SET 7 – due 13 November

1. [10 points]. In class, I took the derivation of the photon propagator to the stage where the gauge fixed action was

$$S = \frac{1}{2}\int d^4 p A_\rho(-p)[p^2 g_{\rho\nu} - (1-\frac{1}{\alpha})p_\rho p_\nu]A_\nu(p). \qquad (1)$$

The propagator is the inverse of the object in square brackets. Writing

$$-iD_{\mu\nu}(p) = i[g_{\mu\nu}X(p) + p_\mu p_\nu Y(p)], \qquad (2)$$

where X and Y are scalar functions of p, go through the index-pushing exercise

$$g_{\mu\nu} = [p^2 g_{\mu\rho} - (1-\frac{1}{\alpha})p_\mu p_\rho][g_{\rho\nu}X(p) + p_\rho p_\nu Y(p)] \qquad (3)$$

to show that the photon propagator is

$$-iD_{\mu\nu}(p) = \frac{i}{p^2}[g_{\mu\nu} - (1-\alpha)\frac{p_\mu p_\nu}{p^2}]. \qquad (4)$$

To show this quickly, consider the two projectors,

$$\Pi^t_{\mu\nu} = \frac{1}{p^2}(g_{\mu\nu}p^2 - p_\mu p_\nu) \qquad (5)$$

and

$$\Pi^l_{\mu\nu} = \frac{p_\mu p_\nu}{p^2}. \qquad (6)$$

Show $\Pi^t_{\mu\lambda}\Pi^t_{\lambda\nu} = \Pi^t_{\mu\nu}$, $\Pi^l_{\mu\lambda}\Pi^l_{\lambda\nu} = \Pi^l_{\mu\nu}$, and $\Pi^t_{\mu\lambda}\Pi^l_{\lambda\nu} = 0$. You will be able to solve Eq. 3 in two lines.

2. [10 points] In class we computed the reaction $e^+e^- \to \mu^+\mu^-$ using the Feynman gauge propagator ($\alpha = 1$ in the language of problem 1.) Repeat the calculation of $e^+e^- \to \mu^+\mu^-$ for arbitrary gauge-fixing term α (as defined in problem 1) and show that the α dependence in fact drops out. You can stop computing once you show this! There is only one Feynman diagram. If the amplitude involved several graphs, only their sum, not the individual terms, would be gauge invariant.