and industry competitiveness has long been a topic of major interest, both in policy circles and in (academic) development economics. The above result suggests that the effects of corruption on competition are more subtle than previously understood. Corruption indeed reduces the level of competition in an industry
when compared to the market solution of free-entry, as the latter would by definition generate no bribe revenue at all for the official. However, corruption does not always completely stifle competition; it does so only for new industries with no shadow firms, a somewhat exceptional case.\textsuperscript{17} Otherwise, the presence of pre-existing competition reduces the effects of corruption on the final market outcome. In fact, in case the initial number of firms is sufficiently high, it may even lead to a final number of firms that exceeds the usual benchmark of the second-best socially optimal number of firms.

As to the converse question of whether competition can alleviate or eradicate corruption, part of a prominent policy debate, the model at hand makes it clear that the market structure and the level of corruption are co-determined endogenously in equilibrium, as previously pointed out by Bliss and Di Tella (1997) in their related model.\textsuperscript{18} Nevertheless, to address this policy question within this model, we may consider total bribe revenue (the returns from corruption), as a measure of the extent of corruption, and the number of firms in the industry as a measure of competition. Then, our result predicts that high levels of competition will be associated with low levels of corruption (i.e., low industry profits). Returning to the original question here, one may be tempted to conclude that competition does alleviate corruption. However, this would be a misplaced conclusion since the key underlying causal factor in this setting is the existing number of firms (including the shadow firms). The higher the latter number, the higher the level of competition. As to the extent of corruption, it should perhaps be viewed as constant and maximal in this setting, since the corrupt official is limited solely by the economic constraints he faces, and by no other factors. In other words, the existing number of firms can drastically curtail the negative effects of corruption on market structure and social welfare, rather than on corruption per se (which is taken as a given here).

In addition, if $\lambda$ is taken as a measure of corruption, then since the equilibrium number of firms in the market is independent of $\lambda$, i.e it does not matter if the official charges 10\% or 90\%, the effect of corruption on market structure and on social welfare will be constant.\textsuperscript{19} Another implication of interest is that the presence of the shadow (or underground) firms serves to limit the corrupt official’s ability to distort market structure, in line with the conclusions of Choi and Thum (2005).\textsuperscript{20} The result also suggests a way to harness an

\textsuperscript{17}Indeed, many of the industries that ordinarily fall into the scope of this model would have been in existence long before a particular corrupt official takes office and tend to include shadow firms. Some examples are taxi cabs, restaurants, pubs, night clubs, medical practices, private detectives, etc.

\textsuperscript{18}However, in contrast to Bliss and Di Tella (1997), the present model does not give rise to any selection effect in industry entry since all firms are assumed identical.

\textsuperscript{19}In the welfare evaluation, the bribe is a simple money transfer and thus has no effect on welfare.

\textsuperscript{20}Friedman et al. (2000) establish empirically that the extent of corruption and regulation (and not taxes) drives firms towards the underground economy. In contrast, our model takes the number of underground firms as a primitive and investigates its effect on entry under corruption.
official’s desire to be corrupt to benefit society as a whole. A benevolent social planner may choose to design an optimal industry, such that the official lets in exactly the same number of firms as would be socially optimal. This result is formalized in the following proposition.

**Proposition 1.** Consider an industry with \( \ell \geq 1 \) pre-existing firms. If \( \ell < \sqrt{\frac{(a-c)^2}{K} - 1} \),

(i) corruption induces the second-best optimal number of firms if and only if \( \ell = \ell^* \) such that
\[
\ell^* + m(\ell^*) = n^*.
\]

(ii) corruption leads to more (less) firms than is socially optimal if
\[
\ell + m(\ell) > (<) n^*.
\]

This Proposition stands in sharp contrast to corruption induced monopoly. It attributes a critical role in understanding the effects of corruption on market structure to a seemingly innocuous parameter of the model: The existing number of firms in an industry. If this takes the particular value \( \ell^* \), then the official will find it in his own interest to simply mimic an honest social planner (while nevertheless collecting a percentage bribe). Furthermore, if the existing number of firms exceeds \( \ell^* \), then corruption will give rise to a final number of firms intermediate between the second-best optimum and free entry.

Proposition 1 also suggests a number of interesting policy options that a benevolent government leader with no direct control over the official’s behavior could implement, depending on the specific conditions in a particular industry. For a mature industry, which is the prototypical case including restaurants and taxi cabs, the initial number of firms may be close enough to \( \ell^* \) that no particular intervention is warranted.

For a new or emerging industry, a benevolent leader will be well advised not to rely on a corrupt official, as the result will be corruption-induced monopoly. Instead, such a leader should contemplate a number of possible fortuitous modifications to this ordinary setting. He may deliver some initial licenses through special channels connected to his office and not the official’s, the aim being to raise the initial number of firms from none or a few to something close or equal to \( \ell^* \). After this step is completed, the entry process may be delegated to an official once more, knowing that his interests are now more or fully aligned with those of society. In addition, our results provide new support for the oft-propounded policy of rotating officials across different industries.

Finally, in terms of casual empirical evidence, the results of this section are fully consistent with an elementary but potent observation: Even in the most corrupt economies, there are
simply not that many monopolies in operation in industries with entry certification! Thus the corruption-induced monopoly theory is contradicted by an obvious stylized fact.

In the next section, we investigate an alternative measure suggested for combating bribery: Setting up competition among corrupt officials.

4 Competition among officials

Competition among officials has long been suggested as an antidote for corruption in the economics literature. Rose-Ackerman (1978) introduced the concept of “overlapping jurisdictions” of licensing authorities, wherein multiple officials would be granted the authority to independently issue licenses for the same industry.\textsuperscript{21} The argument at its core relies on (Bertrand-style) price competition among officials undercutting each other, until eventually bribe rates are driven to zero, spelling an end to corruption. Shleifer and Vishny (1993) propound this view and theoretically analyze overall market performance depending on the “industrial organization of corruption”.

