# 7440 - Intro Material

## What is solid state physics?

- Large collections ($\sim 10^{23}$) of electrons and atomic nuclei that form a solid.

  Solid → { doesn't flow
  holds shape without container
  simplest cases: crystal structure

- Interested in collective properties, states of matter (e.g. metals, insulators, ....)

- More generally, many ideas / tools are directly relevant to other parts of condensed matter physics [soft matter, quantum liquids ($^3$He, $^4$He), cold atomic gases, ...]

- This class: (1) Structure
  (2) Basic electronic properties
  } and probes of these properties

- 7450: Magnetism, Superconductivity, Superfluidity, ....
  Closer to "modern" topics in strongly correlated systems

- Solid state physics (& condensed matter physics more broadly) is different from ~~many subjects~~ subjects in many other physics courses

  (1) Huge area

  (2) Not systematic (especially when first learning)

  (3) Essential role of approximations

But some unifying themes; important ones in this class:

  (i) Role of order & symmetry

  (ii) A lot can be understood by "pretending" electrons don't interact.

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"Theory of Everything" → (for a monatomic solid)

- $N$ electrons $(\vec{r}_i, \vec{p}_i, \sigma_i)$ ; $i = 1, \ldots, N$ (spin)
- $N_I$ ions $(\vec{R}_I, \vec{P}_I)$ ; $I = 1, \ldots, N_I$
  ~ charge $-e$
  ~ charge $Ze$

- $ZN_I - N = 0$ (Charge neutrality)

$$H = \sum_{i=1}^{N} \frac{\vec{p}_i^{\,2}}{2m} + \sum_{I=1}^{N_I} \frac{\vec{P}_I^{\,2}}{2M} + \underbrace{H_{el\text{-}el} + H_{ion\text{-}ion} + H_{el\text{-}ion}}_{\text{Coulomb interactions.}}$$

$$H_{el\text{-}el} = e^2 \sum_{i<j} \frac{1}{|\vec{r}_i - \vec{r}_j|} \text{ , etc.}$$

Typical parameters (c.f. alkali metals: Li, Na, K, ...)

- $n = \frac{N}{V} \sim 10^{22} - 10^{23}\ cm^{-3}$
- $\underbrace{M \sim 10^4 m \text{ (or larger)}}$

One small parameter! $\Rightarrow$ electrons move faster than ions

~~[redacted]~~

$\Rightarrow$ 1st approximation: electrons see ions as a static background.

- Can we say anything else!?!? ~~[redacted]~~

(Looks – and is – extremely complicated.)

- Estimate importance of interactions (electrons first)

  - Kinetic energy $E_K \sim \frac{\hbar^2}{m r_s^2}$ (recall $E_k = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$)

    - $r_s = \left(\frac{3}{4\pi n}\right)^{1/3}$ = characteristic spacing between electrons

      $\left(\frac{4\pi}{3} r_s^3 = \frac{1}{n} = \frac{\text{Volume}}{\text{electron}}\right)$

    - Above estimate is just dimensional analysis

      (Alternatively, each electron is "confined" in sphere of radius $r_s$, so by uncertainty principle $p \sim \frac{\hbar}{r_s}$.)

    - Typical $r_s = 2\text{-}3$ Å. (alkali metals)

  - Potential energy $E_p$

    Dimensional analysis $\Rightarrow E_p \sim \frac{e^2}{r_s}$

    (Electrons interact mostly with their neighbors, since long-ranged Coulomb interaction is screened by background positive charge of the ions.)

Ratio: $\frac{E_P}{E_K} \sim \left(\frac{r_s}{a_0}\right)$; $a_0 = \frac{\hbar^2}{me^2} \approx 0.5$ Å

↳ Bohr radius

- $\left(\frac{r_s}{a_0}\right) \ll 1$ – weak interactions
- Typical metals, $\left(\frac{r_s}{a_0}\right) \sim 1 - 10 \Rightarrow$ neither very weak nor very strong interaction

$\Rightarrow$ difficult problem!

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Redo for ion system:

$$E_P^I / E_K^I \sim \frac{r_s}{a_0^I} \quad ; \quad a_0^I = \frac{\hbar^2}{Me^2} \sim 10^{-4} a_0$$

$$\Rightarrow \left(\frac{r_s}{a_0^I}\right) \sim 10^4 \left(r_s/a_0\right) >> 1$$

$\Rightarrow$ Strong interactions for ions

$\longrightarrow$ To minimize potential energy, ions tend to form a crystal!

"More is different"

• Important Philosophical Point: Even though we can write down essentially exact Hamiltonian, this does not mean we understand the system, even in principle.

~ Exact model is intractable (analytically & numerically)

⇒ Approximation is essential

⇒ Understanding solids (and other condensed matter systems) is a fundamental problem requiring new concepts/tools, not just a complicated application of Schrödinger's equation.

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Outline of the course:

(1) Crystal structure & other related properties

(e.g. lattice vibrations, elasticity, defects, ...)

(2) Add the electrons back in:

- band theory
- physics of metals, semiconductors, insulators