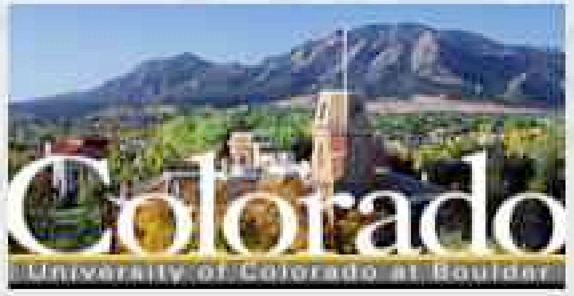
A many particle generalization of the Landau-Zener problem

Victor Gurarie, University of Colorado

collaborators: Alexander Altland, University of Cologne

Anatoly Polkovnikov, Boston University

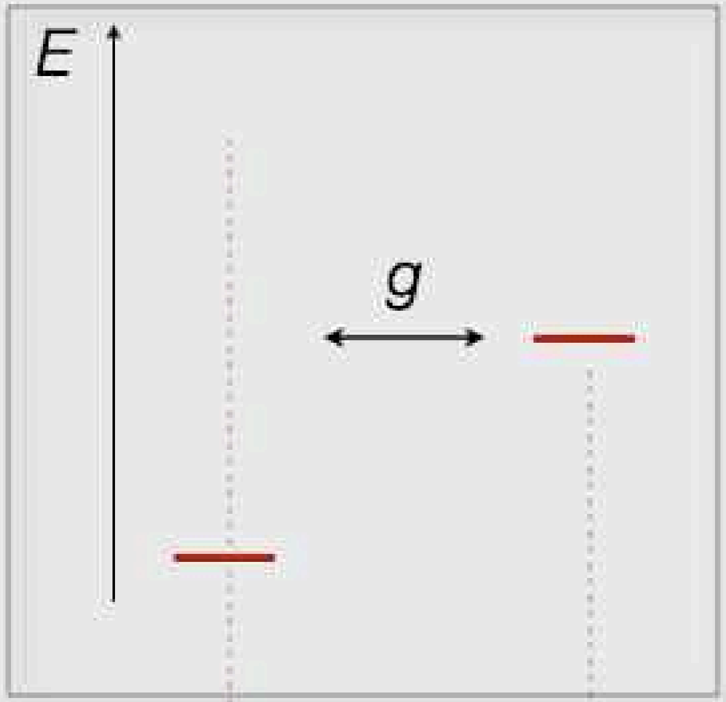


- system
- rate equations/semiclassical analysis
- adiabacity

Boulder, April 9 2009

Landau-Zener (LZ) problem

- two coupled levels subject to weakly time dependent driving



formalize by

$$\hat{H} = \begin{pmatrix} \lambda t & g \\ g & -\lambda t \end{pmatrix}$$

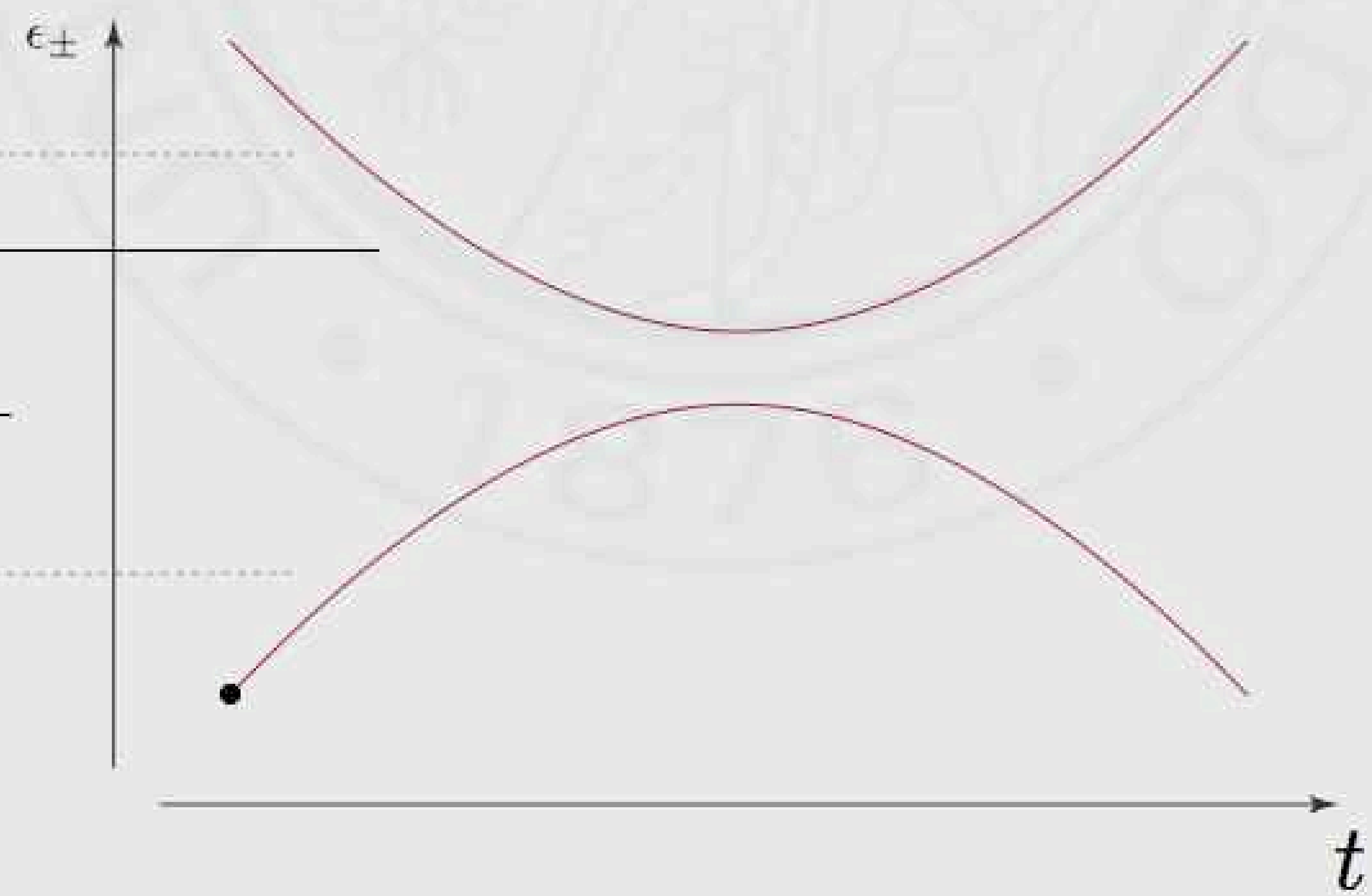
instantaneous levels

$$\epsilon_{\pm} = \pm\sqrt{(\lambda t)^2 + g^2}$$

$$P_{\text{excite}} = e^{-\pi g^2/\lambda}$$

$$P = 1 - e^{-\pi g^2/\lambda}$$

Landau/Zener 1932

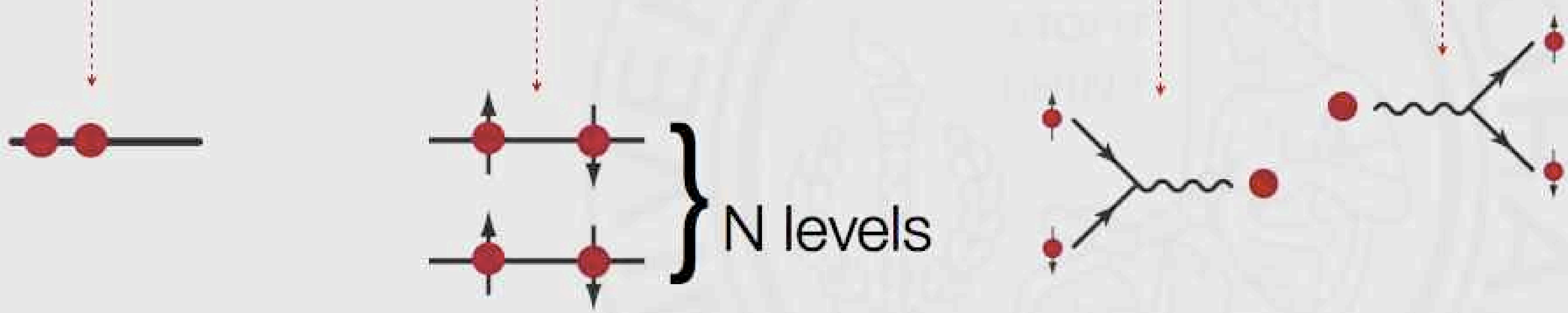


The model: 1. Fermi-Bose

(Time-dependent Dicke model)

$$\hat{H} = -\gamma \hat{b}^\dagger \hat{b} + \frac{\gamma}{2} \sum_{i=1}^{N} \left(\hat{a}^\dagger_{i\uparrow} \hat{a}_{i\uparrow} + \hat{a}^\dagger_{i\downarrow} \hat{a}_{i\downarrow} \right) + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left(\hat{b}^\dagger \, \hat{a}_{i\downarrow} \hat{a}_{i\uparrow} + \hat{b} \, \hat{a}^\dagger_{i\uparrow} \hat{a}^\dagger_{i\downarrow} \right)$$

$$\gamma = \lambda t$$



Particle conservation: $\left\langle \hat{b}^\dagger \hat{b} \right\rangle + \frac{1}{2} \sum_{i=1}^{N} \left\langle \hat{a}^\dagger_{i\uparrow} \hat{a}_{i\uparrow} + \hat{a}^\dagger_{i\downarrow} \hat{a}_{i\downarrow} \right\rangle = N$

Initially, at $t \to -\infty$

$$\frac{1}{2} \sum_{i=1}^{N} \left\langle \hat{a}^\dagger_{i\uparrow} \hat{a}_{i\uparrow} + \hat{a}^\dagger_{i\downarrow} \hat{a}_{i\downarrow} \right\rangle = N$$

$$\left\langle \hat{b}^\dagger \hat{b} \right\rangle = 0$$

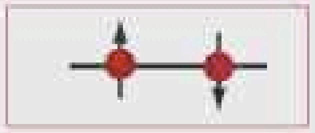
Would like to find, at $t \to +\infty$

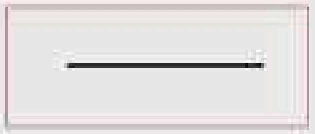
$$n_b = \left\langle \hat{b}^\dagger \hat{b} \right\rangle = ?$$

How many bosons did we create?