While we utilize the concept of multiple licensing authorities with overlapping jurisdiction in a direct manner here, we introduce a novel twist. We posit that officials in fact charge identical bribe rates $\lambda$, but compete in the number of licenses they issue.\textsuperscript{22}

There are multiple reasons for positing that competition in quantity may constitute a plausible alternative, at least for some potential settings captured by this model. The first is that, due to the level of secrecy that typically surrounds such transactions, the bribes actually paid by firms often remain unknown to non-transacting parties. Yet, the standard Bertrand mechanism of price theory naturally relies on the broad-based observability of actual prices. The second reason is that, while this measure may lower the bribes, it need not drive them to zero altogether. In many cases the officials do incur a genuine cost in processing an application, which includes their opportunity cost of filling out paperwork and conducting inspections. Thus they can be expected to charge at least their marginal cost even when Bertrand competition (among officials) has considerable bite.

\textsuperscript{21}The original recommendation is more far reaching than the scope of the current paper. The argument extends to all matters of state where the corruption may hinder the business of the state, including licenses in an industry, the police force, etc.

\textsuperscript{22}Interestingly, such “quantity” competition among officials can be seen as Cournot style competition (as opposed to the Bertrand style competition suggested previously in the literature), in the $m_i$ variables, but with a hyperbolic inverse “demand function” given by the bribe function

$$\lambda p \left[ \left( \frac{a - c}{1 + m_i + m_{-i}} \right)^2 - K \right].$$
Finally, in corrupt environments, equal bribes might also result from a partial collusive agreement between officials, which fixes a common level for the bribe (say, at some historical, pre-competition level) without extending the deal to the number of licensees each official may issue. Partial collusion of this sort seems like a plausible reaction on the part of officials to the institution of competition in their services. We shall see that, in relevant cases, the number of officials will be small enough that partial collusion might well be sustainable.

Alternatively, price competition of a limited sort may easily be accommodated within the present model. The different officials may be seen as offering somewhat differentiated services (in processing time, reputation, advice, ...), in which case they may charge firms different proportions of their equilibrium profits (or different λ’s), as long as all these λ’s remain fixed and not subject to competitive forces. In addition, even when services are homogeneous, secrecy and fear of retribution may provide sufficient friction to prevent firms from shopping around for the lowest bribes, so that different λ’s could still prevail. Our analysis is easily seen to extend to such cases, as long as all the λ’s remain constant.

We now consider the two cases of new and pre-existing industries separately.

### 4.1 The case of a new industry

Formally, we modify the previous model by assuming that there are \( g \geq 2 \) independent (corrupt) officials in charge of regulating entry into a new industry. In the first stage, official \( i \) simultaneously chooses \( m_i \), the number of firms he allows to enter. Each official as before receives bribes as a fixed proportion \( \lambda \) of the firm’s profit for his services. However he must now account for the effects on firm profit of competition from the firms let in by other officials.\(^{23}\) Recognizing these strategic considerations, each official seeks to maximize his bribe income and in equilibrium best responds to the number of licenses issued by his colleagues (as the industry is new, there are no pre-existing firms to consider). Then, official \( i \)’s payoff, as a function of all the numbers of firms chosen by the \( g \) officials, and contingent on a Cournot equilibrium in the second stage involving all firms allowed in, is

\[
\max_{m_i} B_i(m_1, m_2, ..., m_g) = \lambda p m_i \left[ \left( \frac{a - c}{1 + m_i + m_{-i}} \right)^2 - K \right],
\]  

\(^{23}\)In the case of the divisionalization literature Baye, Crocker, and Ju (1996), this is equivalent to the situation where each of the \( g \) existing firms engages in divisionalization.
where \( m_{-i} = \sum_{G\setminus\{i\}} m_i \) is the sum of all other officials' selected numbers of firms and \( p \) is the probability that no audit is conducted and corruption goes unhindered.\(^{24}\)

The following lemma characterizes the unique subgame perfect equilibrium.

**Lemma 2.** In a market with entry cost \( K \), \( g \) licensing authorities and no pre-existing firms, there exists a unique (and symmetric) number of licenses \( \bar{m} \) issued by each official, which is implicitly defined by the following equation

\[
(a - c)^2(1 + (g - 2)\bar{m}) = K(1 + g\bar{m})^3.
\]

In the unique subgame perfect equilibrium, each official chooses to issue \( \bar{m} \) licenses and so the market consists of \( g\bar{m} \) firms. This number is lower than the free entry case and higher than the monopoly that would be imposed when there is a single official doling out licenses.

For the impact on social welfare, we focus on the case of two licensing authorities. Then the number of licenses issued by each agent is given by

\[
\bar{m} = \frac{1}{2} \left[ \left( \frac{(a - c)^2}{K} \right)^{\frac{1}{3}} - 1 \right].
\]

It is then obvious that the total number of firms in equilibrium is the same as the second best optimal number of firms per Mankiw and Whinston (1986), i.e \( 2\bar{m} = n^* \). The following Proposition formalizes this result and establishes the uniqueness of the number of licensing agents that can give rise to the social optimal number of firms.

**Proposition 2.** Competition among officials leads to the socially optimal number of firms if and only if there are two independent officials in charge of issuing licenses.

To benevolent leaders of a country with known widespread corruption in government services, the benchmark of second-best socially optimal number of firms can only form an unattainable ideal. Yet, this simple institutional reform can actually deliver this market ideal as a Nash equilibrium of the relevant strategic game, which in addition does not require any form of costly monitoring or supervision. Such a policy would be particularly beneficial in weak states where eradicating corruption and finding “officials of unquestioned integrity” are often fruitless endeavors, due to political, cultural and historical factors. Once a government sets up such an optimal institution, the rational forces of individual choice on the part of all the players involved in the game yield the socially optimal outcome. In fact, the

\(^{24}\)It is sensible to posit equal treatment, so that either all officials are audited or none are (we will have more to say about audits below).
underlying mechanism utilizes the officials’ compulsion to be corrupt for the benefit of society, in providing a remedy to the inefficiency of the market solution to free entry. Since corrupt behavior is actually part of the optimal solution to the entry problem, there is no need to keep the (costly) random audit scheme in place (i.e., one can set \( p = 1 \) for this model).

