SET 6 – due 4 November

Two problems inspired by the LHC: In the Standard Model the coupling of the (spinless) Higgs boson ϕ to a fermion-antifermion pair is $\mathcal{L} = (m/v)\bar{f}f\phi$ where m is the fermion's mass and v is the so called Higgs vacuum expectation value. It is is related to the Fermi coupling, in weak interactions, $G_F = 10^{-5}$ GeV^{-2}, $1/(2v^2) = G_F/\sqrt{(}2)$, and $v = 246$ GeV.

1) [20 points] Compute the decay rate for Higgs into an $\bar{f}f$ pair. Call the Higgs mass M. Plot your result as a function of the parameter $x = 2m/M$. The formula for the differential decay width is

$$d\Gamma = \sum_{spins} \frac{1}{2M}|\mathcal{M}|^2 \frac{d^4p_1}{(2\pi)^3}\frac{d^4p_2}{(2\pi)^3}(2\pi)^4\delta^4(P - p_1 - p_2)\delta(p_1^2 - m^2)\delta(p_2^2 - m^2) \quad (1)$$

2) [25 points] Now compute the differential cross section for $\bar{f}f \to \phi\phi$. In all your calculation, except of course for the vertices, assume that the fermions are massless. (This amounts to doing the calculation at energies much greater than the fermion masses.) Use of the relativistic invariants s, t, and u will help a lot, and you can use the formula from a previous problem set,

$$d\sigma = \frac{1}{16\pi s^2}|\mathcal{M}|^2 dt. \quad (2)$$

In this case, $s + t + u = 2M^2$ (Check and confirm!) Show that the differential cross section vanishes at threshold, $s = 4M^2$. Higgs pair production has not yet been observed. This problem resembles pair annihilation into photons but is much simpler.
Note that the propagator for a spin-1/2 fermion is

$$S_F(p) = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}. \quad (3)$$