2. Cavity QED representation

- fermionic level *i* is represented by two spin configurations

 (pseudo-) spin up

 (pseudo-) spin down

$$\gamma = \lambda t$$

i

$$\hat{H} = -\gamma\, \hat{b}^\dagger \hat{b} + \frac{\gamma}{2} \sum_{i=1}^{N} \sigma_i^z + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left(\hat{b}^\dagger \sigma_i^- + \hat{b}\, \sigma_i^+ \right)$$

- introduce spin operators as: $\hat{S}^a = \frac{1}{2} \sum_i \sigma_i^a, \ (a = 1, 2, 3)$

$$\hat{H} = -\gamma\, \hat{b}^\dagger \hat{b} + \gamma \hat{S}^z + \frac{g}{\sqrt{N}} \left(\hat{b}^\dagger \hat{S}^- + \hat{b}\, \hat{S}^+ \right)$$

$$\left\langle \hat{S}^z \right\rangle \in \left[-\frac{N}{2}, \frac{N}{2} \right]$$

Initially, at $t \to -\infty$

$$\left\langle \hat{S}^z \right\rangle = \frac{N}{2}$$

$$\left\langle \hat{b}^\dagger \hat{b} \right\rangle = 0$$

Would like to find, at $t \to +\infty$

$$n_b = \left\langle \hat{b}^\dagger \hat{b} \right\rangle = ?$$

How many bosons did we create?

3. Atomic/molecular Bose condesates

$$H = -\gamma\,\hat{b}_a^\dagger \hat{b}_a + \gamma\,\hat{b}_m^\dagger \hat{b}_m + \frac{g}{\sqrt{N}}\left(\hat{b}_a^\dagger \hat{b}_a^\dagger \hat{b}_m + \hat{b}_a \hat{b}_a \hat{b}_m^\dagger\right)$$

$$\gamma = \lambda t$$

Atoms Molecules

Actually realized in Carl Wieman's group, 2005.

Particle conservation: $\left\langle \hat{b}_m^\dagger \hat{b}_m \right\rangle + \frac{1}{2}\left\langle \hat{b}_a^\dagger \hat{b}_a \right\rangle = N$

Initially, at $t \to -\infty$

$$\left\langle \hat{b}_m^\dagger \hat{b}_m \right\rangle = N$$

$$\left\langle \hat{b}_a^\dagger \hat{b}_a \right\rangle = 0$$

Would like to find, at $t \to +\infty$

$$n_b = \left\langle \hat{b}_a^\dagger \hat{b}_a \right\rangle = ?$$

How many atoms did we create?

Simplest case: *N=1*

$$\hat{H} = -\gamma\,\hat{b}^\dagger\hat{b} + \frac{\gamma}{2}\sigma^z + g\left(\hat{b}^\dagger\sigma^- + \hat{b}\,\sigma^+\right)$$

$\gamma = \lambda t$ Time-dependent standard cavity QED

Simple Hilbert space

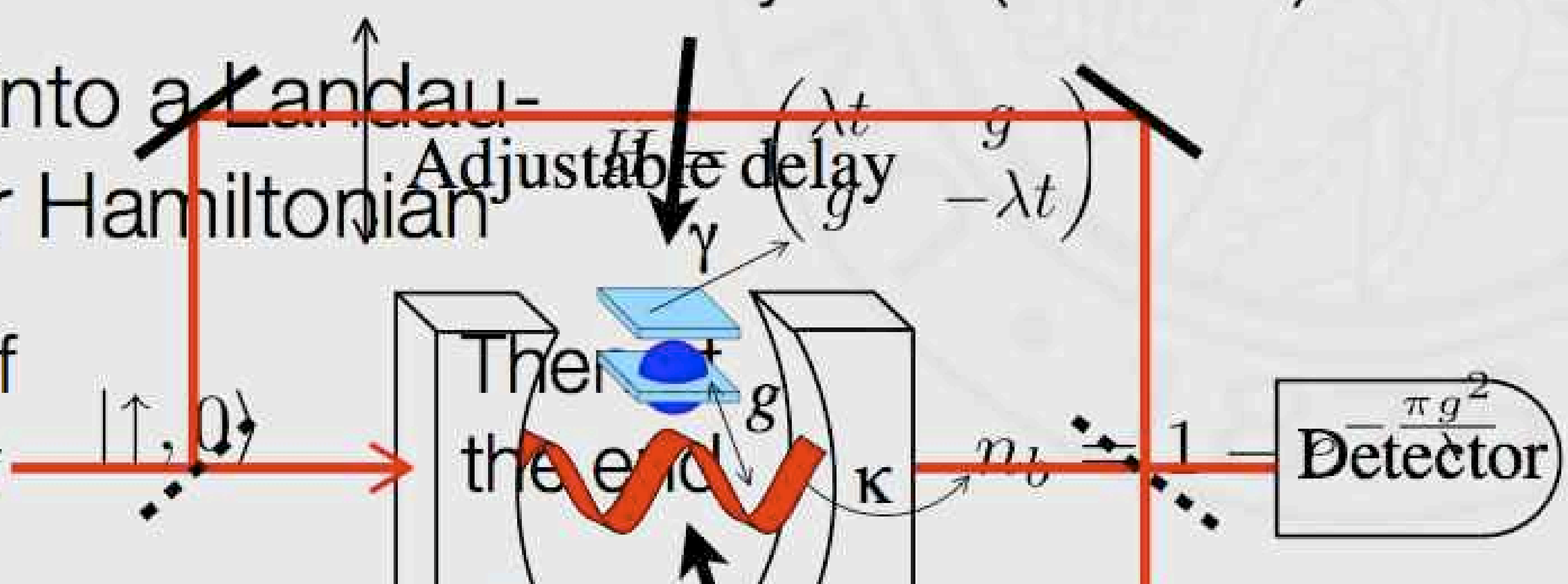
$|\uparrow, 0\rangle$ Spin up, no bosons

$|\downarrow, 1\rangle$ Spin down, 1 boson

Two level system (an atom)

Maps into a Landau-Zener Hamiltonian $H = \begin{pmatrix} \lambda t & g \\ g & -\lambda t \end{pmatrix}$

Thus: if initially $|\uparrow, 0\rangle$ Then at the end $n_b = 1 - e^{-\frac{\pi g^2}{\lambda}}$



Yet: see J. Keeling, VG, PRL (2008) for interesting new physics even in this situation.

A photon (boson)

Case of interest: large N

$$\hat{H} = -\gamma\, \hat{b}^\dagger \hat{b} + \frac{\gamma}{2} \sum_{i=1}^{N} \sigma_i^z + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left(\hat{b}^\dagger \sigma_i^- + \hat{b}\, \sigma_i^+ \right)$$

$$\gamma = \lambda t$$

Initially:

$$\left\langle \hat{b}^\dagger \hat{b} \right\rangle = 0$$

$$\langle \sigma_i^z \rangle = 1$$

~~Conjecture: finally~~

$$\cancel{n_b = N\left(1 - e^{-\frac{g^2}{\lambda}}\right) ???}$$

Wrong!!

Correct formula at $\dfrac{\lambda}{g^2} \ll 1$: $n_b \approx N \left(1 - \dfrac{\lambda}{\pi g^2} \log N \right)$

Derived in: A. Altland, VG, A. Polkovnikov, T. Kriecherbauer, PRA (2009)

Quantum Phase Transition

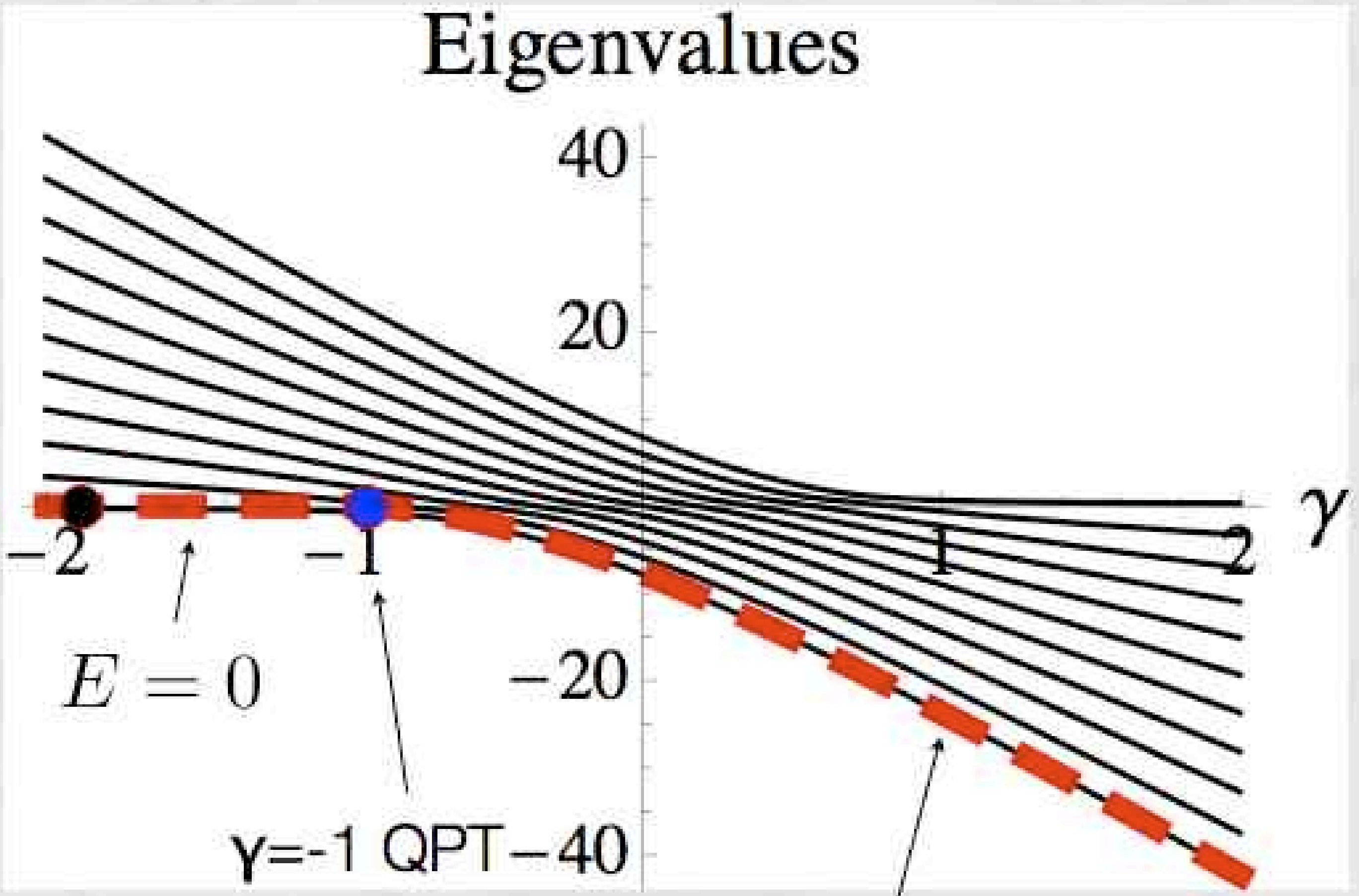
$$\hat{H} = -\gamma\,\hat{b}^{\dagger}\hat{b} + \gamma\hat{S}^{z} + \frac{g}{\sqrt{N}}\left(\hat{b}^{\dagger}\hat{S}^{-} + \hat{b}\,\hat{S}^{+}\right)$$

$$\left\langle \hat{S}^{z}\right\rangle \in \left[-\frac{N}{2}, \frac{N}{2}\right]$$