The intuition from the theory of mergers described in the previous section extends to two officials, each trading off the profit per firm allowed in and the market share of all his firms in the total industry. With exactly two officials, for the case of linear demand and costs (see Section 5), the process of adjusting this trade off creates the right incentives for each official to behave in a way compatible with that of an honest social planner.

As discussed in more detail in a more general context in the next subsection, a direct corollary of this result is that using three or more officials is socially suboptimal, with welfare monotonically declining in the number of officials beyond two (welfare is shown to be concave in the number of firms in the proofs).

The result does not depend on the value of the bribe, which can be interpreted as one possible measure of the level of corruption. In such a case the government may indeed doubly benefit, if it is the case that inter-officials competition lowers the level of corruption. A social planner is then able to reduce both the size and negative impact of corruption, while also avoiding the inefficiency caused by free entry.

This solution shares one drawback with the second-best social optimum: They lead to market outcomes that are not entry proof. Once \( 2\bar{m} \) firms have entered, further entry remains profitable, and more firms would therefore wish to enter. Thus, firms that have paid the bribe to enter must be guaranteed that the officials will not allow further opportunistic rounds of entry and concomitant bribe collection. In other words, there is an issue of credibility, some form of which is common to most transactions under corruption. Within the two stage game model, this issue is assumed away by construction, since officials commit to their chosen numbers of firms in the first stage by the very structure of such games. From a real-life point of view, such an assumption is motivated by reputational concerns on the part of corrupt officials, a feature supported by anecdotal evidence (The Economist, 2012).

Given the novel nature of the exercise, examples of direct application are difficult to find. However a close example can be found in the experience of Singapore’s custom department. Perceived as a corrupt department, the government introduced a second (privately contracted) department with overlapping jurisdiction (Bardhan, 2006). This is believed to have been successful in reducing the level of corruption in the department. Proposition 2 however suggests a more important effect of this change, the introduction of an additional official may additionally have induced the socially optimal number of licenses to be issued.

Overall, this constitutes a novel approach of using market based mechanisms to yield
a socially optimal solution in markets characterized by two sources of inefficiency. This approach provides a single solution to two fundamental problems. In one stroke it allows the social planner to avoid the disastrous consequence of a monopoly in the market due to corruption and it also surprisingly rids the market of inefficiency due to free entry by instituting the right amount of competition among corrupt officials.

4.2 The case of a pre-existing industry

In case the industry has been in existence prior to the appointment of the current officials, there will be some pre-existing (say \( \ell \)) firms operating in the market. This feature will alter the game faced by the officials. While it is postulated here that they will continue to charge a bribe equal to a fixed proportion of firms’ profits in period 2, they must take into consideration the effect of increased competition not only from the firms allowed to enter by their colleagues, but also from the pre-existing firms whom they have no control over.

In this case, official \( i \)'s payoff function (given \( \ell \) pre-existing firms) is

\[
\max_{m_i} B_i(m_1, m_2, \ldots, m_g; \ell) = \lambda p m_i \left[ \left( \frac{a - c}{1 + m_i + m_{-i} + \ell} \right)^2 - K \right].
\]

Although the presence of pre-existing firms leads to an analytically less tractable problem, a clear-cut result still obtains.

**Proposition 3.** In a market with pre-existing firms, the socially optimal number of officials \( g^* \) is either one or two, according as the existing number of firms is high or low.

This finding is quite intuitive in light of the previous results of the paper. As shown formally in the proof in the Appendix, this conclusion rests on two key (comparative statics) facts that are worth pointing out here:

(i) the final equilibrium number of firms in the market is increasing in both the number of existing firms \( \ell \) and the number of officials \( g \), and

(ii) social welfare is strictly jointly concave in the same two variables.

It follows directly from (i) that choosing three or more officials is never optimal, and then from (ii) that one or two officials might be optimal, depending on the value of \( \ell \), keeping in mind that welfare is maximal with \( g = 2 \) officials and \( \ell = 0 \) existing firms (Proposition 2).

Thus, the presence of pre-existing firms alters the final conclusion on the benefits of competition in license provision in corrupt environments in a manner that goes against the
conventional wisdom on the usual benefits of competition: Minimal or no competition is best, and three or more suppliers is always detrimental.

In order to emphasize the point that monopoly in entry-certifying offices can actually be socially optimal, we provide an explicit example.

**Example 2.** Let \( a - c = 40 \) and \( K = 1 \). Short computations (with details left to the reader) establish the following comparisons:

(i) With \( \ell = 3 \) pre-existing firms, welfare decreases from 780.11 with \( g = 2 \) officials to 779.84 with \( g = 1 \) official.

(ii) With \( \ell = 4 \) pre-existing firms, welfare increases from 779 with 2 officials to 782.61 with one official.

(iii) Welfare is maximized by having two officials for \( \ell = 1, 2, 3 \) and one official for \( \ell \geq 4 \). In other words, \( g^* = 2 \) for \( \ell \leq 3 \) and \( g^* = 1 \) for \( \ell \geq 4 \).

In conclusion, monopoly in entry certification is socially optimal whenever there are at least 4 pre-existing firms in the market. Otherwise, duopoly is socially optimal.

