

## The Robustness of Hedonic Price Estimation: Urban Air Quality

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Theoretical and empirical problems with hedonic benefit estimation (Rosen 1974) are becoming increasingly apparent (see, e.g., Atkinson and Crocker 1987; Bartik and Smith 1984; Horowitz 1984; Butler 1982; Roback 1982; Rosen 1979). However, there has been no systematic comparative analysis, using a single data set, of the relative magnitudes of the potential inaccuracy caused by individual problems. In this paper we utilize a detailed data set to address four issues: (1) variable selection and treatment, (2) measurement error, (3) functional form, and (4) error distribution. In particular, this research explores the robustness of hedonic-based estimates of marginal environmental values to alternative specifications and provides insights to guide future hedonic environmental benefit studies.<sup>1</sup>

Section I describes the data set and benchmark results. The quality of the data set enables the exploration of the broad range of issues considered here. Section II deals with variable selection, examining the difference that alternative choices (and treatments) have on the size and significance of the influence of the environment on housing values. Relevant issues are: (1) specifications of the independent variable set; and (2) degree of equilibrium versus disequilibrium in the housing market.<sup>2</sup>

In Section III, building on the analysis of Section II, the issue of accurate measurement of the environmental variables of interest is examined. The procedure follows Klepper and Leamer (1984), and has been used in a similar manner by Atkinson and Crocker (1987).

Functional form is considered in Section

IV. Following Berndt and Khaled (1979) and Halversen and Pollakowski (1981), we employ a model sufficiently general (the quadratic Box-Cox) to include most of the popular forms in the literature (linear, log-linear, semilog, quadratic and translog) as special cases.

Section V examines alternative assumptions regarding the error distribution. If the true error distribution has greater weight in its tails, reliance on the assumption of normality may lead to biased parameter estimates. The minimum absolute deviation estimator provides a comparison for the least squares results. Concluding remarks are offered in Section VI. The analyses demonstrate the importance of specification bias, measurement error bias, and functional form bias in the estimation of hedonic prices for urban air quality.

The results of our inquiry have important policy implications since the range on the estimates of the marginal benefits for environmental commodities, obtained here, is quite large. This implies that the previous estimates of the marginal valuations of nonmarket commodities, generated with the hedonic method,

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<sup>1</sup>The analysis is conducted at the implicit price level so that issues of demand curve identification do not confuse the interpretation of results.

<sup>2</sup>Other issues, not considered here, are nature of the appropriate dependent variable (multiple-listing price offers, assessed valuations, etc.) and multi-market issues. The former is not considered since our data contain actual market transactions, which is surely the preferred alternative. The latter can not be estimated with this single market data set. Moreover, the issue of most direct concern is how convincing the single market studies are, apart from multi-market issues.

could change substantially with alternative estimation techniques. In addition, our findings cast doubt on the results of studies that have utilized hedonic-based marginal prices to evaluate the validity of other nonmarket methods (see, e.g., Brookshire et al. 1982).

### I. DATA AND BENCHMARK HEDONIC EQUATION RESULTS

The data base includes observations from Los Angeles, Orange, Riverside, and San Bernardino counties in California. The dependent variable is the selling price of owner-occupied single family residences, while the impact of urban air quality on the property values is the policy interest. The variables used to represent urban air quality are visibility and total suspended particulate concentrations. These measures are proxies for the aesthetic impact, the physical damage, and the health components of perceived air quality.

The visibility data are modeled from readings taken by weather station personnel at airports and other weather stations in 1978 and 1979. Data on total suspended particulates are modeled from annual averages obtained from California Air Resources Board monitoring stations. These air quality data were assigned to each individual house by constructing isopleth curves over the study region. The measures are, therefore, site-specific and do not necessarily correspond to community or census tract boundaries. They are still, of course, subject to error in measurement, an issue to which we shall return.

In addition to the air quality variables, the independent variable set includes variables which correspond to three levels of spatial aggregation: site, neighborhood, and community. The site characteristic data were obtained from the Market Data Cooperative (a computerized clearinghouse for housing data) and pertain to homes sold in 1979. A large random sample of approximately 1,400 observations was taken from the original data set of over 100,000 observations. The site characteristic data are at the household level and contain information on nearly every important structural and/or quality attribute.

Other variables which could significantly affect a home's sale price are those that reflect

the condition of the neighborhood and community in which it is located. In order to capture and separate those impacts from air quality differences, several neighborhood and community variables were included in the econometric modeling. Neighborhood refers to surrounding census tract and includes variables such as income, ethnic composition, distance to work, and distance to the beach. Information from the 1980 census was utilized. There were over 1,500 census tracts in the study area and the variation in these data is quite substantial. Pertinent community (city level) variables include density measures, average lot size, school quality, crime rate, and distance to the central business district. Also included are a set of zero-one dichotomous variables for county (Los Angeles, Orange, Riverside, San Bernardino) location. These variables are proxies for the taxes and services not otherwise measured.<sup>3</sup>