N+1 - dim Hilbert space

$$|N/2, 0\rangle$$
$$|N/2-1, 1\rangle$$
$$|N/2-2, 2\rangle$$
$$\cdots$$
$$|-N/2, N\rangle$$

N=10, g=1



$$E = -4N\left[9\gamma - \gamma^{3} + 4(\gamma^{2}+3)^{\frac{3}{2}}\right]/27$$

Large N: tuning through a phase transition

Matrix Hamiltonian

$$|N/2-n, n\rangle \equiv |n\rangle$$

$$\hat{H} = -2\gamma n\,\delta_{n',n} + \frac{g}{\sqrt{N}}\left(n\sqrt{N-n'}\,\delta_{n'+1,n} + n'\sqrt{N-n}\,\delta_{n'-1,n}\right)$$

Tuning through a quantum phase transition

A field explored in the literature recently:

Polkovnikov, arxiv:0312144; PRB (2005)
Zurek, Dorner, Zoller: PRL (2005)

A scaling argument due to Polkovnikov gives:

$n_{\mathrm{exc}} \sim \lambda^{\frac{z\nu}{z\nu+1}}$; z and ν are the critical exponents

However, many exceptions

see Polkovnikov, Gritsev, Nat. Phys. (2008)

Bottom line: Landau-Zener exponential approach to the adiabatic limit typically does not work

Back to the model: three regimes

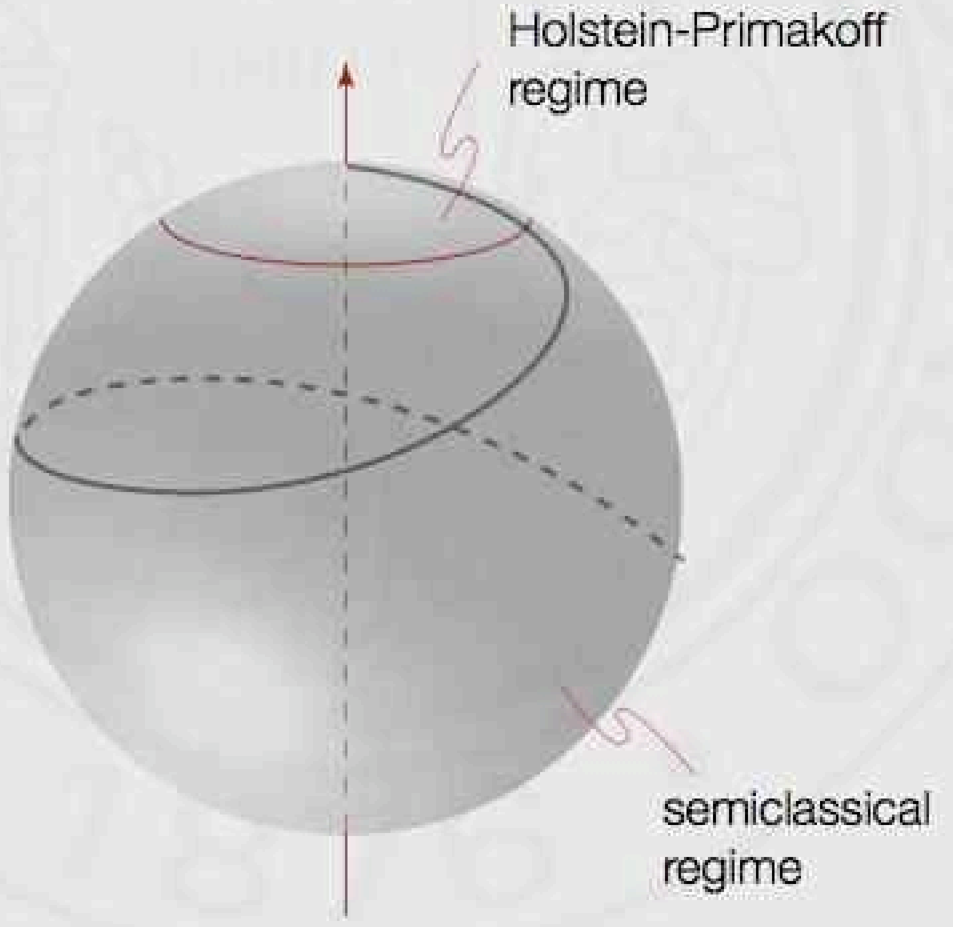
$$\hat{H} = -\gamma\,\hat{b}^\dagger\hat{b} + \gamma\hat{S}^z + \frac{g}{\sqrt{N}}\left(\hat{b}^\dagger\hat{S}^- + \hat{b}\,\hat{S}^+\right)$$
$$\gamma = \lambda t$$

Regimes:

1. Holstein-Primakoff $\lambda \gg \dfrac{\pi g^2}{\log(N)}$

2. Intermediate $\lambda \sim \dfrac{\pi g^2}{\log(N)}$

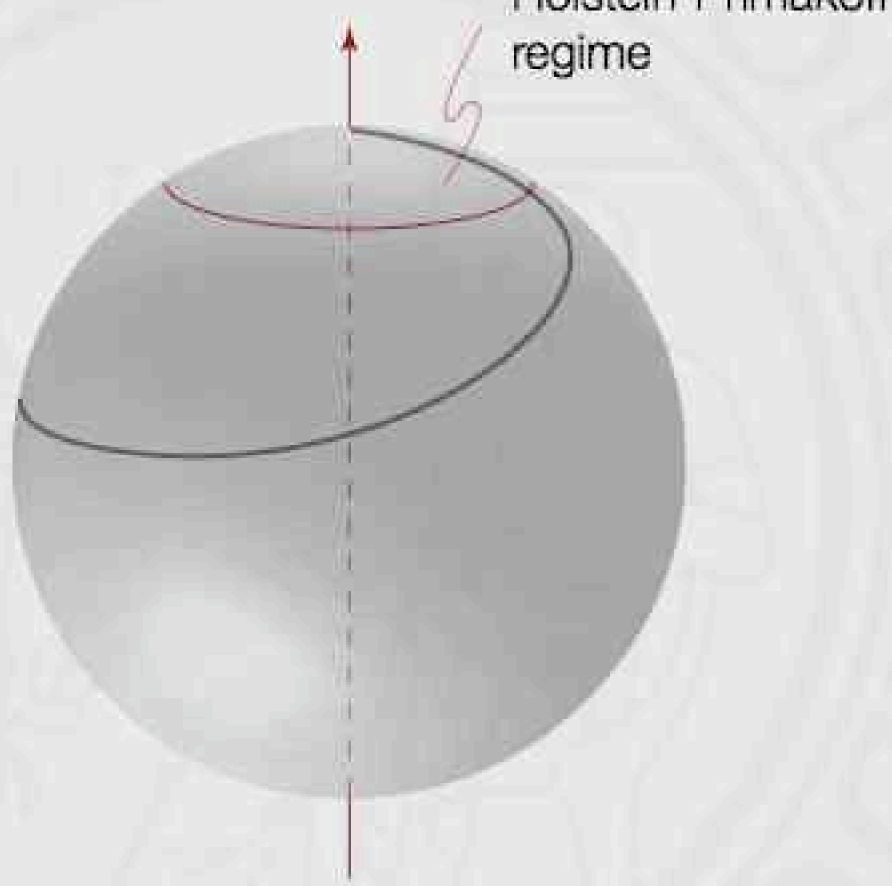
3. Semiclassical (adiabatic) $\lambda \ll \dfrac{\pi g^2}{\log(N)}$



The crossover from HP to semiclassical regimes occur at very slow driving rates!

Holstein-Primakoff regime

$$\hat{H} = -\gamma\, \hat{b}^\dagger \hat{b} + \gamma \hat{S}^z + \frac{g}{\sqrt{N}}\left(\hat{b}^\dagger \hat{S}^- + \hat{b}\, \hat{S}^+\right)$$
$$\gamma = \lambda t$$



Fast driving rate $\lambda \gg \dfrac{\pi g^2}{\log(N)}$

Spin points almost up $\left\langle \hat{S}^z \right\rangle \approx \dfrac{N}{2}$

HP bosons:

$$\hat{S}^+ \approx \sqrt{N}\, \hat{b}_{HP}, \hat{S}^- \approx \sqrt{N}\, \hat{b}^\dagger_{HP}$$

Hamiltonian is now quadratic and solvable

$$\hat{H} = -\lambda t\ \hat{b}^\dagger \hat{b} - \lambda t\ \hat{b}^\dagger_{HP} \hat{b}_{HP} + g\left(\hat{b}^\dagger\, \hat{b}^\dagger_{HP} + \hat{b}\, \hat{b}_{HP}\right).$$

Answer: $n_b = e^{\frac{\pi g^2}{\lambda}} - 1$ Of course, works only if $n_b \ll N$

Yurovsky, Ben-Reuven, Julienne, PRA (2002)

Intermediate regime: rate equations

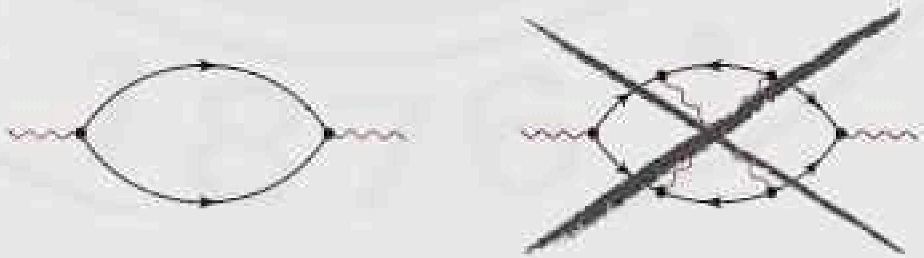
$$\hat{H} = -\gamma \hat{b}^\dagger \hat{b} + \frac{\gamma}{2} \sum_{i=1}^{N} \left(\hat{a}^\dagger_{i\uparrow} \hat{a}_{i\uparrow} + \hat{a}^\dagger_{i\downarrow} \hat{a}_{i\downarrow} \right) + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left(\hat{b}^\dagger \, \hat{a}_{i\downarrow} \hat{a}_{i\uparrow} + \hat{b} \, \hat{a}^\dagger_{i\uparrow} \hat{a}^\dagger_{i\downarrow} \right)$$

▹ rate equation

$$\boxed{d_t n_b = 2\pi g^2 \delta(2\lambda t) \left(n_f^2 \left(1 + n_b\right) - n_b \left(1 - n_f\right)^2 \right)}$$

where $n_b + N n_f = N$

Rate equation can be justified within Keldysh RPA approximation



Answer: $$\boxed{n_b = \frac{N\left(e^{\frac{\pi g^2}{\lambda}} - 1\right)}{2e^{\frac{\pi g^2}{\lambda}} + N}}$$

