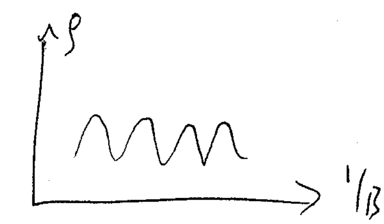
"Quantum Oscillations"

- In large B-fields, clean metallic samples exhibit a variety of oscillatory behaviors, where the oscillations are periodic in $1/B$.

- Examples:
  - ~~$M(B)$~~ $M(H)$ magnetization v.s. applied field H (de Haas – van Alphen effect)
  - Resistivity $\rho(B)$ oscillates: (Schubnikov – de Haas effect)



(crude plot)

- Oscillations arise from oscillatory, singular behavior in the density of states $D(\varepsilon_F, B)$ at the Fermi energy → So <u>any</u> quantity depending on $D(\varepsilon_F)$ exhibits oscillations.

- Remarkably, these phenomena allow <u>direct</u> measurements of Fermi surface geometry.

- We can't understand these effects within semiclassical model → won't get oscillations. So these are quantum effects of B-field.

- First understand for free electron gas in B-field.

Energy spectrum → Landau levels $(\vec{B} \parallel \hat{z})$

$$\varepsilon = \varepsilon_\nu(k_z) = \hbar\omega_c \nu + \frac{\hbar^2 k_z^2}{2m} \quad ; \quad \nu = 0, 1, 2, \ldots$$

$$\hookrightarrow \omega_c = \frac{eB}{mc}$$

- For every ν, k_z, there is very large degeneracy:

$$\#\text{ states at } \nu, k_z = \frac{BL^2}{hc/e} = \#\text{ magnetic flux quanta passing thru area } L^2.$$

- We ignore Zeeman splitting ... will return to its effect later.

- First, find density of states.

- For a 1d system with $\varepsilon(k) = \frac{\hbar^2 k^2}{2m}$,

$$D(\varepsilon) = \frac{1}{\pi\hbar}\sqrt{\frac{m}{2\varepsilon}}\,\Theta(\varepsilon) \quad (\text{D.o.s. per unit length}).$$

- So, in our system, # states per unit energy from νth Landau level is:

$$\underbrace{L^3 D_\nu(\varepsilon)}_{\#\ \text{states/energy}} = \underbrace{\left[\frac{L}{\pi\hbar}\sqrt{\frac{m}{2(\varepsilon-\hbar\omega_c\nu)}}\,\Theta(\varepsilon-\hbar\omega_c\nu)\right]}_{\text{1d}\ \#\ \text{states/energy}} \underbrace{\frac{BL^2}{(hc/e)}}_{\text{extra degeneracy}}$$

$$\Rightarrow \underbrace{D_\nu(\varepsilon)}_{\#\text{states/energy}\cdot\text{volume}} = \frac{eB}{\pi^2\hbar^2 c}\sqrt{\frac{m}{2(\varepsilon-\hbar\omega_c\nu)}}\,\Theta(\varepsilon-\hbar\omega_c\nu)$$

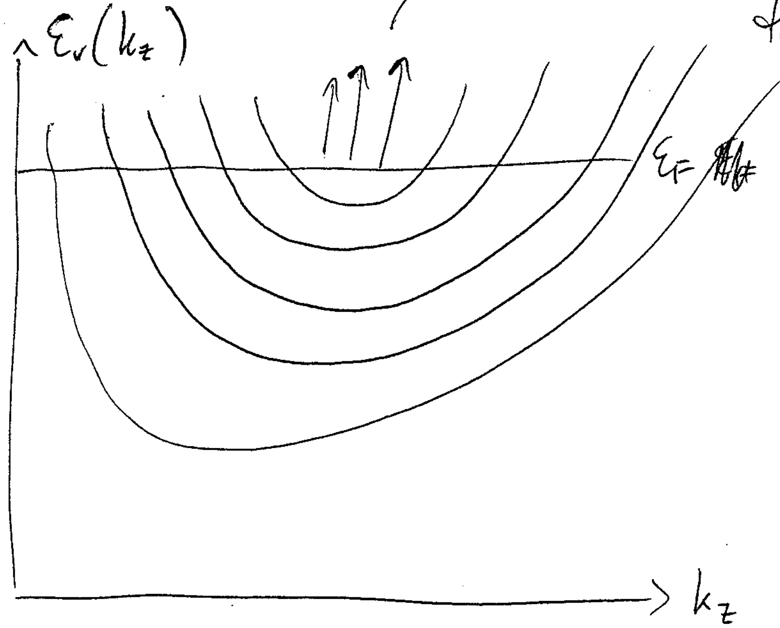
Total D.O.S.: $D(\varepsilon) = \sum_\nu D_\nu(\varepsilon)$

- *Implies*: $D(\varepsilon_F)$ has a $\frac{1}{\sqrt{\varepsilon}}$ singularity

whenever $\varepsilon_F \sim \hbar\omega_c \nu$

As B changes, bottoms of the parabolae move through ε_F.

Picture:



- Now, ε_F should depend only very weakly on B-field, since $\hbar\omega_c \ll \varepsilon_F \Rightarrow$ Ignore B-dependence of ε_F.

- Suppose for some B, $\varepsilon_F = \frac{\hbar e B}{mc}\nu$.

  Find B' so that $\varepsilon_F = \frac{\hbar e B'}{mc}(\nu+1)$ ... this is the "next" value of B when a singularity occurs.