In terms of policy prescriptions, the conclusion of this subsection is important in a number of different dimensions. The first major implication is that the socially optimal design for the entry certification process never calls for instituting three or more certifying offices. From a social standpoint, the ideal “industrial organization of corruption” is always quite simple: Either a duopoly or a monopoly, depending on the existing number of firms in the industry. Raising the extent of competition among corrupt officials beyond duopoly is always detrimental to welfare, the more so the further one gets from duopoly. All this would do is move the industry closer to the free entry solution and away from the second best ideal (e.g., Mankiw and Whinston, 1986). This conclusion is quite remarkable, particularly when one takes into account the fact that the underlying competition is akin to Cournot, rather than Bertrand, competition. In other words, competition proceeds not in terms of bribe setting but rather in the “quantities” of firms allowed in by each official. Recall that the latter feature makes our use of competition among corrupt officials quite distinct from what was widely envisaged in the extant literature. Nevertheless, the conclusion that monopoly could easily emerge as the socially optimal form of entry certification in a corrupt environment is certainly quite provocative and difficult to anticipate on intuitive grounds.

Another implication of this result is that it cautions against the prior recommendation based on the superiority of having two officials for industries with pre-existing firms. As such,
it is yet another reminder of the importance of the distinction between new and existing industries when designing optimal institutions to regulate entry into markets when a necessary certification process must be delegated to corrupt officials. A similar remark applies to the distinction between industries without shadow firms and those with such firms.

Naturally, if one were to take into account the (fixed) costs associated with instituting certifying offices, the inadequacy of oligopoly would become more pronounced.

In terms of limitations, it is fair to observe that this conclusion is predicated on the mode of competition being in the numbers of firms as well as on the lack of full collusive behavior on the part of the officials involved. Admittedly, postulating non-collusive behavior in this particular context is somewhat of a leap of faith. Indeed, these officials typically need not fear any antitrust-like enforcement against full collusive behavior. On the other hand, the social planner will typically hold various suitable means to foster a climate of rivalry between the competing officials. If one were to take into account the possibility of collusive behavior, then a more diffuse industrial organization of corruption might be to the advantage of society, simply because of the well known fact that collusion is more difficult to sustain when more participants are involved.

5 Extensions: Quadratic costs and first-best solution

In this section, we consider two natural extensions to the basic analysis of this paper: Decreasing returns to scale in production (or strictly convex cost function) and first best entry regulation. These two extensions are closely inter-related since first-best planning yields a monopoly under constant (or decreasing) returns to scale. The basic question raised here in one of robustness: To what extent do the main results of Section 4 extend to these new specifications? We shall compare standard free entry, first and second best socially optimal entry, and entry under multiple corrupt officials, for the following specification. Consider an $n$-firm symmetric Cournot oligopoly with inverse demand $P(Q) = \max\{a - Q, 0\}$, and cost function $c(q) = cq + dq^2$, and assume that $a > c > 0, d \geq 0$. This allows us to address both extensions in one stroke, and paves the way for a broad scope of comparisons.

The per-firm output and profit and the social welfare at the unique Cournot equilibrium in a $n$-firm Cournot oligopoly are easily computed as:

$$q_n = \frac{a - c}{n + 1 + 2d}, \quad \pi_n = \frac{(d + 1)(a - c)^2}{(n + 1 + 2d)^2}, \quad W_n = \frac{n(n + 2 + 2d)(a - c)^2}{2(n + 1 + 2d)^2}.$$
We also assume, to ensure that at least one firm is viable in the industry (i.e., \( \pi_1 \geq K \)), that

\[(a - c)^2 \geq 4K(d + 1). \quad (8)\]

We begin with a brief exposition of the various scenarios and their equilibrium outcomes.

5.1 Free entry and second best solution

Under the zero profit condition, \( \pi_n = K \), the free-entry number of firms is the solution to

\[(d + 1)(a - c)^2 = K(n + 1 + 2d)^2.\]

Solving yields

\[n = \frac{\sqrt{d + 1(a - c)}}{\sqrt{K}} - 1 - 2d.\]

Under second best planning, the social planner’s objective function is

\[\max_n \int_0^{nq_n} P(s)ds - cnq_n - dq_n^2 - nK \quad \text{or} \quad \max_n \frac{n(n + 2 + 2d)(a - c)^2}{2(n + 1 + 2d)^2} - nK.\]

The first order condition w.r.t. \( n \) reduces to, upon simplification,

\[(a - c)^2[2d^2 + (n + 3)d + 1] = K(n + 2d + 1)^3. \quad (9)\]

The second best number of firms, \( n_2^* \), is easily shown to be the unique real root of (9).\(^{25}\)

5.2 The first best solution

We postulate here an omnipotent government that is endowed with full control of both the number of firms that will enter an industry and of their market conduct once in. We thus follow von Weizsacker’s (1980) seminal study that pioneered the normative literature on industry entry, also in the same context of Cournot competition with linear demand and quadratic costs. While most often considered only as a useful benchmark for market performance, this form of planning is also of some relevance in some industries that fit the scope of the present study, such as taxi cabs, private schools,...

\(^{25}\)Throughout this section, we shall provide the main equations but withhold all the computational details for the sake of brevity.
The objective function of the planner then is

\[ \max_{n,q} \int_0^{nq} P(s)ds - cnq - dq^2 - nK \]

The first order condition w.r.t. \( q \) equates price and marginal cost, which yields the optimal per firm output and corresponding welfare

\[ q_n = \frac{a - c}{n + 2d} \quad \text{and} \quad W_n = \frac{n(a - c)^2}{2(n + 2d)} \]

The first order condition w.r.t. \( n \) yields the first best number of firms as the solution to

\[ \frac{d(a - c)^2}{(2d + n)^2} - K = 0, \]

or

\[ n^*_1 = (a - c)\sqrt{\frac{d}{K}} - 2d. \] (10)

### 5.3 Entry with \( g \) corrupt officials

Here we consider entry regulation via the independent operation of \( g \geq 1 \) corrupt officials (and, for simplicity, no existing firms). This scenario is the direct analog of that of Section 4.1, the only difference being that the cost function is now quadratic instead of linear.