The estimated hedonic price gradient which serves as a benchmark equation is presented in the first column of Table 1. The equation is representative of the literature, and only a few observations are relevant at this point. First, the specification presented is only one of many possible models. The impact of various included/excluded variables is the concern of the next section. A second observation is the nonlinear semilog specification. Nonlinearity is consistent with the conjecture of Rosen (1974), who noted that consumers cannot always arbitrage by dividing and repackaging house attributes. Third, a significant portion (.73) of the variation in home sale price is explained by the independent variable set. Fourth, ten of the fourteen estimated coefficients are significantly different from zero at the five percent level. The exceptions are house age, pool, air conditioning, and time to work. Fifth, there exist two variables (distance to the beach and distance to business district) that perform in an unexpected manner, that is, their relationship to the dependent variable is contrary to prior expectations. Sixth, the coefficients on the air quality variables are significantly different from zero and

<sup>3</sup>The data and a descriptive appendix are available from the authors.

possess the expected relationship to home sale price.<sup>4</sup>

## II. VARIABLE SELECTION AND TREATMENT

The potential variables for inclusion into the hedonic price equation are divided into "focus," "free," and "doubtful" variables. Focus variables (the measures of environmental

quality) are those of particular policy interest, although they may in fact prove to be doubtful. Free variables (structural traits of a house) are known to affect the dependent variable

<sup>4</sup>Initial estimates, which included NO<sup>2</sup> and a measure of violations of ozone standards among the focus variables, suggested that these pollutants do not have an impact on property values—NO<sup>2</sup> always had a positive re-

TABLE 1  
PRELIMINARY HEDONIC PRICE GRADIENT ESTIMATES  
The Dependent Variable = Natural Logarithm of Home Sale Price (\$100)

Variable	Label	Specification 1	Specification 2
Age of Home in Years	AGE	.0003 (.70)	-.001 (-2.78)
Number of Bathrooms	BATH	.042 (2.93)	.067 (4.68)
Sq. Feet of Living Area	AREA	.00038 (22.24)	.00035 (22.27)
Pool	POOL	.011 (.67)	.034 (2.20)
Fireplaces	FIRE	.083 (7.08)	.070 (6.49)
Existence of a View	VIEW	.214 (8.86)	.126 (5.43)
Air Conditioning	AIR	-.014 (-.98)	-.021 (-1.55)
Distance to Beach	BEACH	.006 (5.04)	-.008 (-4.22)
% White—Census Tract	WHTCT	.006 (11.81)	.0077 (15.32)
Time to Work—Census Tract	WRKCT	-.0003 (-.16)	.005 (2.48)
Distance to Business Dist.	CBD	.003 (3.17)	-.0067 (-5.41)
Lot Size	LOTSZ	.000002 (2.14)	.00003 (8.24)
Visibility	VIS	.066 (15.35)	.0043 (.63)
Total Suspended Part.	TSP	-.012 (-15.63)	-.010 (-13.51)
FBI Crime Index	CRIME		-3.422 (-8.11)
Orange County Dummy	D1		-.339 (13.19)
Riverside County Dummy	D2		-.279 (-3.59)
San Bernardino County Dummy	D3		-.033 (-.71)
Constant		5.83 (74.12)	6.647 (68.79)
R-Square		.737	.776
Residual Sum of Squares		68.269	58.161

Note: *t*-values are in parentheses.

(property value) but are not of special interest. Doubtful variables (e.g., neighborhood income) may or may not affect the dependent variable, and the direction of the effect may be uncertain.

In this study, the focus variables are a measure of visibility (VIS) and total suspended particulates (TSP). The set of free variables is composed of living area (AREA), number of bathrooms (BATH), age of house (AGE), existence of a view (VIEW), lot size (LOTSZ), number of fireplaces (FIRE), presence of a swimming pool (POOL), percentage of the population that is white in the census tract (WHTCT), distance of the census tract to the nearest beach (BEACH), distance of the community to the central business district (CBD), and mean time to work from the census tract (WRKCT).

The doubtful variables are mean census tract income (INCOME), the FBI-reported crime rate for the community (CRIME), a school quality measure (SCHOOL), and a set of county dummy variables for the non-Los Angeles county data (D1 = Orange, D2 = Riverside, and D3 = San Bernardino). The doubtful variables were chosen primarily for illustration. They have been variously included and excluded in previous studies, indicating some uncertainty about their relationship to home prices.<sup>5</sup>

It has been suggested that the distinction between free and doubtful variables is somewhat arbitrary and that the resulting specification searches are therefore too narrow (McAleer et al. 1985). Leamer (1985) argues, however, that theory should provide guidance for the empiricist in drawing just such distinctions. In the hedonic analysis of property values, theory strongly indicates that the structural and access variables are free variables, while the neighborhood traits are doubtful. Empirically, Atkinson and Crocker (1987, 29) find "that the specification uncertainty caused by collinearity is small for structural attributes (e.g. floor space, age, and lot size) but substantial for neighbourhood attributes (e.g. air pollution, school quality, and crime). . . ."

To examine the issue of the bias in the estimated coefficients of the focus variables, we employed the following strategy: housing price was regressed on the focus variables, the

free variables, and every permutation of the doubtful variables. In addition, the focus variables themselves were treated as doubtful. These results are presented in Table 2. Consider first the results for VIS (See columns 1 and 3 of Table 2). VIS appears to be particularly sensitive to the county dummies. Without TSP in the equations, the estimates of the effect of VIS on property values range from positive and significant to negative and significant when the county dummies are entered. When TSP and VIS are both included in the equation, the estimates on the effect of VIS go from positive and significant to insignificant of mixed sign. This finding illustrates the collinearity between VIS and the county dummies and the consequences of trying to use the county dummies to capture the unmeasured influence of tax and public service variation among counties.

