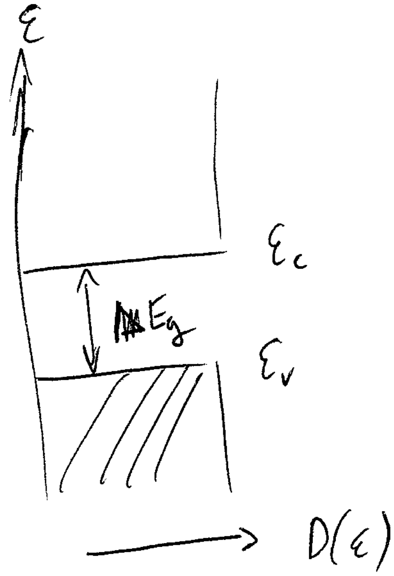
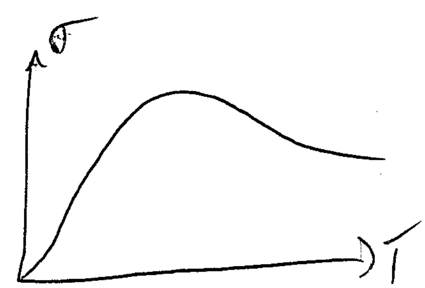
Semiconductors



- Just insulators with (relatively) small band gaps $E_g \lesssim 1\ eV$

- Why distinguish semiconductors & insulators?

  (1) Noticeable conduction @ finite-T



$$\sigma(T) \sim e^{-E_g/2k_BT}$$

@ low T

$$\sigma = \frac{n e^2 \tau}{m}$$

low-T increase comes from this.

(2) Semiconductors are technologically very useful

Why?

(1) Easily manipulated by "doping" w/ impurities. (Small dielectric const. ~large imp. for this)

(2) " " by applied $\vec{E}$-fields.

Examples: Si, Ge

Band structure: Usually describe using effective mass approx.

$$\varepsilon(\vec{k}) = \varepsilon_c + \frac{\hbar^2}{2} k_i M^{-1}_{ij} k_j \quad (\text{conduc band})$$

$$\varepsilon(\vec{k}) = \varepsilon_v - \frac{\hbar^2}{2} k_i M^{-1}_{ij} k_j \quad (\text{valence} \;\text{"})$$

- Can diagonalize M^{-1}, M

→ Measure M^{-1} by cyclotron resonance

(recall $T = \dfrac{2\pi m^* c}{eB}$, $m^* = \left(\dfrac{\det M}{M_{zz}}\right)^{1/2}$)

- Apply microwave field in resonance w/ periodic motion of orbits.

of carriers (clean case).

$$n_c(T) = p_v(T)$$

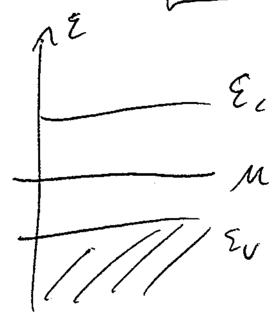
n_c: # el's in cond. band. p_v: # holes in val. band

$$n_c(T) = \int_{\varepsilon_c}^{\infty} d\varepsilon \, g_c(\varepsilon) \frac{1}{e^{(\varepsilon-\mu)/k_BT}+1}$$

$$p_v(T) = \int_{-\infty}^{\varepsilon_v} d\varepsilon \, g_v(\varepsilon) \left(1 - \frac{1}{e^{(\varepsilon-\mu)/k_BT}+1}\right)$$

$$= \int_{-\infty}^{\varepsilon_v} d\varepsilon \, g_v(\varepsilon) \frac{1}{e^{(\mu-\varepsilon)/k_BT}+1}$$

Assume: $k_BT \ll E_g$ and $\underline{\mu \text{ lies near middle of gap}}$:



• Will check consistent.

$$\Rightarrow \quad \varepsilon_c - \mu \gg k_BT$$
$$\mu - \varepsilon_v \gg k_BT$$

$$\Rightarrow n_c(T) \approx \left[\int_{\varepsilon_c}^{\infty} d\varepsilon \, g_c(\varepsilon) \, e^{-(\varepsilon-\varepsilon_c)/k_BT}\right] e^{-(\varepsilon_c-\mu)/k_BT} \equiv N_c(T) e^{-(\varepsilon_c-\mu)/k_BT}$$

$$p_v(T) \approx \left[\int_{-\infty}^{\varepsilon_v} d\varepsilon \, g_v(\varepsilon) \, e^{-(\varepsilon_v-\varepsilon)/k_BT}\right] e^{-(\mu-\varepsilon_v)/k_BT} = P_v(T) e^{-(\mu-\varepsilon_v)/k_BT}$$

