SET 1–due 4 September

We'll start out reviewing the algebra of creation and annihilation operators.

1) A coherent state $|\lambda\rangle$ of a one-dimensional oscillator is defined through

$$a|\lambda\rangle = \lambda|\lambda\rangle \tag{1}$$

where λ is a complex number.

(a) [5 points] Prove that the normalized coherent state is

$$|\lambda\rangle = e^{-|\lambda|^2/2} e^{\lambda a^\dagger}|0\rangle.$$

(Hint: begin with $|\lambda\rangle = \sum_n c(n)|n\rangle$ and find an iterative solution for the $c(n)$'s.)

(b) [5 points] What is $\langle\lambda|H|\lambda\rangle$? In addition, show that $|c(n)|^2$ is Poisson-distributed in n: what is the most probable n?

(c) [5 points] What is $\langle\lambda|x(t)|\lambda\rangle$? An important little lesson, told (like all lessons) cryptically: States of definite photon number have vanishing electric field; if you want to do all of Jackson starting from quantum electrodynamics, you must work with coherent states.

2) [15 points] One can define "propagators" for quantum mechanical systems. (This is the analog of something we'll see later in the course.) (a) [5 points] Evaluate

$$G(t) = \langle 0|x(t)x(0)|0\rangle\theta(t) + \langle 0|x(0)x(t)|0\rangle\theta(-t)$$

where $|0\rangle$ is the ground state for a simple harmonic oscillator of natural frequency Ω and $x(t)$ is the position operator at time t. It may be helpful to recall that operators evolve according to

$$-i\frac{dO}{dt} = [H, O]. \tag{2}$$

(b) [5 points] What is

$$G(\omega) = \int_{-\infty}^{\infty} \exp(i\omega t)G(t)dt? \tag{3}$$

(You may need to introduce convergence factors $\exp(\pm\epsilon t)$ in the integrals, and take the $\epsilon \to 0$ limit.)

(c) [5 points] Because $\Omega > 0$, the answer you found in part (b) is often written as

$$G(\omega) \sim \frac{1}{\omega^2 - \Omega^2 + i\epsilon} \tag{4}$$

where I have not bothered to write the correct prefactors (but you should!). Check that this expression is correct, and then compute $G(t)$ from $G(\omega)$ by inverting the Fourier transform. This is a little exercise in complex variables for you to review.