Let \( m_i \) be the number of firms allowed in by the \( i^{th} \) official, with payoff function (cf. (6))

\[ \max_{m_i} B(m_i, m_{-i}) = \lambda \frac{(1 + d)(a - c)^2 m_i}{(m_i + m_{-i} + 1 + 2d)^2} - \lambda m_i K \] (11)

The first order condition is

\[ \frac{(1 + d)(a - c)^2(m_i + m_{-i} + 1 + 2d)^2 - m_i(1 + d)(a - c)^22(m_i + m_{-i} + 1 + 2d)}{(m + m_{-i} + 1 + 2d)^4} - K = 0, \]

which, upon computation, reduces to (with \( \bar{m} \) as the number of firms per official)

\[ (1 + d)(a - c)^2[1 + 2d + (g - 2)\bar{m}]) = K(1 + 2d + g\bar{m})^3 \] (12)

We shall refer to this situation as the \( g \)-officials scenario, and denote the resulting number of firms by \( M_g = g\bar{m} \), for all \( g = 1, 2, ... \)
5.4 A four-way comparison

We now report a number of observations concerning the comparative performance of the four scenarios in terms of equilibrium entry and welfare. A word of caution is in order for comparisons involving the first best scenario. The overall analysis tacitly assumes that a modification of the $g$-officials scenario wherein firms are regulated to produce at marginal cost in the second stage, instead of their default Cournot outputs, is not a realistic or meaningful scenario (for the environments under study in the present paper). Were it feasible, such a scenario would eradicate corruption as defined here at once since firms would achieve no profit they could share with corrupt officials. For consistency, we thus take the first best scenario to form a useful benchmark for the present setting, but not a realistic option.

For the sake of a complete analysis, we consider all the possible values of $d$, which range from 0 up to the value of $d$ that makes (8) hold with equality, call it $\overline{d}$. Thus $\overline{d}$ is such that the only viable market structure is monopoly. On the other hand, in practice, the relevant range of $d$ is typically much smaller than that, as confirmed by much empirical evidence.\(^{26}\)

For the sake of brevity, the presentation of the results of this section is less formal than in the rest of the paper (only the main equations are provided). All the findings can all be illustrated in Figures 1 and 2. While these are graphed for specific parameter values, $a - c = 20$ and $K = 4$, these values were chosen in a way that all the possible effects show up, in a robust manner. Figure 1 depicts the four solutions (the last one for multiple values of $g$) in terms of the resulting number of firms and Figure 2 the corresponding welfare levels, as a function of $d$. Since all the graphs in each figure converge to the same point as $d \to \overline{d}$, only a left subset of the domain $[0, \overline{d}]$ is shown.

Before expositing the results of economic interest, we begin with some preliminary observations that will help the reader get familiar with the basic implications of Figures 1. The first deals with the common aspects of the curves for all the scenarios.

**Fact 1.** (i) The numbers of firms for all four scenarios (including the $\overline{M}_g$ curves for all $g \geq 1$) are inverse U-shaped in $d$.

(ii) As $d$ increases to $\overline{d}$, the value that makes (8) hold with equality, all the numbers of firms converge to 1, as expected since monopoly is the only viable market structure.

(iii) For each of the scenarios, welfare in globally decreasing in $d$.

The second point reports the effects of increasing competition among corrupt officials.

**Fact 2.** (i) For the $g$-officials scenario, the resulting number of firms $\overline{M}_g$ is increasing in $g$ (or the corresponding curves shift out as $g$ increases in Figure 1).

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\(^{26}\)Indeed, many studies confirm the prevalence of constant returns to scale in industrial production, or of slightly decreasing returns to scale, see e.g., Basu and Fernald (1997).
(ii) As $g \to +\infty$, the $M_g$ curves converge to the free entry curve.

Figure 1: Comparison of total number of firms in market under the 4 scenarios as a function of $d$, $a - c = 20$, $K = 4$.

Figure 2: Comparison of social welfare in market under the 4 scenarios as a function of $d$, $a - c = 20$, $K = 4$.

This result is quite intuitive and follows from well known results on Cournot oligopoly once one observes that the payoff functions (11) can be thought of as Cournot profit functions,
as in footnote 22 (see e.g., Amir and Lambson, 2000).

The first result of definite economic interest deals with the comparison between first best and second best planning scenarios. Despite the extensive literature on entry regulation, this comparison has not been addressed in earlier work.27

**Fact 3.** The number of firms is always lower for the first best scenario than for the second best scenario. However, the former converges monotonically and rapidly to the latter as \( d \) increases. Thus the two planning solutions differ significantly only for small values of \( d \).

This conclusion is not a priori intuitive. It is thus of quite some independent interest for the broad literature on entry regulation. As this result will feature prominently in the comparisons below, it is important to stress that the coincidence between the number of firms under the two planning solutions need not translate into equal levels of social welfare. Indeed, since firms’ market conduct is not the same in the two cases, social welfare is higher in the first best case than in the second best case, the difference between the two being the usual deadweight loss associated with Cournot behavior. It turns out that, for Cournot competition, the first and second best levels of welfare converge to each other fairly rapidly.

We now uncover another coincidence result for the case of constant returns to scale.

**Fact 4.** When \( d = 0 \), the first best solution calls for a monopoly firm, thus coinciding with the solution of the 1-official scenario, i.e., corruption-induced monopoly.28

Despite this coincidence, the intuitive reasons behind the emergence of monopoly in these two scenarios are of course entirely different. In addition, the market conduct of the resulting monopolist is maximally differentiated across the two cases, since the firm produces at marginal cost in the first best case. Accordingly, the welfare level of corruption-induced monopoly is substantially lower than the rst best (maximal) value of welfare.

The next two main results deal directly with the main motivation of this section, namely a robustness analysis of the conclusion of Section 4.

**Proposition 4.** (i) For small values of \( d > 0 \), the 2-officials scenario yields a number of firms that lies between the first and the second best numbers, i.e., \( n_1^* < M_2 < n_2^* \).

(ii) The difference in welfare between the second best and the 2-officials (or 3-officials) scenarios converges to zero as \( d \) increases, and is significantly positive only for small \( d \).