Using result from last HW:

$$g_{c,v}(\varepsilon) = \sqrt{2|\varepsilon-\varepsilon_{c,v}|}\,\frac{m_{c,v}^{*\,3/2}}{\hbar^3\pi^2} \quad ; \quad m^*_{c,v} = \left(\det M_{c,v}\right)^{1/3}$$

$$\Rightarrow \quad N_c(T) = \frac{1}{4}\left(\frac{2 m_c^* k_B T}{\pi \hbar^2}\right)^{3/2}$$

sim. for $P_v(T)$

Note: $n_c p_v = N_c P_v \, e^{-E_g/k_B T} \longrightarrow$ No μ-dependence

• So far haven't used $n_c = p_v$, which gives

$$(m_c^*)^{3/2} e^{-(\varepsilon_c-\mu)/k_B T} = (m_v^*)^{3/2} e^{-(\mu-\varepsilon_v)/k_B T}$$

$$\Rightarrow \quad \mu = \underbrace{\frac{\varepsilon_c+\varepsilon_v}{2}}_{\text{middle of gap}} + \underbrace{\frac{3}{4} k_B T \ln\left(\frac{m_v^*}{m_c^*}\right)}_{\text{small}}$$

"Extrinsic" Case (w/ impurities)

- ~~Will see that now $n_c \neq p_v$, $n_c - p_v = \Delta n$~~

- Main effect is to introduce extra carriers.

e.g. Si doped with P

$Si = [Ne]\, 3s^2 3p^2$

$P = [Ne]\, 3s^2 3p^3 \rightarrow$ 1 extra electron & 1 extra + charge

$\Rightarrow$ Single P impurity introduces localized + charge @ nucleus & extra − charged electron.

$\rightarrow$ Electron binds to P impurity from potential

$$V(r) = -\frac{e^2}{\varepsilon r}$$

ε ← dielectric const.

Also, kinetic energy $-\frac{\hbar^2}{2m}\vec{\nabla}^2 \rightarrow -\frac{\hbar^2}{2m^*}\vec{\nabla}^2$

(simplest case)

$\Rightarrow$ Binding energy $\quad \mathcal{E}_d = \mathcal{E}_c - \dfrac{m^* e^4}{\varepsilon^2 \, 2\hbar^2} = \mathcal{E}_c - \dfrac{m^*}{m}\dfrac{1}{\varepsilon^2} \times 13.6\,\text{eV}$

$\uparrow$

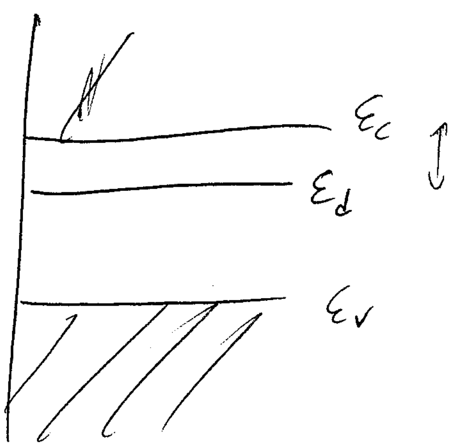
$d \rightsquigarrow$ "donor impurity"

Size of bound state: $r_0 = \dfrac{\hbar^2 \varepsilon}{m^* e^2} = \dfrac{m}{m^*}\varepsilon a_0$

- Typically, $m^* < m$ and $\varepsilon \gtrsim 10$

$\Rightarrow$ r_0 is large compared to lattice spacing

& $\mathcal{E}_c - \mathcal{E}_d$ <u>small compared</u> to E_g.



$\updownarrow$ Since small, very easy to thermally excite bound electrons into conduction band

$\longrightarrow$ n-type semiconductor.

Acceptors e.g. Al in Si

$$Al \rightarrow \ominus \quad \oplus$$

$\oplus$ ← remove 1 electron

$\ominus$ ← fixed - charge

Think of in 2 steps:

(1) Add fixed - charge

(2) Remove electron.