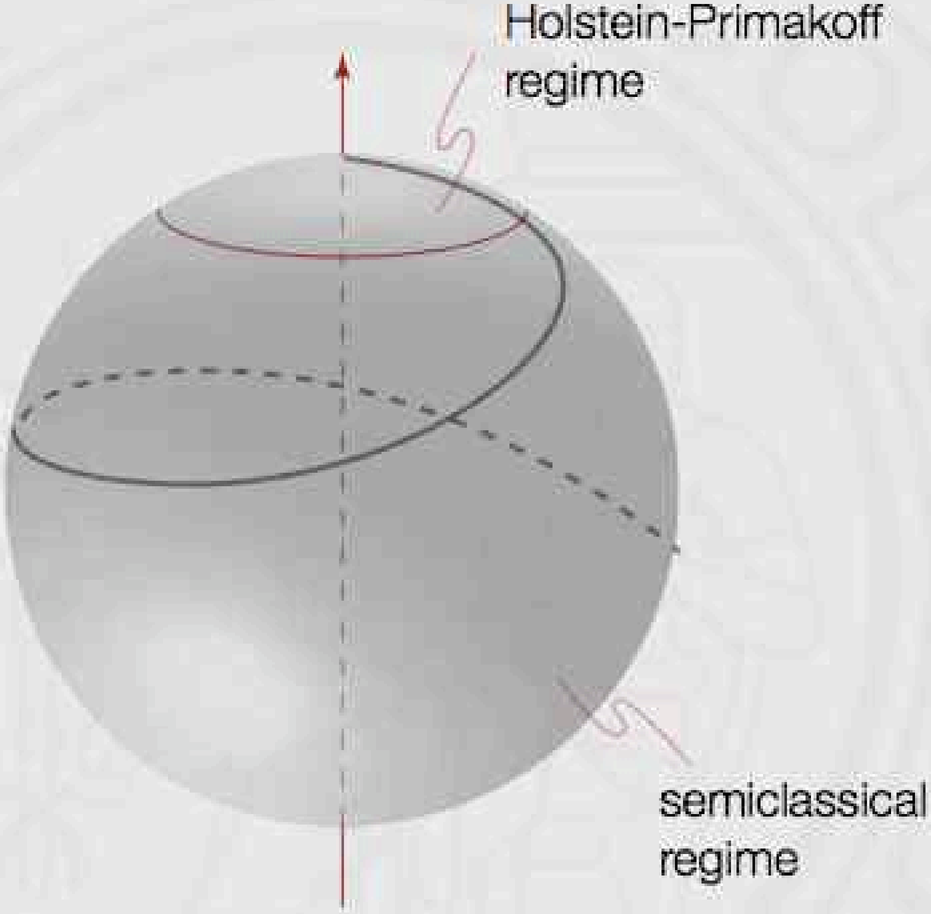
Works if $\lambda \gtrsim \dfrac{\pi g^2}{\log(N)}$

A. Altland, V. Gurarie, PRL (2008)

Semiclassical (adiabatic) regime

$$\hat{H} = -\gamma\,\hat{b}^\dagger\hat{b} + \gamma\hat{S}^z + \frac{g}{\sqrt{N}}\left(\hat{b}^\dagger\hat{S}^- + \hat{b}\,\hat{S}^+\right)$$
$$\gamma = \lambda t$$

$$\hat{b} \to \sqrt{N}\,\sqrt{n},\ \ n \in [0,1]$$
$$\hat{S} \to \sqrt{\frac{N}{2}}\mathbf{n} \quad \mathbf{n} \to \varphi, \theta \text{ spherical angles}$$



The end result: classical problem

$$H = -2\gamma n - 2gn\sqrt{1-n}\cos(\phi)$$

with the equations of motion:

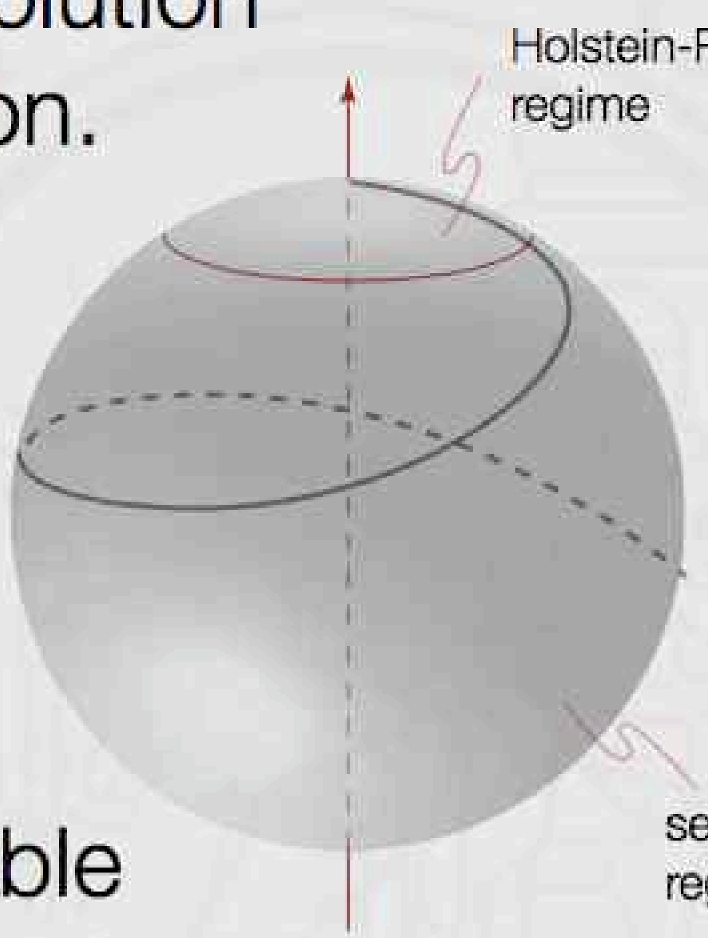
$$\dot{n} = -\partial_\phi H \qquad \dot{\phi} = \partial_n H$$

$$\dot{n} = -2gn\sqrt{1-n}\sin(\phi)$$

Little problem: the solution to these equations is *n(t)=0*!

Trancated Wigner approximation

Need to match the initial quantum evolution with subsequent classical evolution.



Solution: initially *n* is a random variable corresponding to the Wigner function of a harmonic oscillator in the ground state

$$W(n)dn = 2e^{-2Nn}N dn$$

Roughly speaking, initially $n \sim 1/N$

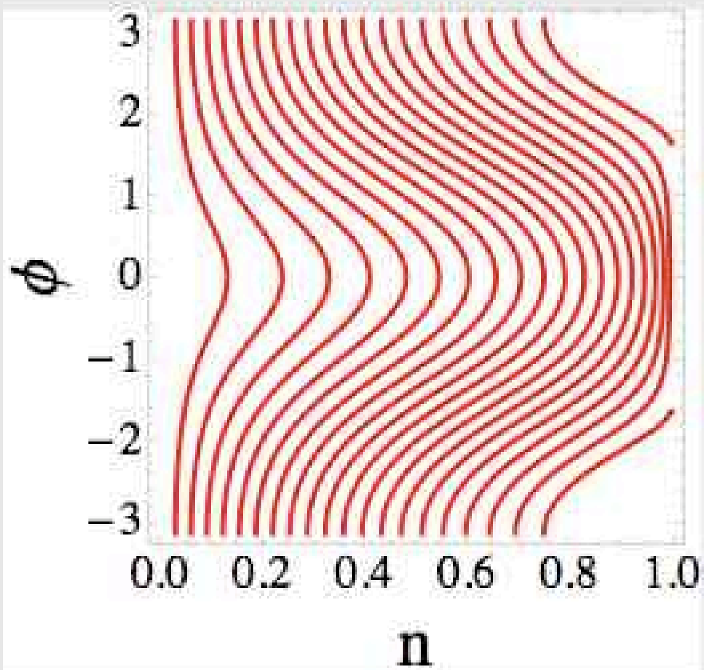
Now need to solve

$$H = -2\gamma n - 2gn\sqrt{1-n}\cos(\phi)$$

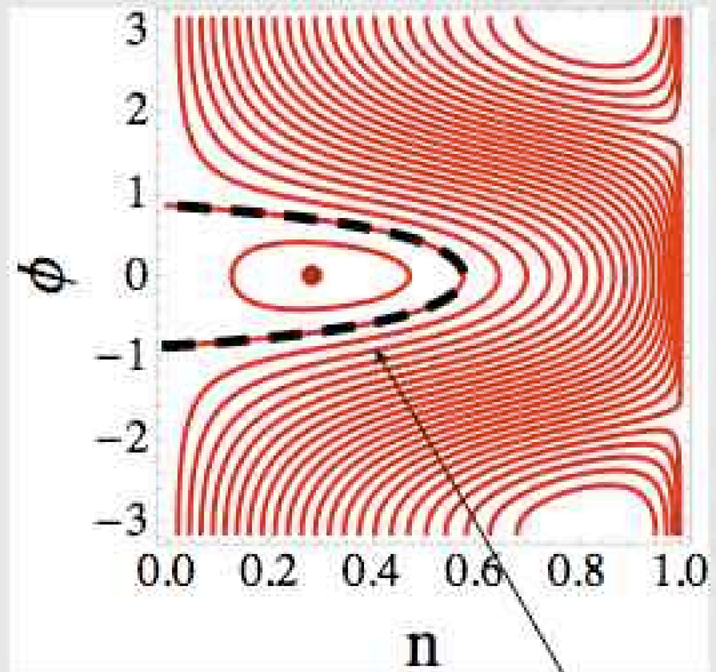
with random initial conditions

Classical phase space

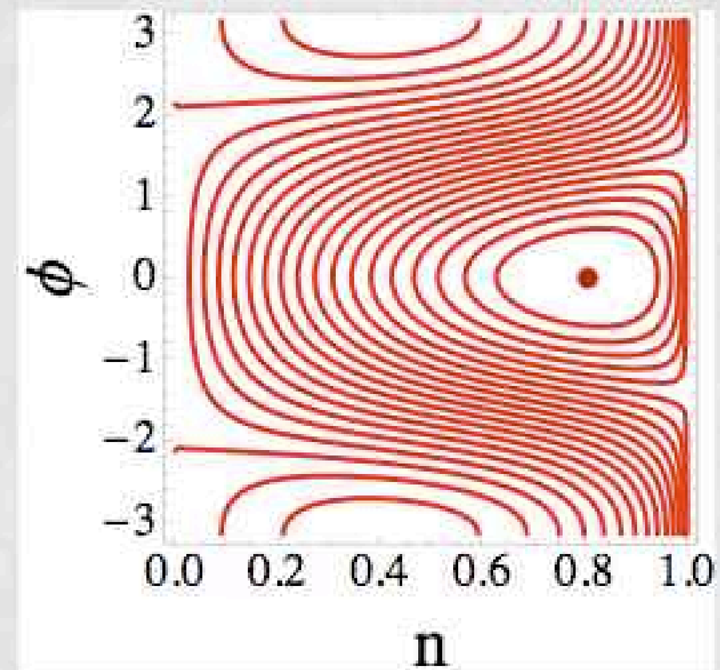
$$H = -2\gamma n - 2gn\sqrt{1-n}\cos(\phi) = \text{const}$$