In other words, under mild diseconomies of scale, instituting competition between two corrupt officials does not give rise to the second best outcome anymore, but sets the resulting

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27It is worthwhile to recall that while von Weizsacker (1980) considered first best regulation, Perry (1984) and Mankiw and Whinston (1986) investigated second best regulation. Even though Suzumura and Kiyono (1987) dealt with the comparison of free entry and both forms of regulation, they did not compare the outcomes of the two planning solutions to each other.

28The same conclusion obviously holds for \( d < 0 \), i.e., in the presence of scale economies in production.
number of firms to somewhere in between the two planner’s solutions. In terms of social welfare, since the 2-officials scenario postulates firms competing a la Cournot, it will always reach a level of social welfare that is lower than the second best solution, which in turn is of course always lower than first best welfare (due to firms producing at marginal cost). However, as part (ii) indicates, the welfare gap narrows quite rapidly as \( d \) increases. While the latter point was unexpected to us on intuitive grounds, it is certainly in line with the coincidence of the two planning solutions for high enough values of \( d \) (Fact 3).

**Proposition 5.** (i) For any value of \( d \geq 0 \), there is a corresponding value of \( g \) such that such that the \( g \)-officials scenario yields the second best number of firms.

(ii) A similar statement holds for the first best number of firms.

(iii) Starting from intermediate values of \( d \), the two critical values of \( g \) (in parts i and ii) are essentially the same.

The conclusion of Part (i) may be viewed as a generalization of the results of Section 4.1. It says that, provided the optimal industrial organization of entry certification (the number of officials, \( g \)) is calibrated to the level of diseconomies of scope in the industry, the key result that competition in entry certification leads to the second best solution is preserved. Thus the main result of Section 4.1 may be seen in light of part (i) as linking the important special case of constant returns to scale (\( d = 0 \)) with \( g = 2 \). Overall then, the conclusion that competition among entry certifying officials is welfare improving extends to the case of decreasing returns to scale and to the case of a first best criterion. However, to reach a social optimum requires calibration of the number of officials to the curvature of costs.

One potential limitation of this generalization is that, while the number of officials required is reasonable for mild diseconomies of scale (or low \( d \)), it might be unrealistically high for intermediate diseconomies (or \( d \)). Nevertheless, it is worth stressing again that the cases of constant or mildly decreasing returns to scale are the most relevant ones for many industries (Basu and Fernald, 1997). Thus, the coincidence results for intermediate or strong diseconomies of scale should be seen as being of limited practical scope.

As to the comparison with first best, although part (ii) appears to be a similar result to part (i), competition in entry certification does not replicate the first best level of social welfare for mild diseconomies of scale, but it does for intermediate or large values of \( d \) (part (iii)), due to near-equivalence with the second best solution.

---

As expected, for \( d = 0 \), the 2-officials scenario yields the second best number of firms (as in Section 4.1).

Adapting results from Baye et al. (1996), one can show that an analogous robustness conclusion extends to the case of non-linear demand as well.
6 Conclusion

This paper joins a small but growing literature that adopts a positive perspective on market performance centered around the integration of corruption—viewed as the sale of government property for selfish gain by mid-level officials—as a key determinant of market structure.

With no existing firms in the market, this scheme will always give rise to a corruption-induced monopoly, and thus typically to drastic under entry. However, in reality, a particular official will be faced with existing firms that are out of his reach and may not be forced to pay a bribe. In such cases, the official’s ability to curtail competition may be drastically reduced, and the outcome may exceed the socially optimal number of firms. This result paves the way for some policy prescriptions on the part of a benevolent head of government, including issuing special licenses from his office to create or increase the number of existing firms before letting a corrupt official take over, and timing officials’ nomination to particular industries with a view to avoid any corruption-induced monopolies (in particular arrange for the transfer of any official shortly after overseeing a new industry launch).

A benevolent leader may also delegate the authority to issue licenses to multiple authorities, who then compete in the numbers of firms they allow in the market while all charging the same type of bribe as before. The main result here is that, with two officials, the outcome will be the socially optimal number of firms. This result constitutes an extreme illustration of the theory of the second-best as envisioned by Lipsey and Lancaster (1957): The two distortions at hand, free entry under imperfect competition and the presence of fully corrupt officials, exactly offset each other and yield the second best solution. With pre-existing firms in the market, the socially optimal number of officials is always either one or two, relatively simple and realistic structure for the socially optimal industrial organization of corruption.

The results of the paper enlighten the much discussed relationship between competition and corruption. As in Bliss and DiTella (1997), the two variables are endogenously and simultaneously co-determined, rendering any comparative statics perspective a rather complex issue. Nevertheless, for the discrete comparison of entry under corruption and entry without corruption, one can conclude that corruption need not stifle competition. In fact, it may even partially or fully correct the natural inefficiency inherent in free entry, depending on the number of initial firms operating in the industry (Mankiw and Whinston, 1986). When no such firms are present, corruption with two competing officials restores full (second-best) efficiency. With sufficiently many pre-existing firms, a monopoly official is socially optimal, a rather surprising conclusion going against conventional wisdom on corruption (Rose-Ackerman, 1978). One key finding of the paper is that the total number of pre-existing firms and shadow firms emerges as a key parameter that may play a critical
role in this complex relationship. Interestingly, the “social evil” of the shadow firms may actually play a positive role in countering the effects of the “social evil” of corruption.

Finally, we show that the main insight of the paper—that competition among corrupt officials can restore social efficiency—essentially carries over in a qualitative sense to a model with quadratic costs and, to some extent, first best entry regulation. A key qualification of this conclusion is that the level of competition in entry certification must be tailored to the level of diseconomies of scale. Along the way, we also uncover a partial coincidence result for first and second best entry, a result of some independent interest.

Our analysis and conclusions suggest some testable implications of potential interest for empirical work on corrupt economies, even though reliable data is difficult to obtain due to secrecy requirements and the developing state of most relevant economies. One testable implication is whether a high rate of turnover of entry-certifying officials within industries with long-lived licenses will be associated with higher levels of competition or better market performances. Another is to conduct a policy evaluation following a reform introducing two competing officials in cases where some data on the previous situation (with one official) is available, thus yielding a natural experiment (e.g., the Singapore Customs example).