(1) $$H = + \frac{\hbar^2}{2m^*}\vec{\nabla}^2 + \frac{e^2}{\varepsilon r} \longrightarrow$$

↓

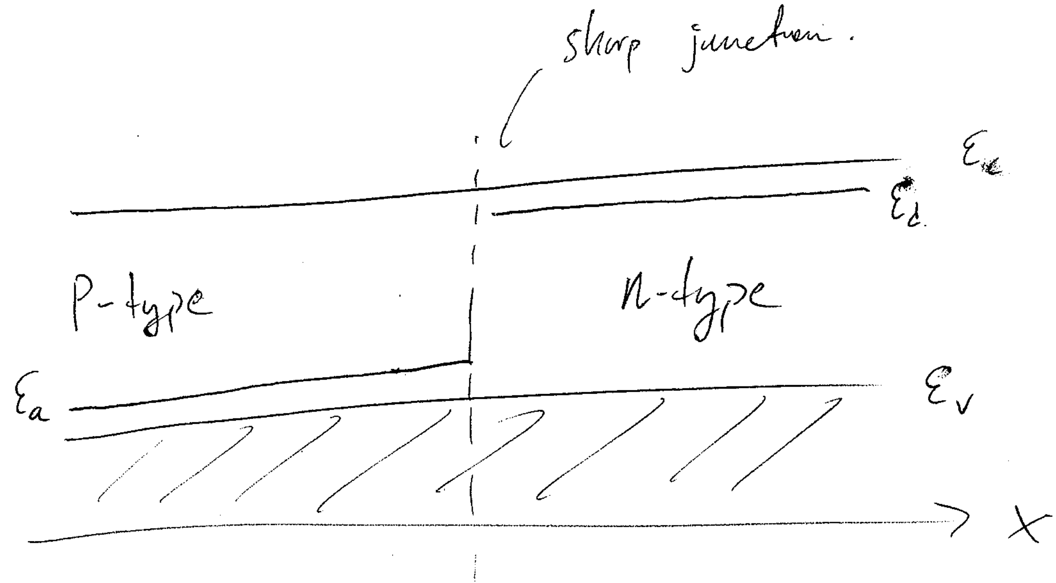
Assume only top of valence band states important

(2) ~~For overall charge neutrality:~~

Argument:
All these states come from valence band → so same # of states as in V.B. So, before step 2, the bnd state is <u>filled</u>.

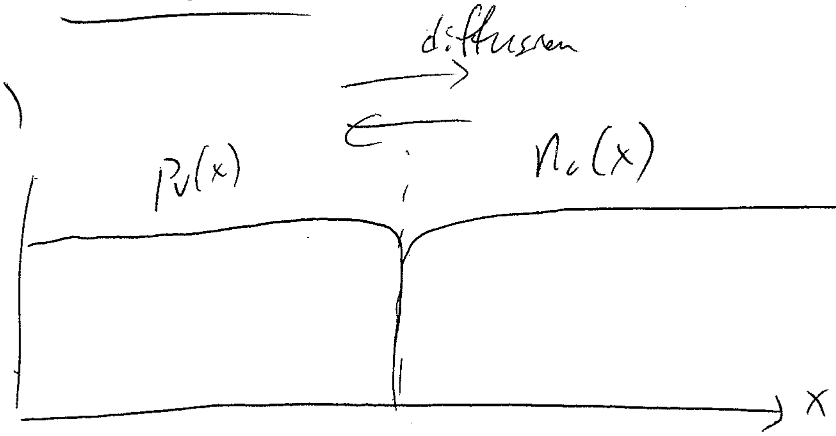
P-N junction (diode)

· First equilibrium.

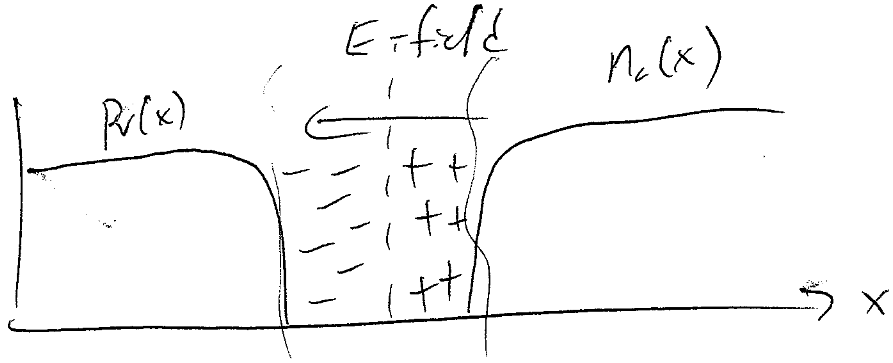


· Carrier densities :