For TSP, the signs of the coefficients are always negative and significant at the five percent level. With only TSP in the equation, the coefficient estimates vary, becoming larger in absolute value when the county dummies are entered. However, when VIS and TSP are entered together, the estimates for TSP exhibit reasonable stability, ranging from  $-.008$  to  $-.012$ .

For the included variables not shown in Table 2, AREA, BATH, FIRE, POOL, and WHTCT are fairly stable and similar to the estimates presented in the benchmark results and in the literature. The estimated effect of

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relationship with home price (perhaps reflecting access to freeways), while the ozone measure had a coefficient which was highly variable and rarely significant.

Therefore, our analysis is limited to the more visible pollutants, visibility and total suspended particulates, with which NO<sub>2</sub> and ozone are positively correlated. In a sense, this is a form of pretesting that implicitly introduces Bayesian priors that are hidden from the reader. To have included a broader range of measures of pollution could only strengthen the conclusions of this research in light of the positive correlation among pollution variables of policy interest.

<sup>5</sup>As a reviewer of this paper pointed out, there are two types of uncertainty or doubt about variables: (1) inclusion or exclusion, and (2) uncertainty about the sign of the variable's coefficient. Our analysis is concerned with the first, since this type seems to be widespread in applied hedonic studies.

AGE ranges from negative and significant to positive and significant, suggesting correlation with one or more of the doubtful variables. The VIEW coefficient is always positive and significant with a range of approximately .25 to .08. BEACH exhibits a negative and significant relationship when either VIS or TSP is entered separately into the equation. However, when both variables are entered, distance to beach is sometimes positive and significant. The effect of distance to beach is critically affected by the county dummies. This indicates that the distance to beach, visibility and county dummies are measuring a similar influence.<sup>6</sup> WRKCT and CBD both exhibit coefficients that go from negative to positive as various doubtful variable combinations are used.

A specification issue that has received scant attention in the literature is the degree of equilibrium versus disequilibrium in the housing market (Mäler 1977). This has direct implications for the manner in which the lot size variable is treated. If the housing market is in short-run equilibrium only, then it is appropriate to enter lot size as simply another independent variable. This is, in fact, what is

done in virtually all of the hedonic literature. The sole exception, Wieand (1973), is not very clear regarding his reasons for including the lot size variable.

In the environmental application, the benefits of clean air are dependent on *where* one is located but not on how large a lot one purchases. The value of clean air is a certain amount whether one is on a small lot or a large lot—if a large lot is purchased, then one is paying *more* for environmental quality than if a small lot is purchased. This is not true for structural traits (e.g., a swimming pool of constant size costs no more on a relatively small lot than it does on a large lot), but is true for all amenities exhibiting public good characteristics. If the urban economy is characterized as being in long-run equilibrium, then the hedonic prices (the partial derivative of the hedonic equation) for all public good ameni-

<sup>6</sup>In a report to the California Air Resources Board (Trijonis et al. 1984), a similar result with respect to the presence of air conditioning was noted. The importance of collinearity was diminished, however, as the sample size was increased.

TABLE 2  
ESTIMATED COEFFICIENTS FOR TSP AND VIS BY POSSIBLE COMBINATIONS OF DOUBTFUL VARIABLES

Doubtful Variables	VIS Only*	TSP Only*	VIS and TSP Together**	
			VIS	TSP
None	.030	-.006	.066	-.012
INCOME	.022	-.004	.050	-.009
CRIME	.036	-.005	.073	-.012
SCHOOL	.030	-.006	.066	-.012
D1, D2, D3	-.046	-.010	.004***	-.011
INCOME, CRIME	.024	-.004	.054	-.009
INCOME, SCHOOL	.022	-.004	.051	-.009
INCOME, D1, D2, D3	-.040	-.008	-.005***	-.008
CRIME, SCHOOL	.036	-.005	.074	-.012
CRIME, D1, D2, D3	-.042	-.010	.004***	-.010
SCHOOL, D1, D2, D3	-.046	-.011	.004***	-.011
INCOME, CRIME, SCHOOL	.024	-.004	.054	-.009
INCOME, CRIME, D1, D2, D3	-.038	-.008	-.004***	-.008
INCOME, SCHOOL, D1, D2, D3	-.040	-.008	-.005***	-.008
CRIME, SCHOOL, D1, D2, D3	-.042	-.010	.004***	-.010
INCOME, CRIME, SCHOOL, D1, D2, D3	-.039	-.008	-.004***	-.008

\*The hedonic equation is estimated with only one focus variable.

\*\*The hedonic equation is estimated with both focus variables.

\*\*\*Indicates that the *t*-ratio is less than 2.00.

ties (environmental quality as well as included neighborhood traits) should vary with lot size.<sup>7</sup> An empirical analysis of this hypothesis requires that the public good amenities be multiplied by lot size in the hedonic equation.

To test the influence of LOTSZ, the benchmark specification was estimated with the public good amenities multiplied by LOTSZ. The price differentials for a one unit change in VIS and TSP, holding all other characteristics at their mean, were calculated in order to assess the empirical importance of the equilibrium issue. The price differential for VIS changes from \$6819 in the benchmark equation to \$6744 in the equation with LOTSZ interactions, a seemingly insignificant amount. The treatment of LOTSZ is more important for TSP, as the hedonic price for TSP is reduced by approximately fifty percent. Overall, the coefficients are qualitatively robust to alternative treatments of LOTSZ, but the issue certainly merits further examination as there is no reason to expect this result to hold in other locales.