$\gamma = -1.5$

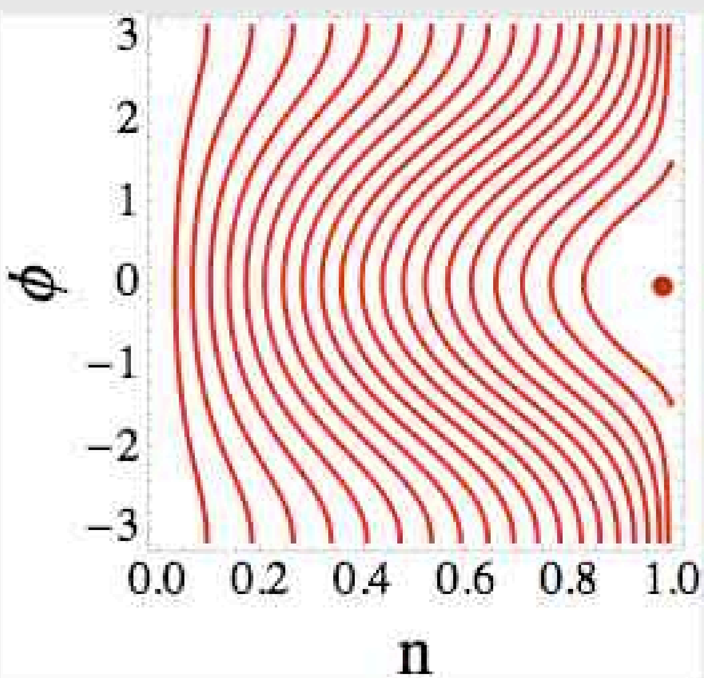


$\gamma = -.65$

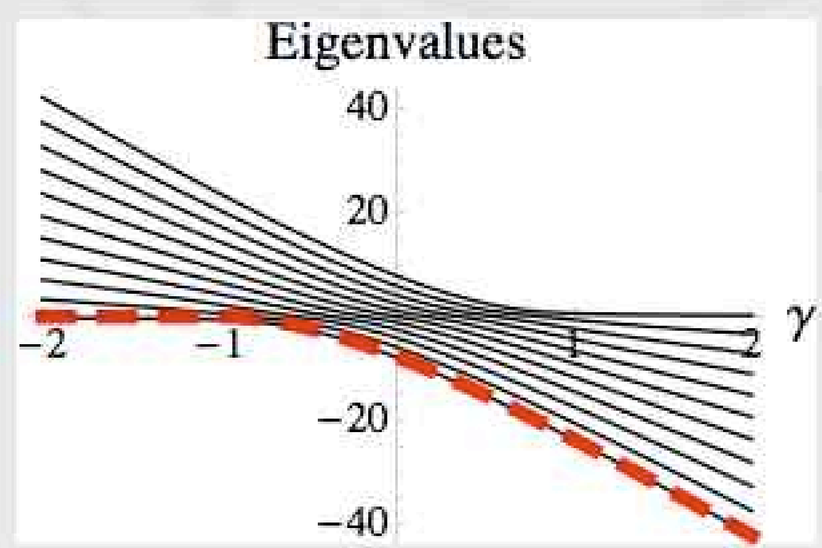


$\gamma = .5$

Critical trajectory



$\gamma = 2$



Adiabatic Invariants

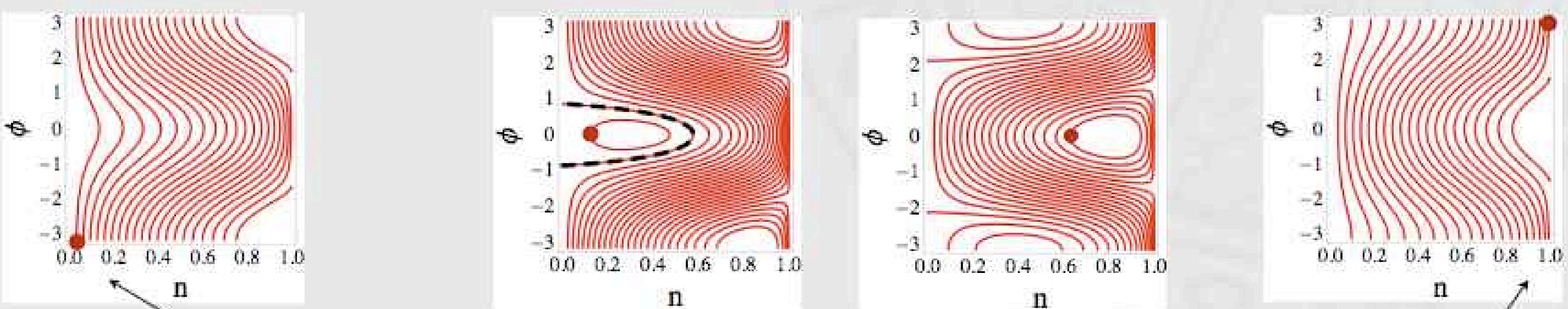
$$I = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi\, n$$

Adiabatic evolution is best captured by adiabatic invariants

As a first approximation, they do not change during the evolution

Recently emphasized in related context by Q. Niu et al (PRL and PRA, 2000-2002)

$\gamma = -1.5$ $\quad\quad$ $\gamma = -0.65$ $\quad\quad$ $\gamma = 0.5$ $\quad\quad$ $\gamma = 2$



$$t \to -\infty : \; I_{\rm ini} = n_{\rm ini} \sim \frac{1}{N}$$

$$t \to +\infty : \; I_{\rm final} = 1 - n_b/N$$

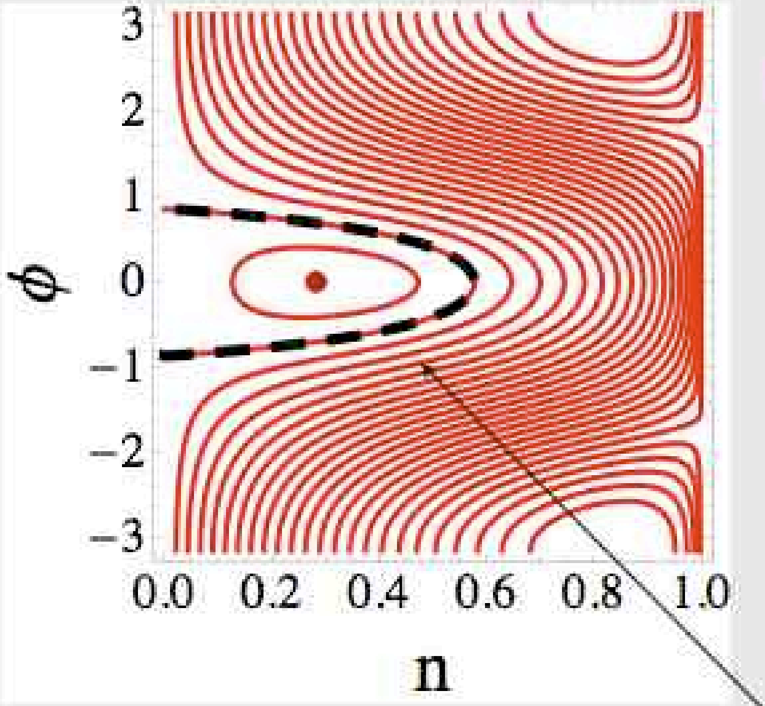
In the deep adiabatic regime, λ very small, $I_{\rm final} - I_{ini} = 0$ $\quad I_{\rm final} \approx 0 \quad n_b \approx N$

Change in adiabatic invariants

Landau and Lifshits, Classical Mechanics, <u>last sections</u>:
For a <u>smooth</u> evolution, $I_{\text{final}} - I_{\text{ini}} \sim e^{-\frac{1}{\lambda}}$

Then $n_b \sim N(1 - e^{-\frac{1}{\lambda}})$ That's Landau-Zener formula!

If an evolution crosses a "critical" trajectory (whose frequency vanishes), then $I_{\text{final}} - I_{\text{ini}} \sim \lambda^{\alpha}$ Then $n_b \sim N(1 - \lambda^{\alpha})$



$\gamma = -.65$

Our case is this critical case. Calculations following the adiabatic invariant theory give

$$n_b \approx N\left(1 - \frac{\lambda}{\pi g^2}\log N\right)$$

A. Altland, VG, A. Polkovnikov, T. Kriecherbauer, PRA (2009)

Critical trajectory (it's existence is related to the QPT)

Relationship with the Painlevé II

A. P. Itin, P. Törmä, arxiv:0901.4778

In the vicinity of a critical point $n = 0,\ \phi = 0, \gamma = -1$

a substitution $\begin{matrix} Y \sim n\sin(\phi/2) \\ s \sim \gamma + 1 \end{matrix}$ leads approximately to

$$\frac{d^2Y}{ds^2} = sY - Y^3$$

This is the Painlevé II equation describing a particle moving in a potential $U(x) = \frac{Y^4}{4} - s\frac{Y^2}{2}$

The solutions to the Painlevé II equation are well known, leading to an improved result

$$n_b = N\left(1 - \frac{\lambda}{\pi g^2}\log\left[\frac{N\lambda e^{\gamma_{\mathrm{Euler}}}}{2\pi g^2}\right] + \dots\right)$$

Crossover to the super-adiabatic regime

$$n_b = N\left(1 - \frac{\lambda}{\pi g^2}\log\left[\frac{N\lambda e^{\gamma_{\mathrm{Euler}}}}{2\pi g^2}\right] + \ldots\right)$$

This could work only if $\lambda \gtrsim \frac{1}{N}$

Indeed, smaller **λ** should lead to such a slow evolution that individual levels are resolved, leading back to the Landau-Zener formula. This regime is unaccessible in the large *N* limit.

