SET 3 – due 30 September

1.) [25 points] Consider a system of two kinds of particles 1 and 2 (of masses m_1 and m_2) with an interaction Hamiltonian density

$$\mathcal{H}_I = g\phi_1^\dagger(x)\phi_1(x)\phi_2(x)$$

where ϕ_2 is self-adjoint ($a_2^c = a_2$). Find the scattering amplitude for $\phi_1(k_1) + \phi_1(k_2) \to \phi_1(k_3) + \phi_1(k_4)$ to lowest nontrivial order. Take your time with this, see how all the creation and annihilation operators collapse, and explain everything carefully. Sketch the relevant Feynman graph(s).

2.) [15 points] A useful set of relativistic invariants for the reaction $1+2 \to 3+4$ are $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, $u = (p_1 - p_4)^2$. (a) [5 points] Check that $s + t + u = \sum_i m_i^2$ in the general case. (b) If energy and momenta greatly exceed masses, an often used approximation for the differntial cross section for a relativistic scattering problem is to assume that all particles are massless. Find a representation for the differential cross section for a reaction $1 + 2 \to 3 + 4$ in terms of s, t, and u in that approximation. Begin with the general formula given in the notes,

$$d\sigma = \frac{|M|^2}{2E_1 2E_2 v_{rel}}(2\pi^4)\delta^4(p_1 + p_2 - p_3 - p_4)\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}. \tag{1}$$

Your answer should look like

$$d\sigma = C\frac{1}{s}|M|^2\frac{dt\ du}{s}\delta(s + t + u) \tag{2}$$

or

$$\frac{d\sigma}{dt} = C\frac{1}{s^2}|M|^2 \tag{3}$$

where C is a dimensionless constant for you to find. Note that this result is valid in any reference frame; it is composed of invariants. (I evaluated the phase space integration in a convenient frame and rewrote the answer in terms of invariants. Also – distinguishable particles, assume M has no azimuthal dependence. Note $v_{rel} = 2$ in this limit.)

Keep this result for use later in the course. With massive particles, the delta function generalizes to $\delta(s + t + u - \sum_i m_i^2)$, no surprise. $d\sigma/dt$ is a relativistic generalization of $d\sigma/d\cos\theta$.

And a comment on the $2E_1 2E_2 v_{rel}$. Ryder remarks that this is equal to $4B$ where $B = [(p_1 \cdot p_2)^2 - m_1^2 m_2^2]^{1/2}$. Peskin and Schroeder write it as $4(E_2 p_1 - E_1 p_2) = 4\epsilon_{\mu x y \nu} p_1^\mu p_2^\nu$. "This is not Lorentz invariant, but it is invariant to boosts along the z-direction. In fact, this expression has exactly the transformation properties of a cross-sectional area."

3) [20 points] Evaluate the propagator for a nonrelativistic field

$$\psi(x,t) = \int \frac{d^3k}{(2\pi)^{3/2}} a(k) \exp(-i(E(k)t - \vec{k}\cdot\vec{x})) \tag{4}$$

from the formal definition of the time-ordered product. Express your answer in position-time space (this should look very familiar, it is the formula used in the spreading of a wave packet) and in momentum-frequency space (which should also be unsurprising). Here $E(k) = k^2/(2m)$, of course.