Due to the obvious importance of variable selection and treatment (especially the role of the county dummies), two specifications will be used for subsequent analyses: the benchmark specification, referred to as specification 1 in Table 1, and specification 2, also reported in Table 1, which includes the county dummies and CRIME.

### III. MEASUREMENT ERROR

The issue of measurement error has a long history in econometric theory, although techniques attempting to correct for measurement error have rarely found their way into applied work in economics, in part due to the difficulties in treating the problem.

We employ a variation of the methodology suggested by Klepper and Leamer (1984) and first used in the hedonic literature by Atkinson and Crocker (1987). The goal of the analysis is to identify the importance of measurement error in any variable with respect to the estimation of the coefficients of interest (VIS and TSP). More precise measurements would be necessary if the variability in parameter estimates were too great.

The Klepper-Leamer approach extends

Frisch (1934) who shows that bounds on maximum likelihood (ML) parameter estimates can be obtained by "reverse" regressions. That is, a regression equation is estimated for each variable potentially measured with error and the least squares fit is obtained. Then the coefficients of interest are solved for, and the minimum and maximum values over the separate regressions form the bound. When there are several variables (Frisch considered the simple regression case), the ML estimate is to be found in the "core" of the separate estimates only if the separate estimates of the parameters have the same sign. Otherwise the ML estimates could be any value, depending upon the correlation between the measured and the true variable. If it is suspected that measurement error may be present in any of the variables in a hedonic price study, the first step is to run the reverse regressions and examine the signs of the coefficients.

As can be seen in Table 3, the results from the reverse regressions with the Los Angeles data set indicate that the coefficients for the focus variables VIS and TSP change sign and are, therefore, unbounded in both specifications. The variables assumed to be measured with error (WHTCT, WRKCT, BEACH, LOTSZ, VIS, TSP, and CRIME) were chosen primarily for illustration. However, with the exception of LOTSZ, they represent the neighborhood, location, and environmental variables. These variables can be considered proxy measures and are, therefore, measured with error by definition.

The situation may not be as bleak as it first appears. If the true relationship between price and the list of explanatory variables is imprecise, the maximum possible value for  $R^2$  in the absence of any measurement error must lie between the actual  $R^2$  and 1.0. The regression coefficients from the reverse regressions need to be adjusted for this fact. Let  $R^{*2}$  represent some maximum  $R^2$ , where  $R^2 < R^{*2} < 1.0$ , such that the adjusted reverse regressions will lie in the same orthant as the direct regression, since if  $R^2 = R^{*2}$  all the reverse regressions are identical to the direct regression. The crucial

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<sup>7</sup>This result is formally treated in Parsons (1986).

question becomes, what part of the gap between  $R^2$  and 1.0 could be attributed to measurement error? Then, if we thought it plausible that  $R^{*2}$  (in the absence of measurement error) could not be greater than the sum of .77 (for our example) and this amount, the direct regression would again bound the set of ML estimates.

Another approach to understanding the relationship between measurement error and the direction of effects of explanatory variables is to answer the following question: what is the minimum correlation necessary between the actual and true variables before at least the signs of the coefficients are unambiguous? To answer the question, we assume the relationship between the measured and the true variables is given by

$$X_{ij} = Z_{ij} + v_{ij}$$

where  $X_{ij}$  is the measured value of the  $j$ th observation on the  $i$ th explanatory variable,  $Z_{ij}$  is its true counterpart, and  $v_{ij}$  is white noise. In the normal errors in variables model,  $Z_{ij}$  is drawn from a normal distribution with mean  $\mu_i$ ,  $v_{ij}$  is added to it to form  $X_{ij}$ , and the  $v_{ij}$  are assumed to be uncorrelated with the  $X_{ij}$ . Let the squared correlation coefficient between

$Z_{ij}$  and  $X_{ij}$  be represented by  $\rho_i$ . If this correlation is known, the ML estimator is  $\beta = (X'X - E)^{-1}X'Y$  where  $E$  is a diagonal matrix whose  $i$ th diagonal element is  $1 - \rho_i$ . Note that as the  $\rho$ 's approach one,  $E$  approaches the zero matrix and  $\beta$  becomes the least squares estimator. This is as it should be, since  $\rho_i = 1$  implies that the observed variable measures the true variable without error and, under normality, ML and least squares coincide.

It is, of course, impossible to know (or even estimate)  $\rho_i$ , since  $Z_{ij}$  is not observed. The usual approach to estimation is to try to find an instrumental variable that is highly correlated with  $Z_{ij}$  but uncorrelated with  $v_{ij}$ . This is a difficult task, as with any instrumental variable estimation. Since we are examining parameter variability here, we have the luxury of specifying potential values for the  $\rho$ 's and examining the parameter estimates of interest.