(1)

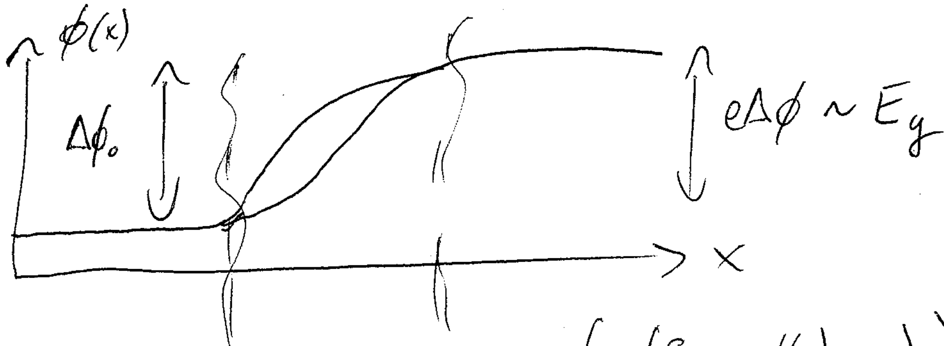


(2)



· E field opposes diffusion → reaches equilibrium.

Let $\phi(x)$ be electrostatic potential



Model: $$n_c(x) = N_c(T)\exp\left(\frac{-(\mathcal{E}_c - e\phi(x) - \mu)}{k_B T}\right)$$

(slowly varying potential)

Now, $$p_v(x) = p_v(-\infty)\exp\left[\frac{-(e\phi(x) - e\phi(-\infty))}{k_B T}\right]$$

$\Rightarrow$ $p_v(x) \approx 0$ in depletion layer.

Apply voltage

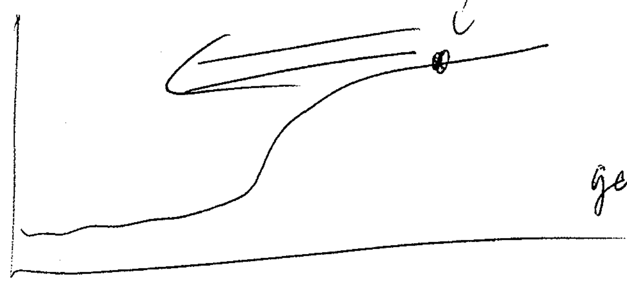
$V > 0$ means higher + $\phi(x)$ on p-type side

$$\boxed{\Delta\phi = \Delta\phi_0 - V}$$

→ $V > 0$ tends to shrink depletion layer
$V < 0$ enlarges it.

Consider hole current → 2 kinds:

(1) J_h^{gen}

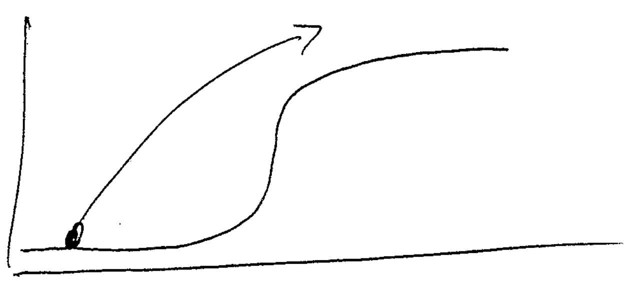


thermally excited hole

gets sucked over to p-side

- Not sensitive to V, since any hole ~~off~~ on the n-side gets sucked over

(2) J_h^{rec}



- Hole jumps over ~~b~~barrier from $p \to n$.
- Sensitive to V

- Expect $J_n^{rec} \approx J_{n0}^{rec} \, e^{eV/k_BT}$

$J_n = J_n^{rec} - J_n^{gen}$ and $J_n^{gen} \approx$ const. (no V-dep.)

$J_n(V=0) = 0.$

$\Rightarrow J_n = J_n^{gen}\left(e^{eV/k_BT} - 1\right)$

- Same analysis works for electrons.