## 7 Appendix

This appendix contains the proofs of the results, given in the same order as in the text. For the sake of brevity, most of the computational details are left to the interested reader.

### Proof of Lemma 1

We first show that the objective $B(m; \ell)$ is a strictly quasi-concave function of $m$. To this end, it is sufficient to show that the second derivative of $B(m; \ell)$ is strictly negative whenever the first derivative vanishes. To shorten notation, let $A := (a - c)^2$ and set $\lambda p = 1$ w.l.o.g.

The first derivative of $B(m; \ell)$ is

$$
\frac{\partial B(m; \ell)}{\partial m} = \frac{A}{(1 + \ell + m)^2} - K - \frac{2mA}{(1 + \ell + m)^3}
$$

and the second derivative is

$$
\frac{\partial^2 B(m; \ell)}{\partial m^2} = \frac{-4A}{(1 + \ell + m)^3} + \frac{6mA}{(1 + \ell + m)^4}.
$$
Now consider

\[
\left. \frac{\partial^2 B(m; \ell)}{\partial m^2} \right|_{\partial B'(m)/\partial m = 0} = \frac{-4A}{(1 + \ell + m)^3} + \frac{3A}{(1 + \ell + m)^3} - \frac{3K}{1 + \ell + m}
\]

\[
= \frac{-A}{(1 + \ell + m)^3} - \frac{3K}{1 + \ell + m}
\]

\[
< 0
\]

Hence \(B(m; \ell)\) is strictly quasi-concave in \(m\). It follows that \(B(m; \ell)\) has a unique argmax.

To guarantee the interiority of the maximum of \(B(m; \ell)\), it is sufficient to show that its derivative is (strictly) positive at the left boundary (\(m = 0\)). In other words,

\[
\left. \frac{\partial B(m; \ell)}{\partial m} \right|_{m = 0} = \frac{A}{(1 + \ell)^2} - K > 0
\]

since (by assumption) \(\ell < \sqrt{\frac{A}{K}} - 1\), i.e., that the number of pre-existing firms is less than the number of firms under free-entry.

To show that \(m \in [0, 1 + \ell)\) when \(K > 0\), it is sufficient to show that the derivative of \(B(m; \ell)\) is strictly negative at \(m = 1 + \ell\), i.e., that

\[
\left. \frac{\partial B(m; \ell)}{\partial m} \right|_{m = 1 + \ell} = \frac{A}{(2(1 + \ell))^2} - K - \frac{2(1 + \ell)A}{(2(1 + \ell))^3} = -K < 0.
\]

Therefore, we always have \(m < 1 + \ell\).

Finally, when \(K = 0\), we have \(\left. \frac{\partial B(m; \ell)}{\partial m} \right|_{m = 1 + \ell} = 0\), so that (by the strict quasi-concavity of \(B(m; \ell)\)) \(m = 1 + \ell\) is the unique solution. \(\square\)

**Proof of Proposition 1**

Since (5) amounts to finding the zeros of a third-degree polynomial, solving yields two complex roots and one real root. It can be verified that the unique real root is

\[
m(\ell) = \frac{(AK)^{\frac{3}{2}} \cdot \left[ 9L + \sqrt{\frac{3(A+27KL^2)}{K}} \right]^{\frac{3}{2}} - 3^{\frac{3}{2}} A^{\frac{3}{2}}}{(3K)^{\frac{3}{2}} \cdot \left[ 9L + \sqrt{\frac{3(A+27KL^2)}{K}} \right]^{\frac{3}{2}}} - L, \quad \text{for } K > 0 \quad (15)
\]
where $A = (a - c)^2$ and $L = 1 + \ell$. Let $G(\ell) := \ell + m(\ell)$. It is easy to see that $G(\ell)$ is a continuous and strictly increasing function in $\ell$ since

$$G'(\ell) = 1 + \frac{3^3 AK \left(9 + \frac{54\sqrt{3}L}{2\sqrt{A+27KL^2}}\right)}{9 \left[AK^2 \left(9L + \sqrt{\frac{3(A+27KL^2)}{K}}\right)\right]^{\frac{2}{3}}} + \frac{3^3 A^2 K^2 \left(9 + \frac{54\sqrt{3}L}{2\sqrt{A+27KL^2}}\right)}{9 \left[AK^2 \left(9L + \sqrt{\frac{3(A+27KL^2)}{K}}\right)\right]^{\frac{2}{3}}} - 1 > 0$$

Recall that $\bar{n} = 1 \leq n^* \leq \hat{n}$, and observe that $\bar{n} = 1 = G(0) < G(\hat{n}) = \hat{n}$ (the latter inequality being due to the fact that $m(\hat{n}) = 0$ by definition of $\hat{n}$). Hence, by the intermediate value theorem and the continuity and strict monotonicity of the function $G(\ell)$, there exists a unique (real number) $\ell^*$ such that $G(\ell^*) = n^*$.

The second part follows directly from the strict monotonicity of the function $G(\ell)$.

**Proof of Lemma 2**

The first-order condition in (6) reduces to

$$\frac{A(1 - m_i + m_{-i})}{(1 + m_i + m_{-i})^3} = K. \quad (16)$$

This implicitly defines a reaction function $m_i = r(m_{-i})$, which is the same function for all players by symmetry. By differentiating and collecting terms, one can easily check that $r'_i(m_{-i}) > -1$, so that the function $m_i + r_i(m_{-i})$ is increasing in $m_i$. By a well known argument (see Theorem 2.1 in Amir and Lambson, 2000), this property implies that no asymmetric equilibrium in the $m_i$'s is possible.