Focusing on WHTCT, WRKCT, BEACH, LOTSZ, VIS, TSP, and CRIME, estimates for  $\beta$  are presented in Table 4, assuming  $\rho_i \leq 1$  for all  $X_i$  that are assumed to be measured with error and with  $\rho_i = 1$  for all  $X_i$  that are assumed to be measured accurately—AREA, BATH, AGE, FIRE, AIR, POOL, VIEW, CBD, D1, D2, and D3. The estimates vary substantially,

TABLE 3  
SIGNS OF THE COEFFICIENTS ON THE VARIABLES POSSIBLY MEASURED WITH ERROR BY  
ALTERNATIVE DEPENDENT VARIABLES (REVERSE REGRESSION RESULTS)

Independent Variable	Dependent Variable							
	P	WHTCT	WRKCT	BEACH	LOTSZ	VIS	TSP	CRIME
Specification 1								
WHTCT	+	+	-	+	+	-	+	na
WRKCT	-	+	-	-	-	-	+	na
BEACH	+	+	+	+	+	+	+	na
LOTSZ	+	+	+	+	+	+	+	na
VIS	+	-	+	+	+	+	+	na
TSP	-	-	+	-	-	-	-	na
Specification 2								
WHTCT	+	+	+	+	-	-	+	-
WRKCT	+	+	+	-	-	-	+	-
BEACH	-	-	+	-	-	+	+	-
LOTSZ	+	-	-	+	+	+	-	+
VIS	+	-	-	-	+	+	+	+
TSP	-	-	-	+	+	-	-	+
CRIME	-	+	+	-	-	-	-	-

even when  $\rho_i$  is .95. This indicates that any inferences drawn from the hedonic estimates are quite suspect.

The a priori expectations of the property value impact of VIS and TSP are a positive influence for VIS and negative influence for TSP. This prior is violated in both specifications. However, it is difficult to analyze the importance of measurement error to the estimated coefficients on VIS and TSP in Table 4, since all the  $\rho$ 's change together. Therefore, estimates for the coefficients on VIS and TSP were obtained by changing only one  $\rho$  at a time and assuming the other variables are measured accurately. These are presented in Table 5.

In specification 1, the estimates for the coefficient on VIS and TSP are quite stable when WHTCT, WRKCT, and LOTSZ are measured with error. They seem more sensitive to measurement error in BEACH, VIS, and TSP. Although the coefficient on TSP remains relatively constant for different  $\rho$ 's on VIS (see the fifth row of Table 5), its value is less than half the value obtained with no measurement error. Similarly, the VIS coefficient is quite stable as TSP's correlation changes, but is much smaller than the .066 estimate

reported in Section I. The conclusions are basically the same for specification 2, although WRKCT seems to cause VIS to fall quite substantially.

An important observation from Table 5 is that our priors (in terms of expected signs on estimated coefficients) are realized when WHTCT, WRKCT, BEACH, LOTSZ, and CRIME are presumed to be measured with error. They are not when either VIS or TSP are modeled as measured with error. This indicates that the relative emphasis of future environmental economic research should concentrate on the accurate measurement of the environmental quality variables. At a minimum, others can use this technique to assess the importance of measurement error in their data.

#### IV. EFFECT OF FUNCTIONAL FORM

Economic theory usually has little to say about correct functional form. For the situation of hedonic gradients, which are by theory already in reduced form and hence are solutions of several equations, even less can be presumed. It would seem, then, unreasonable to impose an a priori structure on the data.

TABLE 4  
ESTIMATED COEFFICIENTS FOR THE VARIABLES POSSIBLY MEASURED WITH ERROR BY  
VARIOUS VALUES FOR  $\rho$

$\rho$	WHTCT	WRKCT	BEACH	LOTSZ	VIS	TSP	CRIME
Specification 1							
.5	-.00034	-.0001	.0089	-.6E-6	-.003	.0003	na
.6	-.00046	.00018	.0142	-.87E-6	-.0045	.0004	na
.7	-.00075	.00048	.0354	-.18E-5	-.009	.0009	na
.8	-.00012	-.0013	.0797	-.3E-5	-.010	-.0013	na
.9	-.0015	-.00073	-.020	.6E-6	-.006	.0001	na
.95	-.004	-.0035	-.018	.16E-6	-.028	.00012	na
.97	-.0073	.0474	-.013	-.2E-6	.128	.013	na
.99	-.0185	-.0233	-.0211	.1E-7	-.036	.0154	na
Specification 2							
.5	-.00035	-.16E-4	.006	-.4E-6	-.0017	.0003	-.032
.6	-.00045	-.00005	.0085	.27E-6	-.0027	.0004	-.05
.7	-.00064	-.00016	.0143	.3E-6	-.0042	.00059	-.118
.8	-.0012	-.0010	.045	.5E-5	-.014	.0013	-.667
.9	-.0013	.0026	-.039	-.14E-4	.014	.0003	1.526
.95	-.0041	.0045	-.020	-.00002	.0102	.002	2.164
.97	-.0116	.0121	-.0171	.006	.011	.0048	4.573
.99	.0046	-.0403	-.0619	.00037	.051	.0022	-39.147



Berndt and Khaled (1979) and Halvorsen and Pollakowski (1981) combine several ideas in the literature to produce a model sufficiently general to include many of the most popular specifications (including linear, log-linear, semilog, quadratic, and translog). Since all of the popular functional forms are included in the most general functional form, the quadratic Box-Cox, conventional tests of hypotheses are available (i.e., likelihood ratio tests). Although estimation is by maximum likelihood, the likelihood function condenses considerably so that the computational burden is not onerous.