Some algebra shows:

$$\frac{1}{B'} - \frac{1}{B} = \frac{1}{\nu B} = \frac{e\hbar}{mc\,\varepsilon_F}$$

$$\Delta\left(\frac{1}{B}\right) = \frac{e\hbar}{mc\,\varepsilon_F} \quad \text{periodicity !}$$

This can be written:

$$\Delta\left(\frac{1}{B}\right) = \frac{2\pi e}{\hbar c}\,\frac{1}{\pi k_F^2} = \frac{2\pi e}{\hbar c}\,\frac{1}{A_e}$$

- A_e is the area of the Fermi surface cross section with extremal (in this case maximum) area.

- This result actually generalizes to band electrons ...

extremal X-section with area πk_F^2.

A

Quantum Osc. Band Electrons

- Semiclassics $\rightarrow$ Re-quantize (approximately)
- Closed orbit, periodic motion with frequency $\nu = \frac{1}{T}$

  $\sim$ harmonic oscillator

$$\Rightarrow \varepsilon_{\nu+1}(k_z) - \varepsilon_\nu(k_z) = h\nu = \frac{h}{T(\varepsilon_\nu(k_z), k_z)}$$

Recall $T(\varepsilon, k_z) = \frac{\hbar^2 c}{eB} \frac{\partial}{\partial \varepsilon} A(\varepsilon, k_z)$

$$\left.\frac{\partial}{\partial \varepsilon} A(\varepsilon, k_z)\right|_{\varepsilon = \varepsilon_\nu(k_z)} \simeq \frac{A(\varepsilon_{\nu+1}(k_z), k_z) - A(\varepsilon_\nu(k_z), k_z)}{\varepsilon_{\nu+1}(k_z) - \varepsilon_\nu(k_z)}$$

(Since $\Delta\varepsilon \ll \varepsilon_F$)

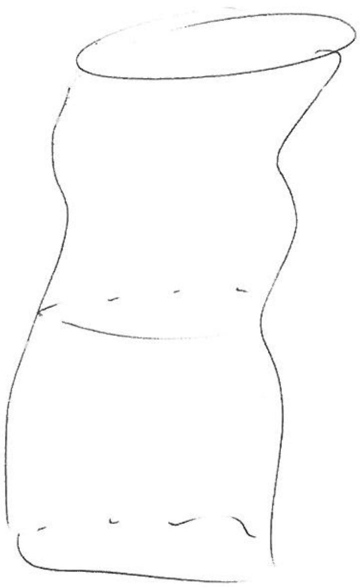
$$\Rightarrow \boxed{\Delta A = A(\varepsilon_{\nu+1}(k_z), k_z) - A(\varepsilon_\nu(k_z), k_z) = \frac{2\pi e B}{\hbar c}}$$

B

$$\Rightarrow \quad A(\varepsilon_\nu(k_z), k_z) = (\nu + \lambda)\, \Delta A$$

assume indep. of k_z, B.

Orbits with fixed ν, as fn. of k_z form a "tube"

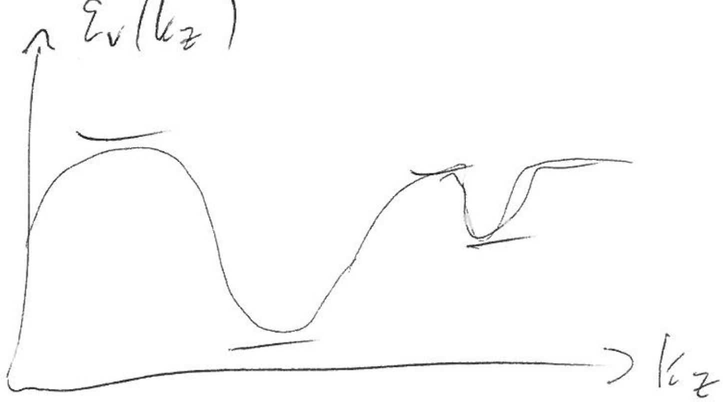


ε const

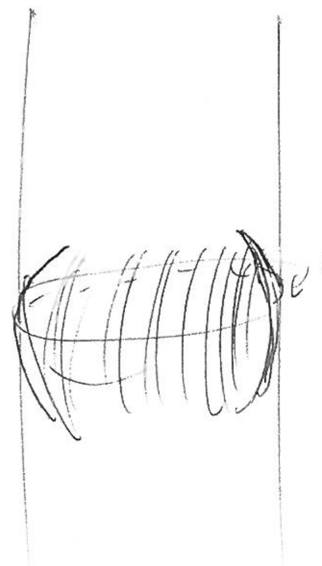
$\updownarrow \; \varepsilon = \varepsilon(k_z)$

($d=1$ "band")

Suppose



(c)



$$\frac{\partial^2 \varepsilon_\nu(k_z)}{\partial k_z^2} > 0$$

Happens for some B, ν

$$(\nu+\lambda)\frac{2\pi e B}{\hbar c} = \overset{\nu+1}{\overset{\parallel}{(\nu'+\lambda)}}\frac{2\pi e B'}{\hbar c}$$

$$\frac{\nu+\lambda}{B'} = \frac{\nu+1+\lambda}{B}$$

$$(\nu+\lambda)\frac{2\pi e}{\hbar c}B = A_e(\varepsilon_F)$$

$$(\nu+1+\lambda)\frac{2\pi e}{\hbar c}B' = A_e(\varepsilon_F)$$

$$\frac{1}{B} = \frac{(\nu+\lambda)\frac{2\pi e}{\hbar c}}{A_e(\varepsilon_F)} \qquad \frac{1}{B'} = \frac{(\nu+1+\lambda)\frac{2\pi e}{\hbar c}}{A_e(\varepsilon_F)}$$

$$\Rightarrow \Delta\left(\frac{1}{B}\right) = \frac{2\pi e/\hbar c}{A_e(\varepsilon_F)}$$