Summary of the analytic results

$$\hat{H} = -\gamma\,\hat{b}^\dagger\hat{b} + \gamma\hat{S}^z + \frac{g}{\sqrt{N}}\left(\hat{b}^\dagger\hat{S}^- + \hat{b}\,\hat{S}^+\right)$$
$$\gamma = \lambda t$$

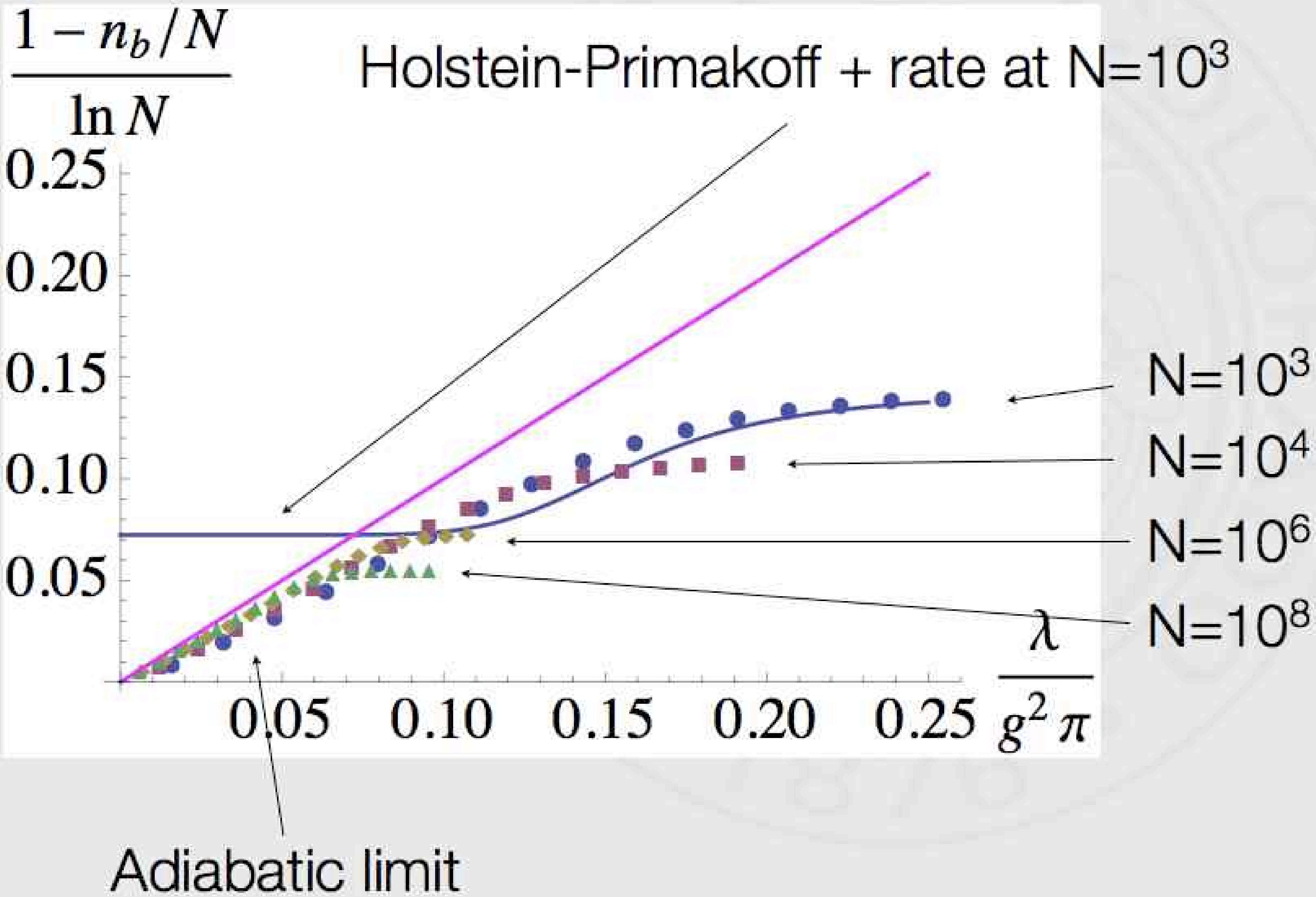
Regimes:

1. Holstein-Primakoff $\lambda \gg \dfrac{\pi g^2}{\log(N)}$ $\qquad n_b = e^{\frac{\pi g^2}{\lambda}} - 1$

2. Intermediate $\lambda \sim \dfrac{\pi g^2}{\log(N)}$ $\qquad n_b = \dfrac{N\left(e^{\frac{\pi g^2}{\lambda}} - 1\right)}{2e^{\frac{\pi g^2}{\lambda}} + N}$

3. Semiclassical (adiabatic) $\lambda \ll \dfrac{\pi g^2}{\log(N)}$ $\qquad n_b \approx N\left(1 - \dfrac{\lambda}{\pi g^2}\log N\right)$

Comparison with numerics



Distribution function

$$\hat{H} = -\gamma\, \hat{b}^{\dagger}\hat{b} + \frac{\gamma}{2}\sum_{i=1}^{N}\sigma_i^z + \frac{g}{\sqrt{N}}\sum_{i=1}^{N}\left(\hat{b}^{\dagger}\sigma_i^- + \hat{b}\,\sigma_i^+\right)$$

$$\gamma = \lambda t$$

Initially:

$$\left\langle \hat{b}^{\dagger}\hat{b} \right\rangle = 0$$

$$\langle \sigma_i^z \rangle = 1$$

We would now like to find, at t→+∞ the full probability distribution function of observing exactly n_b bosons

$$P(n_b)$$

HP: $\lambda \gg \frac{\pi g^2}{\log(N)}$

Exact solution:

$$P(n_b) = e^{-n_b e^{-\frac{\pi g^2}{\lambda}} - \frac{\pi g^2}{\lambda}}$$

Adiabatic: $\lambda \ll \frac{\pi g^2}{\log(N)}$

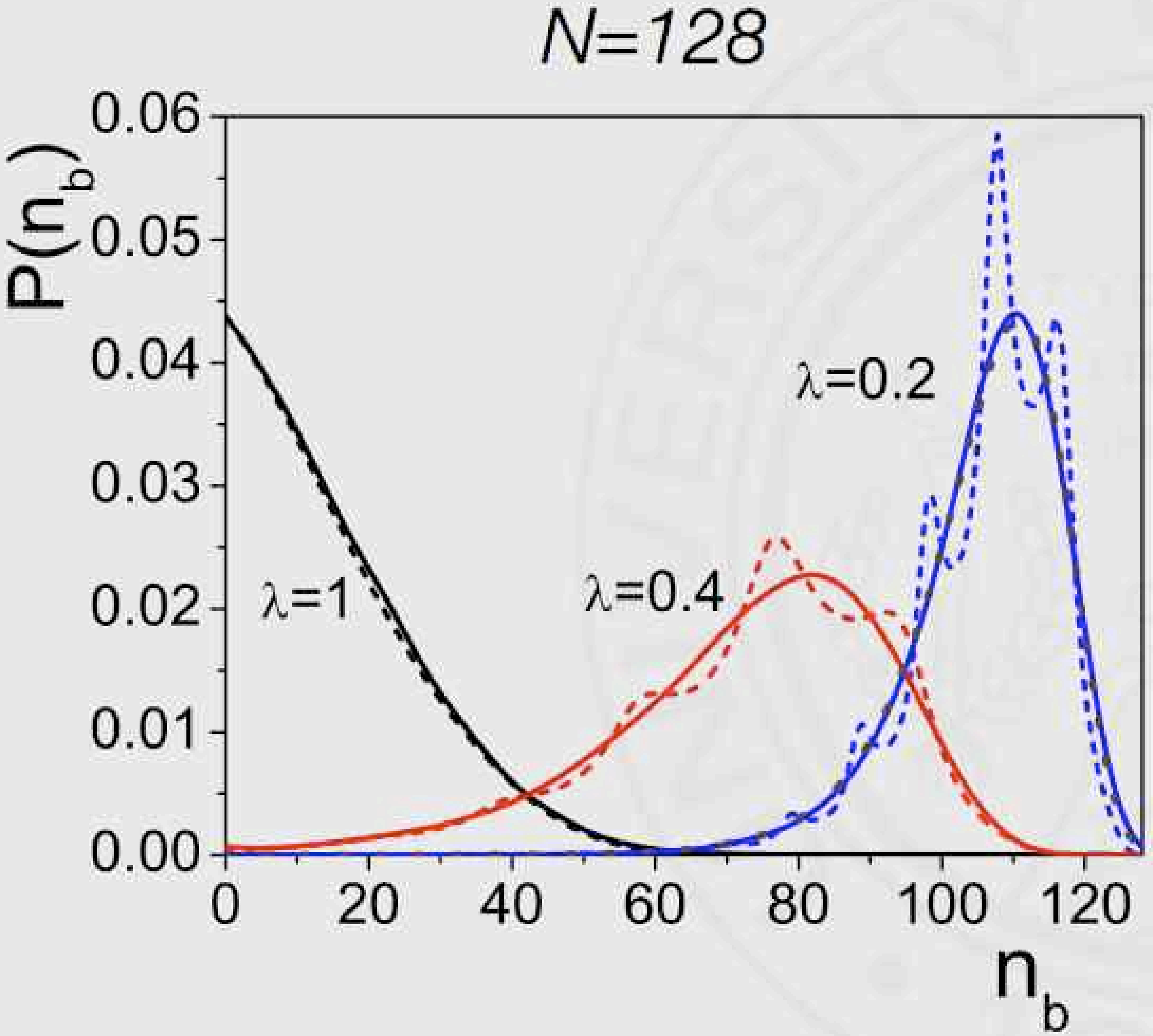
$$n_b = N\left(1 - \frac{\lambda}{\pi g^2}\log\left[\frac{N\lambda}{2\pi g^2 n_{\mathrm{ini}}}\right] + \ldots\right)$$

$$P(n_b) = \frac{N\lambda}{2\pi g^2}\, e^{-\frac{\pi g^2}{\lambda}\left(1-\frac{n_b}{N}\right)}$$

$$P(n_{\mathrm{ini}}) = 2e^{-2n_{\mathrm{ini}}}$$

Gumbel distribution

Comparison with numerics



HP: $\lambda \gg \frac{\pi g^2}{\log(N)}$

Adiabatic: $\lambda \ll \frac{\pi g^2}{\log(N)}$

$$P(n_b) = e^{-n_b e^{-\frac{\pi g^2}{\lambda}} - \frac{\pi g^2}{\lambda}}$$

$$P(n_b) = e^{-\frac{\pi g^2}{\lambda}\left(1-\frac{n_b}{N}\right) - \frac{N\lambda}{\pi g^2} e^{-\frac{\pi g^2}{\lambda}\left(1-\frac{n_b}{N}\right)}}$$

Conclusions

$$\hat{H} = -\gamma\,\hat{b}^{\dagger}\hat{b} + \gamma\hat{S}^{z} + \frac{g}{\sqrt{N}}\left(\hat{b}^{\dagger}\hat{S}^{-} + \hat{b}\,\hat{S}^{+}\right)$$
$$\gamma = \lambda t$$