The quadratic Box-Cox model is

$$P(\theta) = a_0 + \sum_{i=1}^m a_i X_i(\pi) + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m b_{ij} X_i(\pi) X_j(\pi)$$

$$P(\theta) = \begin{cases} (P^\theta - 1)/\theta & \text{for } \theta > 0 \\ \ln(P) & \text{for } \theta = 0 \end{cases}$$

$$X_i(\pi) = \begin{cases} (X_i^\pi - 1)/\pi & \text{for } \pi > 0 \\ \ln(X_i) & \text{for } \pi = 0, \end{cases}$$

where  $P$  = house price,  $X_i$  = the value for the  $i$ th characteristic of the house, and  $\theta$  and  $\pi$  are the Box-Cox transformation parameters.

We consider three pairs of nested specifications, shown by the "tree" diagram (see Fig-

TABLE 5  
ESTIMATED COEFFICIENTS FOR VIS AND TSP WHEN OTHER VARIABLES ARE ASSUMED TO BE MEASURED WITH ERROR BY VARIOUS VALUES OF  $\rho$

Variables Measured with Error	$\rho$				
	.5	.6	.7	.8	.9
Specification 1					
WHTCT	.075 -.013	.075 -.013	.075 -.013	.075 -.013	.076 -.013
WRKCT	.066 -.012	.066 -.012	.066 -.012	.066 -.012	.066 -.012
BEACH	.050 -.009	.048 -.008	.046 -.008	.038 -.006	.196 -.036
LOTSZ	.067 -.012	.067 -.012	.066 -.012	.066 -.012	.066 -.012
VIS	-.002 -.005	-.002 -.005	-.0034 -.005	-.005 -.005	-.011 -.005
TSP	.030 .0002	.030 .0002	.030 .0003	.029 .0004	.027 .0009
Specification 2					
WHTCT	.042 -.012	.042 -.012	.043 -.012	.044 -.012	.048 -.012
WRKCT	.0067 -.01	.0067 -.01	.0067 -.01	.0068 -.01	.007 -.01
BEACH	.028 -.013	.028 -.013	.029 -.013	.031 -.013	.038 -.014
LOTSZ	.0034 -.011	.0034 -.011	.0034 -.011	.0034 -.011	.0032 -.011
VIS	-.000043 -.01	-.00005 -.01	-.00007 -.01	-.0001 -.01	-.0002 -.01
TSP	-.043 .00012	.043 .00015	.043 .0002	.043 .0003	.043 .0006
CRIME	.004 -.011	.004 -.011	.004 -.011	.004 -.011	.004 -.011

Note: In each row of the table, the other variables are considered to be measured accurately.

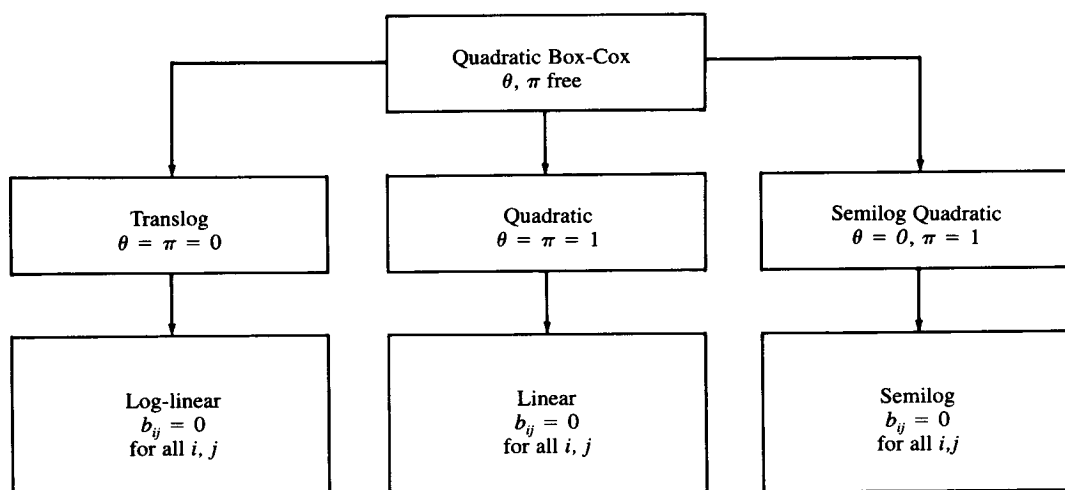


FIGURE 1

ure 1), with the necessary parameter restrictions. A two-step technique is implemented for maximizing the likelihood function. First, values for  $\theta$  and  $\pi$  are chosen and the data are transformed. Then the concentrated log-likelihood is evaluated:

$$L(\theta, \pi) = -\frac{1}{2} n \log \sigma^2(\theta, \pi) + (\theta - 1) \sum_{i=1}^n \log P_i$$

where  $n$  is the sample size,  $\sigma^2(\theta, \pi)$  is the OLS estimate of the variance of the transformed data, and  $P_i$  is the  $i$ th observation on price. A search over a two-dimensional grid for the largest value of  $L(\theta, \pi)$  produces the maximum likelihood estimates of  $\theta$ ,  $\pi$ , and the  $a$ 's and  $b_{ij}$ 's. (An even more general model is possible if it is specified that there is a different transformation for different exogenous variables, although this would greatly increase computer costs as the search would be conducted over an  $(M + 1)$ -dimensional grid.)

