IQHE (& FQHE)

- In variety of semiconductor systems, can make 2DEG

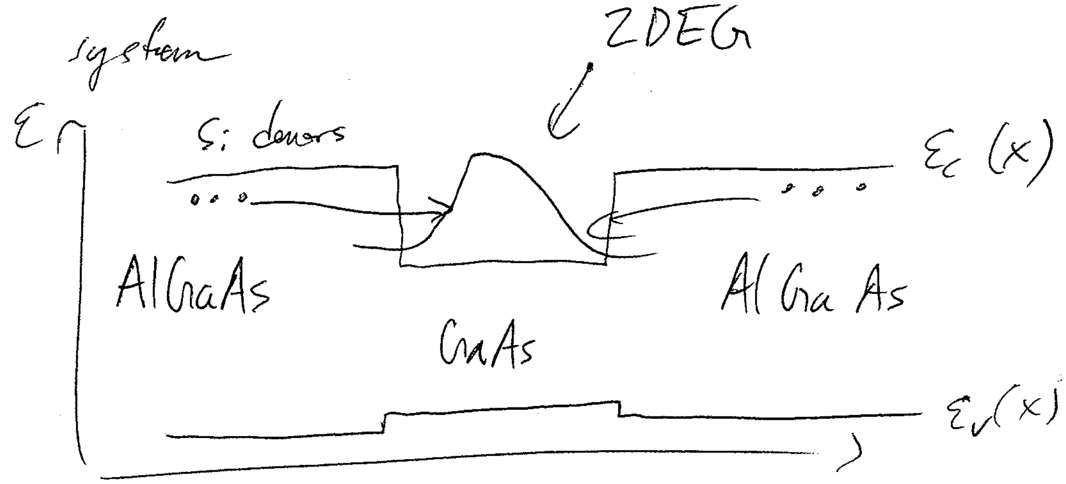
- High-B, low-T, clean sample, ~~σ_{xy}~~ $\Big/ \; \overset{\text{intact}}{\sigma} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \frac{ne^2}{h} \Rightarrow \rho = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \frac{h}{ne^2}$

$\cancel{\sigma_{xy}}\ \sigma_{yx} = \frac{ne^2}{h}$; $n = 1, 2, \ldots$ → accurate to ~ part in 10^{10} (limited by ability to measure ~~[illegible] [illegible]~~ resistance accurately)

$\boxed{\text{Show data}}$

- Theory: Quantization is exact (as $T \to 0$), even for real systems, with all inherent complications.

Example system



- $d=2$ is crucial. One reason:

- $R = \rho L^{(2-d)} \rightarrow d=2$, R and ρ ~~have~~ have same units.

  - Means there's hope that don't need to know details of sample geometry to determine ρ from R, etc.

- IQHE can be understood in terms of non-int. e's in B-field. But impurities are actually important.

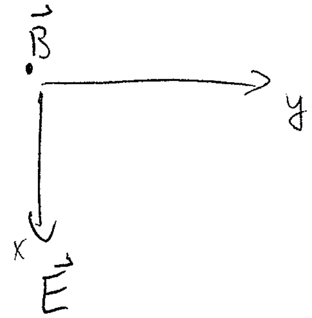
Consider clean system w/o impurities.

- ~~Define~~ e^- density n.

- In lab frame, $\vec{v} = 0 \Rightarrow \vec{J} = 0$; $\vec{E} = 0$, $\vec{B} = B\hat{z}$

- Transform to moving frame (lowest order in $\frac{\vec{v}}{c}$):

  ~~n~~ $\vec{J} = -ne\vec{v}$, $\vec{B} = B\hat{z}$, $\vec{E} = -\frac{1}{c}\vec{v}\times\vec{B}$



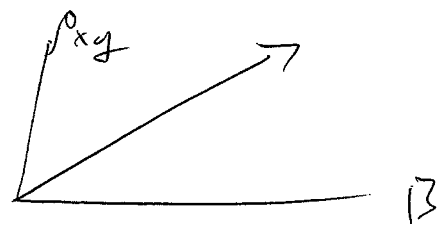
$\vec{J} \sim -\vec{v}$

• In this frame E & B forces cancel so get straight-line current flow.