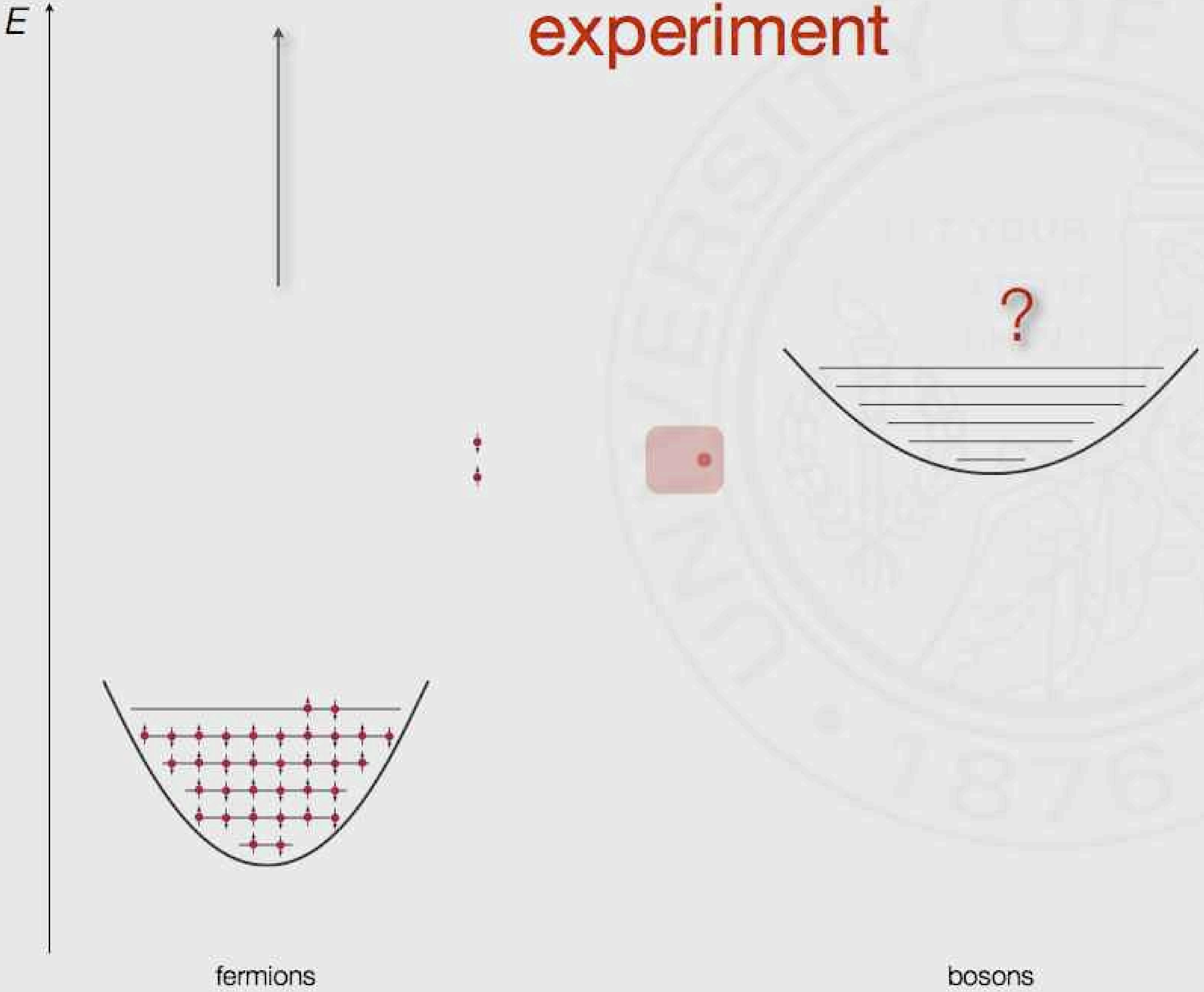
- In a large *N* many-body system, it is hard to reach the adiabatic regime (possible consequences for the adiabatic quantum computing)

- To be adiabatic, **λ** << $g^2/Log(N)$

- Quasiclassical evolution must be supplemented by the quantum initial conditions to study the adiabatic regime

- Very broad distribution of boson numbers despite the applicability of the quasiclassical approximation

Molecule creation in a Feshbach resonance experiment



The model

$$H = \sum_{\mathbf{p},\sigma=\uparrow,\downarrow} \frac{p^2}{2m}\, \hat{a}^\dagger_{\mathbf{p}\sigma}\hat{a}_{\mathbf{p}\sigma} + \sum_{p}\left(\frac{q^2}{4m} - 2\gamma\right)\hat{b}^\dagger_{\mathbf{q}}\hat{b}_{\mathbf{q}} + \frac{g}{\sqrt{V}}\sum_{\mathbf{p},\mathbf{q}}\left(\hat{b}_{\mathbf{q}}\;\hat{a}^\dagger_{\mathbf{q}/2+\mathbf{p}\uparrow}\hat{a}^\dagger_{\mathbf{q}/2-\mathbf{p}\downarrow} + h.c.\right)$$

$$\gamma = \lambda t$$

fermionic atoms

bosonic molecules

interconversion

Initially, at t→-∞, no bosons while fermions fill the Fermi sea up to ε_F, having density n_F.

Finally, at t→+∞, how many bosons we create, as a function of the rate **λ**?

Dimensionless parameters

$\gamma_W = g^2 m^2 / n_F^{1/3}$ Resonance width

$\Gamma = \pi g^2 n_F / \lambda$ Landau-Zener parameter

$$n_b = n_F f\left(\Gamma, \gamma_W\right)$$

Fast rate

$$H = \sum_{\mathbf{p},\sigma=\uparrow,\downarrow} \frac{p^2}{2m}\, \hat{a}^\dagger_{\mathbf{p}\sigma}\hat{a}_{\mathbf{p}\sigma} + \sum_{p}\left(\frac{q^2}{4m} - 2\gamma\right)\hat{b}^\dagger_{\mathbf{q}}\hat{b}_{\mathbf{q}} + \frac{g}{\sqrt{V}}\sum_{\mathbf{p},\mathbf{q}}\left(\hat{b}_{\mathbf{q}}\, \hat{a}^\dagger_{\mathbf{q}/2+\mathbf{p}\uparrow}\hat{a}^\dagger_{\mathbf{q}/2-\mathbf{p}\downarrow} + h.c.\right)$$

$$\gamma = \lambda t$$

~~conjecture:~~ $\cancel{n_b = n_F\left(1 - e^{-\Gamma}\right)}$

$$\Gamma = \pi g^2 n_F/\lambda$$

Wrong!

Dobrescu, Pokrovsky, Phys. Lett. A (2006)

Fast rate λ

$$n_b = n_F\left(\Gamma - \frac{88}{105}\Gamma^2 + \dots\right)$$

compare with

$$n_F\left(1 - e^{-\Gamma}\right) = n_F\left(\Gamma - \frac{\Gamma^2}{2} + \dots\right)$$

Simplified model (valid for narrow resonance only)

Broad resonance: hopeless (even the time-independent problem can be solved only numerically). Concentrate on narrow resonances.

$$\hat{H} = -2\gamma \hat{b}^\dagger \hat{b} + \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{\mathbf{p}\sigma} \hat{a}_{\mathbf{p}\sigma} + \frac{g}{\sqrt{N}} \sum_p \left(\hat{b}^\dagger \hat{a}_{-\mathbf{p}\downarrow} \hat{a}_{\mathbf{p}\uparrow} + \hat{b}\, \hat{a}^\dagger_{\mathbf{p}\uparrow} \hat{a}^\dagger_{-\mathbf{p}\downarrow} \right)$$

$$\gamma = \lambda t$$

This model applies only to the narrow resonance case, since for broad resonance the bosonic momentum dependence becomes important $\gamma_W \ll 1$

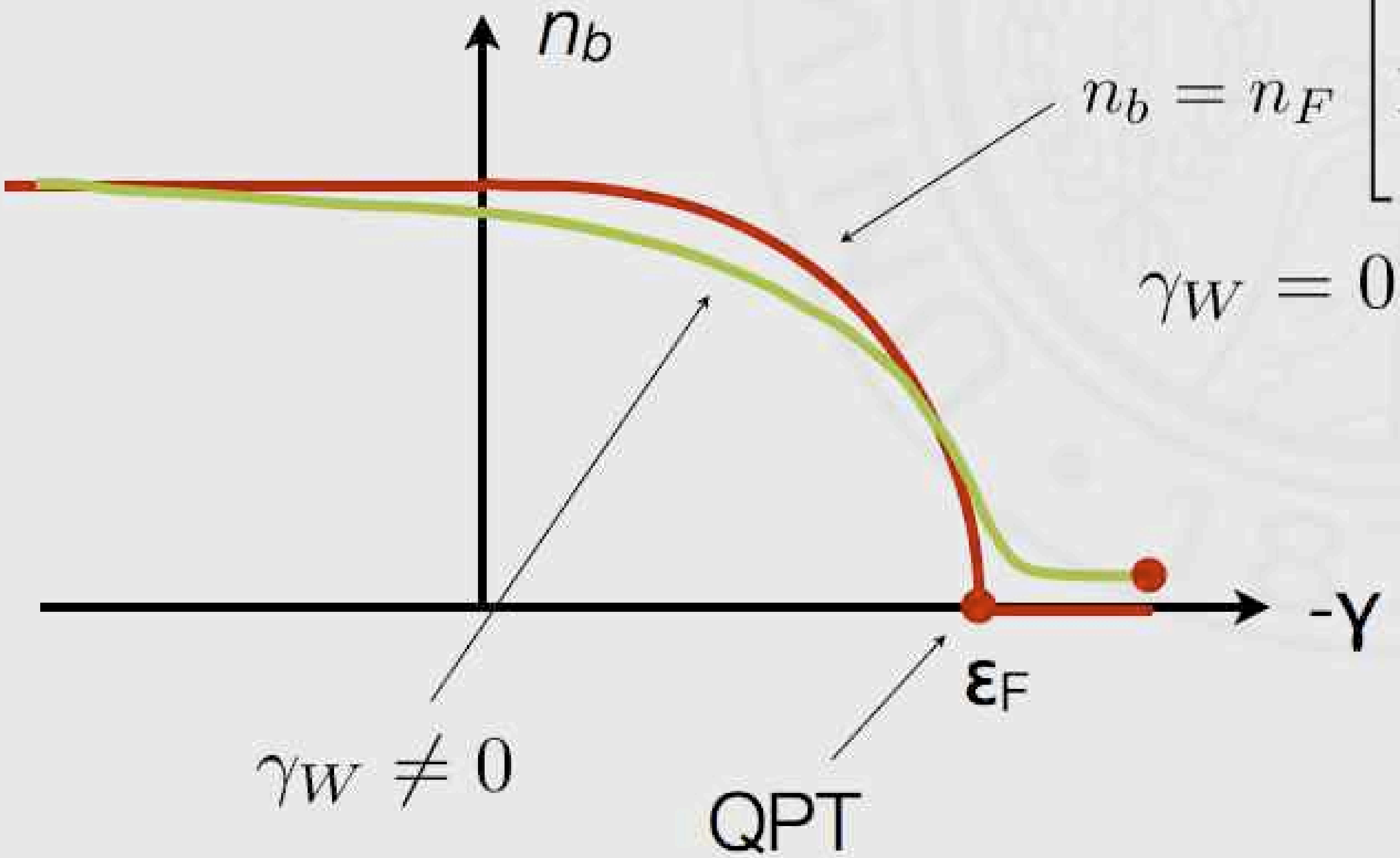
$$\gamma_W = g^2 m^2 / n_F^{1/3}$$

J. Levinsen, VG, PRA (2006)

Narrow resonance: close to a QPT

$$\hat{H} = -2\gamma \hat{b}^\dagger \hat{b} + \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{\mathbf{p}\sigma} \hat{a}_{\mathbf{p}\sigma} + \frac{g}{\sqrt{N}} \sum_p \left(\hat{b}^\dagger \hat{a}_{-\mathbf{p}\downarrow} \hat{a}_{\mathbf{p}\uparrow} + \hat{b}\, \hat{a}^\dagger_{\mathbf{p}\uparrow} \hat{a}^\dagger_{-\mathbf{p}\downarrow} \right)$$