Having obtained the maximized value of the log-likelihood function and resulting parameter estimates, it is a straightforward exercise to test hypotheses concerning more restrictive functional forms. To do so,  $\theta$  and  $\pi$  are set to their respective values under the null hypothesis,  $a$ 's and  $b$ 's are again estimated,

and the new value of the likelihood is computed. Then, minus twice the difference between the constrained log-likelihood and the maximum value of the log-likelihood has, asymptotically, the chi-squared distribution with two degrees of freedom if the null hypothesis is true. The most restrictive specifications (log-linear, linear, and semi-log) may be tested either unconditionally or against their parent members (translog, quadratic, and semilog quadratic, respectively).

In the first specification the optimum optimum is found where  $\theta = .10$  and  $\pi = 1.10$ . This yields a value for the likelihood function of  $-7328.14$ . The values for the log-likelihood, excluding a constant, for the translog, quadratic, and semi-log quadratic are  $-7341.36$ ,  $-7859.77$ , and  $-7334.02$ , respectively. Calculation of the relevant chi-square statistic implies that all of these forms can be rejected at the .01 level. In addition, rejection of their parent members suggests that the lower order forms (log-linear, linear, and semi-log) can also be rejected at the .01 level.<sup>8</sup>

<sup>8</sup>The values, excluding a constant, of the log-likelihood for the log-linear, linear, and semi-log are  $-7543.71$ ,  $-8142.78$ , and  $-7528.71$ , respectively. Either an unconditional test or a test against their parent members suggests rejection.

The implications of the functional form analysis for our second specification are quite similar. The optimum optimum for the quadratic Box-Cox occurs at  $\theta = 1.13$  and  $\pi = -.35$ . The value of the log-likelihood function is  $-7243.26$ . As in the previous case, the values of  $\theta$  and  $\pi$  corresponding to the other estimated forms lie outside the confidence region suggesting rejection of the forms at the .01 level.

The empirical analysis to this point implies that most of the functional forms encountered in the environmental literature can be rejected for our data set. However, the relative impact of functional form variation on hedonic prices has yet to be investigated. Thus, if the estimated prices are relatively insensitive to functional form, then the commonly used forms may provide relatively precise benefit estimates. In this case, the more complex forms may be unnecessary. Note that the testing of estimated parameters may require that the standard errors be adjusted, depending on

how the parameters are estimated (Spitzer 1984).

The relative sensitivity of the estimated hedonic prices for VIS and TSP is illustrated in Table 6 for specifications 1 and 2. The hedonic prices were found by differentiating each form with respect to VIS and TSP. These expressions were then used to predict hedonic prices for each home in the original data set by applying the derivative equations to each observation. Calculated in this fashion, the hedonic prices could provide data for the second stage of the Rosen procedure.

The summary statistics presented in Table 6 indicate that the hedonic prices of VIS and TSP from specification 1 are somewhat stable. The mean hedonic price of visibility ranges from approximately 60 to 100, whereas that for TSP ranges from approximately 7 to 16. The ratios of highest to lowest mean values are then 1.67 and 2.33 for the two amenity variables, respectively. In each case, the quadratic Box-Cox yields the lowest mean price

TABLE 6  
MEANS AND STANDARD DEVIATIONS FOR THE PREDICTED HEDONIC PRICES OF VIS AND TSP BY  
FUNCTIONAL FORM OF THE HEDONIC PRICE EQUATION

Functional Form	Variable	Specification 1		Specification 2	
		Mean	Std. Dev.	Mean	Std. Dev.
Quadratic Box-Cox	VIS	60.1	87.8	781	1824
	TSP	7.0	28.0	5370	5045
Translog	VIS	97.6	85.6	31.9	65.6
	TSP	9.9	12.1	8.1	11.3
Quadratic	VIS	100.5	116.68	42.6	173.9
	TSP	11.6	44.53	11.9	30.1
Semilog Quadratic	VIS	78.9	107.6	30.1	242.9
	TSP	10.5	31.3	17.4	52.8
Log-linear	VIS	70.6	27.7	10.4	4.2
	TSP	11.6	6.1	10.6	5.8
Linear	VIS	87.7	0	15.4	0
	TSP	16.4	0	15.0	0
Semilog	VIS	66.8	35.2	4.4	2.3
	TSP	11.8	6.2	10.5	5.6

Note: Prices stated in \$100s. TSP figures are for a reduction of TSP.

indicating that previous analyses may have overestimated the hedonic price of environmental improvements.

However, the results for specification 2 completely contradict those for specification 1; that is, for the second specification the mean hedonic prices are much more sensitive. The functional form results are seen to intensify the problem of collinearity, encountered earlier, between the county dummies and the pollution measures. The VIS and TSP mean prices vary by ratios of 177/1 and 659.7/1, respectively. Even if the extremely volatile quadratic Box-Cox functional form is excluded, the mean VIS hedonic price varies by a ratio of 9.65/1. In addition, the results for specification 2 indicate that the quadratic Box-Cox produces the largest hedonic prices. Thus, analyses that utilize restricted functional forms could underestimate benefits of environmental improvements.<sup>9</sup>

## V. ROBUST ESTIMATION

There is a growing awareness in the applied economics literature that the extensive reliance on the assumption of normality may bias parameter estimates. It is argued that the normal distribution has too little weight in its tails, so that great importance is placed on outlying observations. Recent work in the area include attempts at the detection of influential observations (Belsley, Kuh, and Welsch 1980), nonparametric maximum likelihood estimation (Manski 1975; Cosslett 1983), and alternative fitting criteria that minimize the effect of outliers (Koenker and Basset 1982; Guilkey and Waldman 1985). It is this last approach that we follow here.