$$|\vec{E}| = \frac{vB}{c}, \quad |\vec{J}| = nev$$

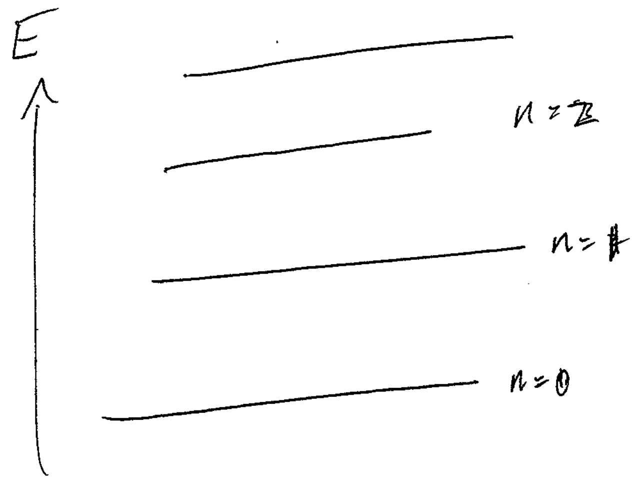
$$\Rightarrow \cancel{\rho_{xy}}\ \sigma_{yx} = \frac{nec}{B}$$ ~~and ρ_{xy} =~~

If assume $\sigma_{xx} = 0$, then $\rho_{xy} = \frac{B}{nec}$



• This <u>only</u> depended on trans. invc. $\rightarrow$ need something to break trans. invc.

Spectrum (Landau levels)



- $E = \hbar\omega_c\left(n + \frac{1}{2}\right)$; $n = 0, 1, \dots$

- Degeneracy: $N_{deg} = N_{\Phi} = \dfrac{B \cdot L^2}{(hc/e)}$

Suppose μ sits just above $n = n_{max}$, then

$$N_{el} = N_{\Phi}(n_{max}+1) = \frac{BL^2}{(hc/e)}(n_{max}+1)$$

$$\Rightarrow \quad n = \frac{N_{el}}{L^2} = \frac{B}{(hc/e)}(n_{max}+1)$$

$$\cancel{R_{xx} = \frac{1}{nec}} \quad \cancel{\rho_{xy} = \frac{B}{nec}} \quad \sigma_{xy} = -\frac{nec}{B} = \frac{e^2}{h}(n_{max}+1)$$

- But, if ~~we~~ slightly change B, μ jumps drastically

  → Need fine-tuning of B.

- Resolution: The Localized midgap states.

A

$$\vec{B} = (\partial_x A_y)\hat{z} = B\hat{z}$$

$$\vec{A} = Bx\hat{y}$$

$$\vec{p} = -i\hbar\vec{\nabla}$$

$$H = \frac{1}{2m}p_x^2 + \frac{1}{2m}\left(p_y + \frac{eB}{c}x\right)^2$$

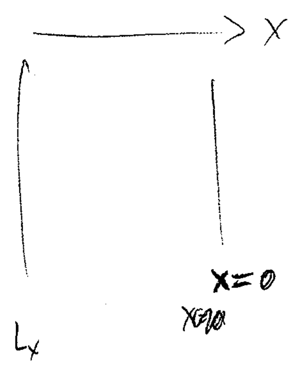
$$\psi(x,y) = e^{iky} f_k(x)$$

$$\Rightarrow \quad H(k) = \frac{1}{2m}p_x^2 + \frac{1}{2m}\left(\hbar k + \frac{eB}{c}x\right)^2$$

$$= \frac{1}{2m}p_x^2 + \frac{1}{2}m\omega_c^2\left(x + k\ell^2\right)^2$$

where $\omega_c = \frac{eB}{mc}$; $\ell^2 = \frac{\hbar c}{eB}$

$$X_k = -k\ell^2 \qquad k = \frac{2\pi n}{L_y}$$



$$L_x = k\ell^2$$

$$k = 0, \ldots, \frac{L_x}{\ell^2}$$

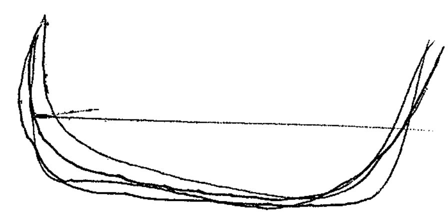
$\Rightarrow$ n_{max} is s.t. $\frac{2\pi n_{max}}{L_y} = \frac{L_x}{\ell^2}$ $\Rightarrow n_{max} = \frac{L_x L_y}{2\pi\ell^2} = N_\Phi$