Hence, we can assume symmetry, i.e. $\frac{m_{-i}}{g-1} = m_i := \bar{m}$ and simplify (16) to obtain

$$A \left[1 + (g - 2)\bar{m}\right] = (1 + g\bar{m})^3 K. \quad (17)$$

Solving yields two complex roots and one real root, the latter being

$$\bar{m} = \frac{(AKg)^{\frac{1}{3}} \left(9 + \sqrt{\frac{3(27Kg+8A-Ag^3+6Ag^2-12Ag)}{Kg}}\right)^{\frac{2}{3}} + 3^{\frac{1}{3}} A^{\frac{2}{3}} (g - 2)}{3^{\frac{2}{3}} g \left[K^2 g^2 \left(9 + \sqrt{\frac{3(27Kg+8A-Ag^3+6Ag^2-12Ag)}{Kg}}\right)\right]^{\frac{1}{3}}} - \frac{1}{g} \quad \text{for } K > 0 \quad (18)$$

This completes the proof of Lemma 2.
Proof of Proposition 2

Assuming that there are $g$ identical corrupt officials, where $i$ chooses $m_i$ firms to enter the market, the social welfare function is defined as (where $\pi^*$ is the profit per firm)

$$\tilde{W}(g, m) = \sum_{i=1}^{g} m_i \pi^* + \int_{0}^{Q^*} [P(Q) - P(Q^*)] dQ.$$ 

Substituting in the equilibrium profit and output from the second stage, we find

$$\tilde{W}(g, m) = \left( \sum_{i=1}^{g} m_i \right) \left[ \frac{(a - c)^2}{(1 + \sum_{i=1}^{g} m_i)^2} - K \right] + \frac{1}{2} \left[ \frac{(a - c) \sum_{i=1}^{g} m_i}{1 + \sum_{i=1}^{g} m_i} \right]^2.$$ 

Denoting the total number of firms in the market as $n = \sum_{i=1}^{g} m_i$, welfare can be written as

$$W(n) = n \left[ \frac{(a - c)^2}{(1 + n)^2} - K \right] + \frac{1}{2} \left[ \frac{(a - c)n}{1 + n} \right]^2 \quad (19)$$

Differentiate (19) with respect to the number of firms $n$ to get

$$W'(n) = \frac{(a - c)^2 - K (1 + n)^3}{(1 + n)^3} = 0. \quad (20)$$

The second order condition $W''(n) < 0$ is easily seen to hold, so welfare is strictly concave in the total number of firms in the market. By symmetry, the socially optimal number of firms let in by each official satisfies $m^* = n/g$, and (20) yields

$$m^*(g) = \frac{1}{g} \left[ \left( \frac{(a - c)^2}{K} \right)^{\frac{1}{3}} - 1 \right]. \quad (21)$$

Comparing (21) with (17), we have $\bar{m} = m^*$ if and only if $g = 2$. Therefore allowing exactly two officials to certify entry produces the socially optimal number of firms in the market. \qed

Proof of Proposition 3

The first order condition of (7) reduces to (with $A := (a - c)^2$)

$$\frac{A(1 + \ell + m_{-i} - m_i)}{(1 + \ell + m_i + m_{-i})^3} = K.$$
With the same argument as in the proof of Lemma 2, one can rule out asymmetric equilibria in the $m_i$'s. Given symmetry, i.e. letting $\frac{m_i}{g-1} = m_i := \bar{m}$, and simplifying yields

$$A[1 + \ell + (g - 2)\bar{m}] = K(1 + \ell + g\bar{m})^3.$$  \hfill (22)

For clarity, denote the solution to (22) with its explicit arguments as $\bar{m}(\ell, g)$. Let $N(\ell, g) := \ell + g\bar{m}(\ell, g)$ denote the equilibrium total number of firms in the market. We first show that $N(\ell, g)$ is strictly increasing in both $\ell$ and $g$.

To this end, standard comparative statics on (22) with respect to $\ell$ (i.e. differentiating once w.r.t. $\ell$) yields

$$\frac{\partial \bar{m}(\ell, g)}{\partial \ell} = \frac{A - 3K(1 + \ell + g\bar{m})^2}{3Kg(1 + \ell + g\bar{m})^2 - A(g - 2)}.$$  \hfill (23)

Via routine calculations, it is easy to check that $\frac{\partial \bar{m}(\ell, g)}{\partial \ell} > -\frac{1}{g}$. Hence $\frac{\partial N(\ell, g)}{\partial \ell} = 1 + g\frac{\partial \bar{m}(\ell, g)}{\partial \ell} > 0$.

Next, differentiating (22) once w.r.t. $g$ yields

$$\frac{\partial \bar{m}(\ell, g)}{\partial g} = \frac{A\bar{m} - 3K\bar{m}(1 + \ell + g\bar{m})^2}{3Kg(1 + \ell + g\bar{m})^2 - A(g - 2)}.$$  

As before, it is easy to check that $\frac{\partial \bar{m}(\ell, g)}{\partial g} > -\frac{\bar{m}}{g}$. Hence $\frac{\partial N(\ell, g)}{\partial g} = \bar{m} + g\frac{\partial \bar{m}(\ell, g)}{\partial g} > 0$.

Hence we have shown that the final number of firms $N(\ell, g)$ is strictly increasing in both $\ell$ and $g$. We next make use of this result to finish the proof of the Proposition.

Social welfare is given by

$$W = (\ell + mg) \left[ \frac{A}{(1 + \ell + mg)^2} - K \right] + \frac{1}{2} A \left[ \frac{\ell + mg}{1 + \ell + mg} \right]^2 .$$

Recall from Proposition 2 that welfare is maximal when $\ell = 0$ and $g = 2$, i.e. when a total of $2\bar{m}(0, 2)$ firms are in the market. As welfare is strictly concave in the number of firms $N = \ell + mg$ (as in the previous proof), it is strictly concave jointly in the pair $(\ell, g)$. Therefore, since $N(\ell, g)$ is strictly increasing in both $\ell$ and $g$, we can conclude that when $\ell > 0$, welfare is maximized by having either $g = 2$ or $g = 1$, according as $\ell$ is low or high.
References