The assumption of normally distributed regression disturbances may be loosely justified by appealing to a variant of the central limit theorem. This appeal requires as preconditions a correctly specified model, no omitted variables, and, as components of the disturbance, many small, independently distributed random variables uncorrelated with the explanatory variables in the model. If these preconditions are not met, disturbances will not be normally distributed and consequently least square estimation will no longer be optimal.

In an attempt to assess the importance of the least squares estimation methodology applied in most studies of hedonic markets, we reestimated our basic specifications employing the fitting criterion of "minimum absolute deviations" (MAD). Algebraically, for the model  $Y_i = \beta'X_i + \epsilon_i$ ,  $i = 1, \dots, n$ , the

criterion is to choose  $\beta$  such that  $\sum_{i=1}^n |Y_i - \beta'X_i|$

is minimized. The idea is that outlying observations are given more weight, but only in proportion to their distance from the center rather than the square of the distance. The results are presented in Table 7.

The estimated coefficients for VIS and TSP for specification 1 are .065 and  $-.0096$  respectively, which are to be compared to .066 and  $-.012$  using least squares (see Table 1). For specification 2, the MAD coefficient esti-

<sup>9</sup>Although these hedonic prices are not strictly interpretable as benefits, it is seemingly the case that high (hedonic) prices produce higher benefit estimates.

TABLE 7  
HEDONIC PRICE EQUATION ESTIMATES  
USING THE MINIMUM  
ABSOLUTE DEVIATION CRITERION

Variable	Specification 1	Specification 2
AREA	.00042	.00041
BATH	.0242	.0325
AGE	.00036	-.001
LOTSZ	.0000012	.000032
FIRE	.0795	.0602
AIR	.0059	-.0045
POOL	.01262	.0277
VIEW	.2454	.160
WHTCT	.005	.0067
WRKCT	-.0032	.0037
BEACH	.0042	-.0070
CBD	.0041	-.0048
VIS	.065	.0141
TSP	-.0096	-.0097
CRIME		-3.225
D1		-.29
D2		-.2065
D3		-.0211
Constant	5.757	6.492
Sum of Residuals	215.3	199.6

mates are .014 and  $-.0097$  compared to .004 and  $-.011$  under least squares.

Except for VIS in specification 2, these estimates do not differ greatly. Since VIS appears to be less reliable than TSP as a measure of pollution, these results are somewhat encouraging. This is because there are at least two benefits to using least squares estimation: it is the commonly available computer package, and the calculation of standard errors is straightforward. The drawback, and the reason for interest in robust methods, is the possibility of parameter estimate bias, a possibility present in our data to only a limited extent.

## VI. CONCLUDING REMARKS

This paper has investigated the empirical importance of four econometric issues in the application of the hedonic technique to value changes in environmental amenities. The econometric issues examined were variable selection, measurement error, functional form, and alternative distributional assumptions.

With respect to variable selection, we found that the coefficients of one measure of air quality, an index of visibility (VIS), were qualitatively sensitive to which subset of doubtful variables were included in the analysis, while the coefficients of another measure, total suspended particulates (TSP), were reasonably stable. The coefficients of VIS were variously negative and significant, negative and insignificant, and positive, both insignificant and significant. The coefficient estimates for TSP were contained in a moderately narrow interval, particularly when VIS was also included in the equation. However, the impact of TSP on property values was seen to vary somewhat depending on how the state of equilibrium in the land market was considered.

Potential measurement error in the control variables did not affect the estimates of the coefficients of VIS and TSP as seriously as potential measurement error in these focus variables. But small measurement error in these variables (very likely in practice) renders even qualitative estimates dubious. This indicates the need for additional efforts to ensure that the environmental quality variables are carefully measured. Future research hop-

ing to arrive at accurate environmental values must begin with correct characterizations of the pollution data.

The more general functional forms were seen to significantly outperform the more restrictive functional forms commonly utilized in the literature. However, even within the most general functional form (the quadratic Box-Cox), the choice of specification greatly affected the results. That the quadratic Box-Cox gave low estimated environmental values for specification 1 and very high estimated values for specification 2 highlights the importance of choice of included variables.

Finally, the minimum absolute deviation estimator was employed as an alternative to least squares. Using the minimum absolute deviation estimation technique reduces the influence of outlying observations. However, MAD estimation, with one exception, did not have an important effect on estimated environmental values. This result may be specific to our data set, suggesting that this robustness exercise be conducted in future environmental benefit estimation.

A very large number of studies (see the review by Bartik and Smith [1984]) have attempted to value hedonically priced traits. To properly estimate the impact of air quality on property values with this method would require: (1) a complete set of covariates; (2) accurate measures of these covariates, particularly the focus variables; (3) the selection of the appropriate relationship between price and the attributes; and (4) the correct stochastic assumptions. To do otherwise is to risk basing important policy decisions on evidence which is misleading at worst and unconvincing at best.

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