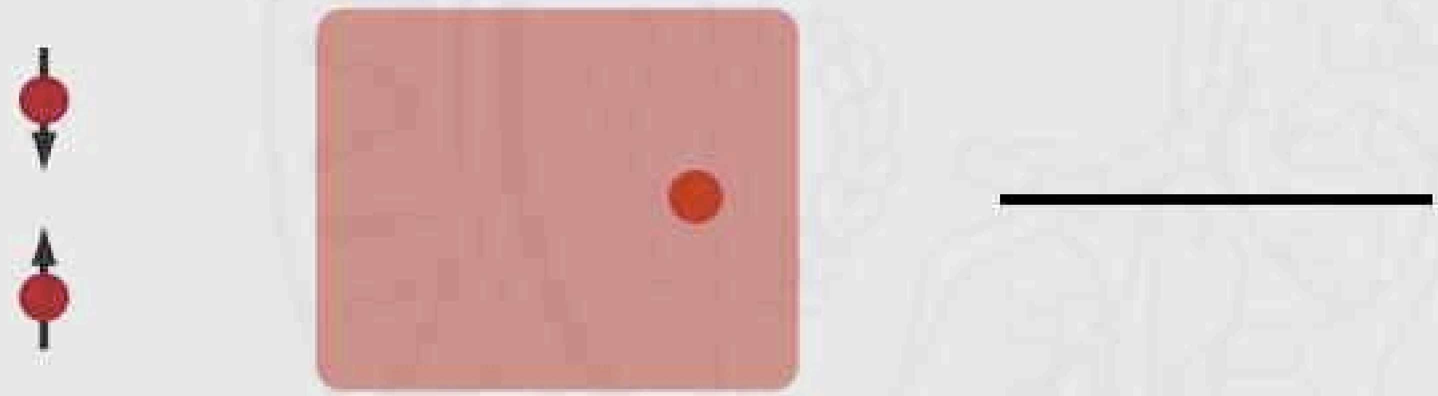
$$\gamma = \lambda t$$



$$n_b = n_F \left[1 - \left(\frac{\gamma}{\epsilon_F} \right)^{\frac{3}{2}} \right]$$

$$\gamma_W = 0 \ (g = 0)$$

The conversion, one pair at a time



$$\hat{H} = -2\gamma \hat{b}^\dagger \hat{b} + \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{\mathbf{p}\sigma} \hat{a}_{\mathbf{p}\sigma} + \frac{g}{\sqrt{N}} \sum_p \left(\hat{b}^\dagger \hat{a}_{-\mathbf{p}\downarrow} \hat{a}_{\mathbf{p}\uparrow} + \hat{b}\, \hat{a}^\dagger_{\mathbf{p}\uparrow} \hat{a}^\dagger_{-\mathbf{p}\downarrow} \right)$$

$$\gamma = \lambda t$$

When $-\epsilon_F \leq \gamma \leq 0$

only those fermions are in resonance whose momentum

$$p^2/(2m) = -\gamma$$

The transition happens one pair at a time

$$\hat{H} = -2\gamma \hat{b}^\dagger \hat{b} + \sum_{p,\sigma=\uparrow,\downarrow} \frac{p^2}{2m} \hat{a}^\dagger_{\mathbf{p}\sigma} \hat{a}_{\mathbf{p}\sigma} + \frac{g}{\sqrt{N}} \sum_p \left(\hat{b}^\dagger \hat{a}_{-\mathbf{p}\downarrow} \hat{a}_{\mathbf{p}\uparrow} + \hat{b}\, \hat{a}^\dagger_{\mathbf{p}\uparrow} \hat{a}^\dagger_{-\mathbf{p}\downarrow} \right)$$

$$\gamma = \lambda t$$

When $-\epsilon_F \le \gamma \le 0$

only those fermions are in resonance whose momentum

$$p^2/(2m) = -\gamma$$

$$n_p^f = e^{-\frac{\pi g^2 n_b(p)}{\lambda}}$$

$$\frac{dn_b(x)}{dx} = e^{-\frac{g^2 \pi n_b(x)}{\lambda}} - 1$$

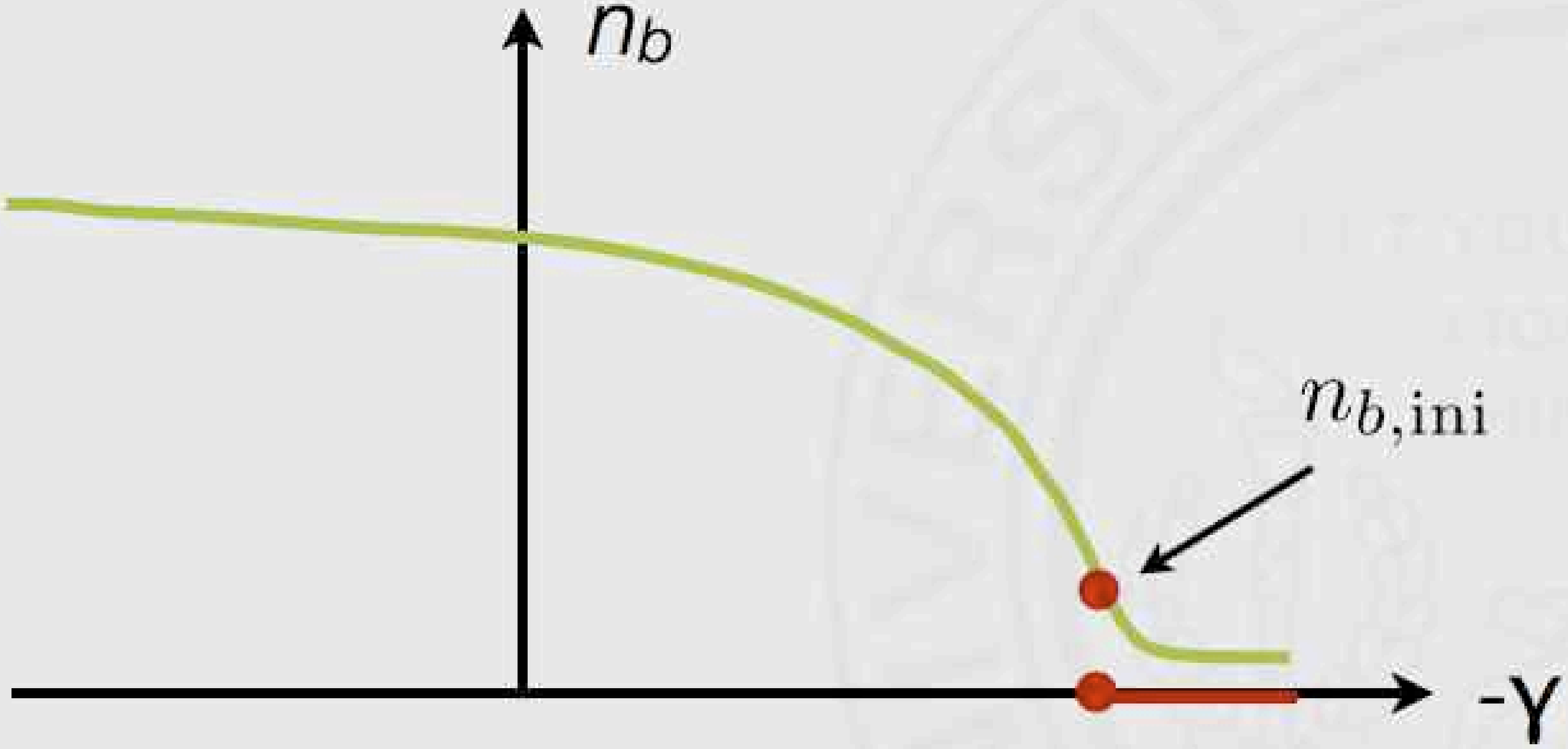
As before, a trivial solution

$$n_b(x) = 0!$$

$$x = p^3/(6\pi^2)$$

$$x \in [0, n_F]$$

Solution



$$n_{b,\mathrm{ini}} \sim n_F \gamma_W$$

$$n_b = n_F \left(1 - \frac{1}{\Gamma} \log \frac{1}{\Gamma \gamma_W} + \ldots \right) = n_F \left(1 - \frac{\lambda}{\pi g^2 n_F} \log \left[\frac{\lambda}{\pi g^2 n_F \gamma_W}\right] + \ldots \right)$$

Compare with $n_b \approx N \left(1 - \frac{\lambda}{\pi g^2} \log N\right)$

VG, arxiv:0812.4474

Conclusions II

$$H=\sum_{\mathbf{p},\sigma=\uparrow,\downarrow}\frac{p^2}{2m}\,\hat{a}^\dagger_{\mathbf{p}\sigma}\hat{a}_{\mathbf{p}\sigma}+\sum_{p}\left(\frac{q^2}{4m}-2\gamma\right)\hat{b}^\dagger_{\mathbf{q}}\hat{b}_{\mathbf{q}}+\frac{g}{\sqrt{V}}\sum_{\mathbf{p},\mathbf{q}}\left(\hat{b}_{\mathbf{q}}\,\hat{a}^\dagger_{\mathbf{q}/2+\mathbf{p}\uparrow}\hat{a}^\dagger_{\mathbf{q}/2-\mathbf{p}\downarrow}+h.c.\right)$$

$$\gamma=\lambda t$$

$$n_b=n_F\left(1-\frac{\lambda}{\pi g^2 n_F}\log\left[\frac{\lambda}{\pi g^4 n_F^{\frac{2}{3}} m^2}\right]+\dots\right)$$

- It is as hard to create molecules in this system as it is in the many-body time-dependent Dicke model.

- The adiabatic limit is approached linearly in driving rate, not exponentially as in the usual LZ problem

- Although there is no QPT, the system is in the vicinity of a QPT, thus similar physics to the Dicke model

The end