LLL wavefns:

$$\psi_k(\vec{r}) = \frac{1}{\sqrt{\pi^{1/2} L \ell}}\, e^{iky}\, e^{-\frac{1}{2\ell^2}(x+k\ell^2)^2}$$

$$\frac{1}{\pi^{1/2}\ell}\int dx\; e^{-\frac{1}{\ell^2}(x+k\ell)^2} = 1 \checkmark$$

Suppose add potential $V(x)$

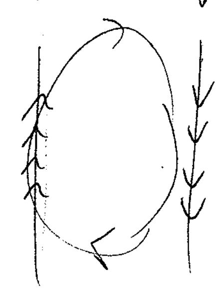


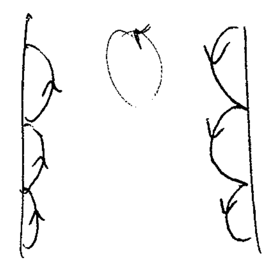
$$H(k) = \frac{1}{2m}p_x^2 + \frac{1}{2}m\omega_c^2 (x+k\ell)^2 + V(x)$$

→ slowly varying

$$E_k \approx (n+\tfrac{1}{2})\hbar\omega_c + V(-k\ell^2)$$

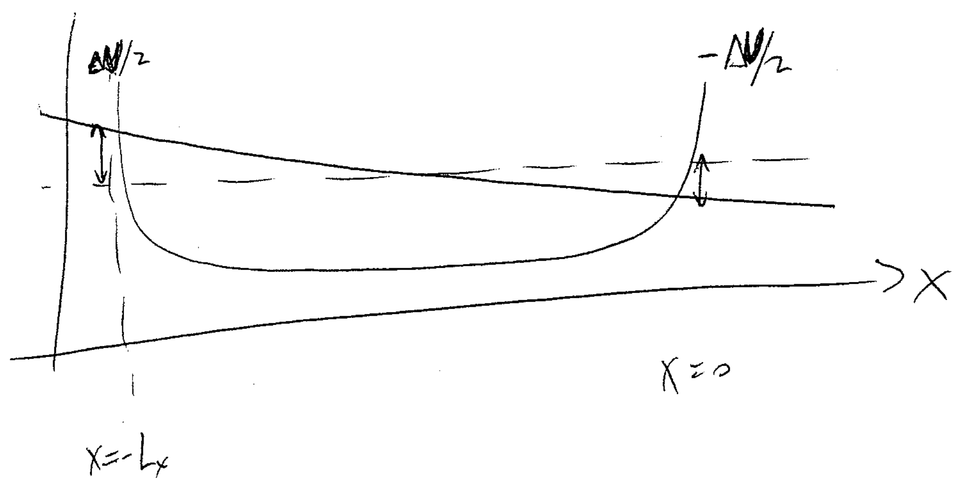
$$\vec{v} = \frac{1}{\hbar}\frac{\partial E_k}{\partial k}\hat{y} = -\frac{\ell^2}{\hbar} V'(-k\ell^2)\,\hat{y}$$





(C)

Suppose apply ~~voltage~~ potential across X-direction.



$\frac{2\pi}{L_y}\ell^2$

Corresponding voltage $e\Delta V = \Delta U \equiv e(V(x=0) - V(x=-L_x))$

- Number of extra occupied states on left:

$$\left[\left|\frac{dU}{dx}\right| \frac{1}{\Delta U/2}\right]^{-1} \cdot \frac{L_y}{2\pi \ell^2}$$

$$= \frac{1}{2\pi\hbar|\vec{v}|} \frac{\Delta U}{2} \cdot L_y$$

Current is: $\left(\frac{1}{2\pi\hbar|\vec{v}|}\Delta U \cdot L_y\right)\frac{e|\vec{v}|}{L_y} =$